Chapter 8

Conclusions and future work

In this final chapter, we summarize all the problems which we have studied in this thesis and highlight the principal results. The significance of the results towards furthering our understanding of the basic dynamics and control of classical and neoclassical tearing modes is also discussed. An attempt is made to identify some future problems that can be addressed with further refinements of our tools and model equations.

8.1 Summary and Conclusions

In this thesis we have investigated some aspects of the physics of neoclassical tearing modes with a view to improving our understanding of their nonlinear dynamics and finding better ways of controlling them.

A large part of the thesis has been devoted to exploring the interaction of sheared equilibrium flows with NTMs - an area of tokamak physics that has not received much attention in the past. For the next generation of advanced long pulse tokamak experiments, such as EAST (China), SST-1 (India), KST (Korea) and eventually ITER, this can become an important issue due to their high $J$ and the possibility of plasma rotation due to internal dynamics or external excitation. We have studied this problem systematically by carrying out both detailed numerical simulations and some appropriate analytical modeling.

Our main results are contained in Chapters 2,3,5,6 and 7.
In chapter 2 we have considered the effect of pressure gradients within the island which can arise due to local heating of the island region by ECRH or any other means. This contribution generally gets cancelled for symmetric island shapes. However as experimental observations show there is always some degree of asymmetry arising from a host of physical effects. We show in our model calculation that when the next order magnetic shear terms in the equilibrium magnetic field are retained then this term survives and generates a self-consistent bootstrap current within the island which is comparable to the usual current perturbation calculated from the resistivity change mechanism. Their combined contribution in the island evolution equation substantially reduces the saturation width of the island. Based on this fact we have suggested the use of modulated neutral beams as an alternative scheme for neoclassical tearing mode control. In this case pressure gradients within the island can be changed by controlled delivery of both density and energy at appropriate phases and amplitudes. Our preliminary estimates show that such a scheme is feasible and of comparable efficacy to the ECRH scheme in terms of power requirements and other parameters.

Chapter 3 contains our first analytical results concerning the influence of flows on NTMs. In this we have investigated the possible nonlinear saturated states of neoclassical tearing modes in the presence of equilibrium sheared flows. A center manifold reduction method is used to reduce the MHD equations to a set of amplitude equations when the system parameters are close to their marginal values. The possible time asymptotic states of the NTMs in presence of flows have been found out by carrying out a bifurcation analysis of these amplitude equations. Our results show that interesting time asymptotic nonlinear states like single saturated magnetic islands, frequency locked states and oscillating magnetic island states can exist for different parametric regimes.

In subsequent chapters we have looked at the problem of shear flow - NTM interaction in a more
detailed manner by carrying out time evolution studies. The basic model equations adopted for this purpose have been the generalized reduced MHD equations that are valid for arbitrary aspect ratios and depend for their validity on the smallness of the $k_y/k_z$ ratio. Chapter 4 gives a detailed description of this model as well as that of the numerical code NEAR which has been used to solve the equations.

In chapter 5 we have presented the numerical results that have been obtained using the NEAR code. Our time evolution simulation studies of the tearing modes have been done using different flow profiles. The results indicate that differential flow has a strong stabilizing influence on the nonlinear evolution of both classical and neoclassical tearing modes whereas negative velocity shear has a destabilizing effect.

The source of the stabilizing influence of differential flow has been primarily traced to two physical factors - the pressure-curvature term and the toroidal coupling. The pressure curvature term has a stabilizing influence (when the overall curvature is favorable) even in the absence of flow. We have confirmed this by artificially turning this term off in the code. We have further numerically confirmed that the stabilizing effect of this term is enhanced by the presence of differential flow due to equilibrium modifications of the pressure profile caused by the centrifugal effects of flow. However the measured enhancement is not sufficient to fully account for the amount of flow induced stabilization. The additional stabilization comes about from the influence of differential flow on toroidal mode coupling. A semi-quantitative measure of this effect has been obtained by a simple model application of the $\Delta^\prime$ matrix theory and the results are seen to agree with the numerical findings quite well in the linear regime. Flow induced changes observed in the linear regime are found to continue in the nonlinear regime where differential flows lead to lower island saturation levels whereas usual negative flow shear appears to oppose this stabilizing trend. The positive flow shear on the other hand further stabilizes the modes. So there is a strong dependence of tearing modes' stability on the sign and magnitude of flow shear. This dependence must in some way be related to the resistive inner layer and/or the ideal outer layer dynamics of the tearing modes. In the next two chapters we have explored this connection by examining the effect of flow on the inner and
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outer layer dynamics.

In chapter 6, we have investigated analytically the effect of the flow shear on neoclassical tearing modes within the framework of the generalized Rutherford equations, essentially by including appropriately the inertial contribution in the quasi-linear calculation. Both parallel and perpendicular drift components of the flow have been retained and they are seen to give rise to two new contributions which have a similar dependence on the island width \( W \) as the NTM driving term. One of the terms is always destabilizing in nature irrespective of the sign of the flow gradient. However, the parallel flow term contribution depends on the sign of the flow gradient. For a typical tokamak flow profile this term is destabilizing; however it raises the interesting possibility that with an appropriately modified flow profile it might be possible to stabilize the NTM. The stabilizing effect of a positive flow shear, as represented by this term, is in agreement with our numerical simulation results of chapter 5.

The Rutherford model calculation done in chapter 6 assumes the matching stability parameter \( \Delta' \) to be constant and to be given from an ideal outer region without flow. But as is well known, in ideal MHD an equilibrium sheared flow can significantly influence the outer layer dynamics and change the \( \Delta' \) accordingly. In chapter 7 we have addressed this issue and tried to quantitatively assess the flow induced changes in \( \Delta' \). For this we have derived a generalized Newcomb equation in a cylindrical geometry that includes flow contributions as well as finite pressure terms. This model outer layer equation provides a means for estimating \( \Delta' \) in the presence of sheared flows, particularly for large aspect ratio machines.

The numerical results suggest that the combination of the magnetic and velocity profile variations along with finite \( \beta \) effects can profoundly influence the magnitude of \( \Delta' \) and consequently the stability of the tearing mode. This global dependence of \( \Delta' \) needs to be appropriately accounted for when estimating stability thresholds or saturation widths of magnetic islands in the nonlinear Rutherford theory. Our present calculations were done with simple model profiles and in a limited parametric space to highlight.
the sensitivity of $\Delta'$ to equilibrium profile parameters. A more direct utility of our equation would be to estimate $\Delta'$ using realistic equilibrium profiles obtained from MHD equilibrium codes.

8.2 Outlook for future work

In this thesis we have found several interesting results pertaining to the dynamics of neoclassical tearing modes and in particular their interaction with equilibrium sheared flows. Our numerical and analytic calculations have shown that flows can have a significant influence on the stability of both classical and neoclassical tearing modes. This is of great relevance to some of the present day and future tokamak experiments. We already see evidence of the growing importance of flows as well as NTMs in the increasing attention they are receiving in the literature. Flows are important in other contexts as well, e.g. in the control of resistive wall modes, and so are likely to be used in many tokamak experiments. In view of this we feel that our present studies and the tools developed for analytical and numerical investigations can be expanded and built upon to carry out further research in this direction. In the following we discuss some possible refinements of our study as well as possible future directions of research.

1. As is well known, in the absence of flow there are two main kinds of resistive instabilities - those with a small value of $\Delta'$ - often called the constant $\psi$ modes that occur for mode numbers $m > 2$ and the large $\Delta'$ modes - the $m = 1$ resistive mode. In our thesis we have restricted ourselves to a study of the interaction of flows with the $m \geq 2$ modes since the most common NTM modes have $m = 2$ or $m = 3$. The $m = 1$ mode is not directly involved in NTM physics but is responsible for the phenomenon of sawtooth oscillations. There is however strong experimental evidence that sawteeth can trigger NTMs by providing the necessary sized seed island. The interaction of sheared flow with the $m = 1$ mode may therefore have some bearing on NTM evolution and is therefore worthy of investigation. We believe that the NEAR code is capable of exploring this aspect and a
systematic investigation of this problem can be undertaken.

2. We now discuss some limitations of our present work and possible future refinements that can introduce new elements of physics. In our numerical model we have assumed that the mass density is constant. Accordingly we are not evolving the continuity equation and in the process we have left out some important physics e.g. that of drift tearing modes. These modes can bring additional island rotation through $\omega^*$ effects. So one of the obvious extension of our numerical work is to evolve the continuity equation and study the evolution of drift tearing modes.

3. Another possible improvement can be effected in the area of the choice of closure for the neoclassical viscous stress tensor terms in generalized reduced MHD equations of chapter 1. Ideally one should try out different forms of heuristic closure and compare them in the context of proper representation of neoclassical physics. Such an exercise is presently planned for NIMROD but can be implemented far more easily on NEAR. Identification of the right closure is very important for correct simulation of NTMs and needs to be addressed in the future.

4. The use of various extended MHD models is an active area of research where one seeks to include some important kinetic effects within the MHD framework. The study of the interaction of NTMs with energetic particles through the intervention of fishbone activity is one such class of problems that needs future attention. This will require suitable expansion of the GRMHD model with concomitant changes in NEAR.

5. Another important physics issue is the determination of the threshold island size for the excitation of neoclassical tearing modes. This is a very complex issue and poorly understood at present.
Various analytic models have tried to quantify the threshold but there is no general consensus. One of the notions is that coupling between modes may play a very important role here. There is scope to address this issue within the framework of GRMHD and NEAR. The effect of flows on the NTMs excitation is also a very important issue which we have not examined and worth looking into.

6. While calculating $\Delta'$ in chapter 7 using the generalized Newcomb equation we have neglected toroidal effects which is a good approximation for large aspect ratio tokamaks. But for realistic aspect ratios toroidal effects are important and should be incorporated in the generalized Newcomb equation. The influence of such toroidal corrections on $\Delta'$ would then provide a more realistic input for the Rutherford model.

The above are a few simple extensions and refinements to our present work which can lead to interesting new investigations. The larger issue of a more complete understanding of the interaction of equilibrium flows in tokamaks with various MHD and resistive MHD modes is a complex and challenging problem and likely to remain an active area of research for quite some time to come.