CHAPTER 5

Inventory Systems with Disasters

5.1. Introduction

In all inventory models discussed earlier in this thesis we have not brought in the role of perishability and disasters. In several practical situations, these factors play important roles in decision making. For example, in a firm where there is a possibility of occurrence of disaster, it is to be decided about the maximum quantity that can be kept so that the inventory lost due to disasters is minimum and at the same time efficient running of the system is ensured.

Krishnamoorthy and Varghese [43] analyzed an inventory model where the items are damaged due to decay and disasters. They assumed that the lead time is zero and the service time is negligible. Arivarignan et.al [1] discussed a continuous review \((s, S)\) inventory system with perishable items, where lead time and life time of items are exponentially distributed. They obtained both steady state and transient solutions. An extensive survey on perishable inventory can be seen in Nahmias [52]. Subsequently there followed several further investigations. Nevertheless these were all on with negligible service times. Krishnamoorthy and Anbazhagan [30] discussed a system with finite capacity for waiting space where the inventory is served according to an exponentially distributed time. Further they assume perishability of items on stock.

In this chapter we consider two models of \((s, S)\) inventory systems where the commodities are destroyed by disasters. Customers arrive to a single server counter according to a Poisson process with rate \(\lambda\). Service times of customers are independent and identically distributed exponential random variables with parameter \(\mu\). Lead time follows an exponential distribution with parameter \(\beta\). The interval between disasters have exponential distribution with parameter \(\delta\). Each customer requires one unit of item. As a result of service, when the inventory level reaches \(s\) we place an order for \(Q = S - s\)
quantity of the item. If disaster occurs when the inventory level is between 0 and \( s \) there is no need to place the order again, and we place an order if the inventory level is between \( s + 1 \) and \( S \). That is only one existing order is allowed. We assume that customers register their names for the product. Since there is a chance of disasters physical presence of customers at the service station is thus avoided. In Model I we assume that customers do not join the system when the inventory level is zero: whereas in model II customers are assumed to join the system even when the inventory level is zero.

5.2. Mathematical Description of Model I

Let \( N(t) \) be the number of customers in the system and \( I(t) \) be the inventory level at time \( t \). We assume that disaster destroys all the inventoried items present at that epoch; however it is assumed that the customers are not affected by disaster. Customers do not join the system when the inventory level is zero. Those who are already present stays there. It follows that the \( \{((N(t), I(t)), t \geq 0) \) is a LIQBD process on the state space \( \{(i, j); i \geq 0, 0 \leq j \leq S\} \). The infinitesimal generator \( \overline{Q} \) of the process is a block tridiagonal matrix having the following form:

\[
\overline{Q} = \begin{bmatrix} A_{00} & A_0 & 0 & 0 & 0 & \cdots \\ A_2 & A_1 & A_0 & 0 & 0 & \cdots \\ 0 & A_2 & A_1 & A_0 & 0 & \cdots \\ 0 & 0 & A_2 & A_1 & A_0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots \end{bmatrix} \tag{5.2.1}
\]

where the blocks \( A_{00}, A_0, A_1, A_2 \) are square matrices of order \((S + 1)\); they are given by

\[
A_0 = \begin{bmatrix} 0 & 0 \\ 0 & \lambda I_S \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 \\ \mu I_S & 0 \end{bmatrix}
\]
where $\Omega = \lambda + \beta + \delta, \omega = \lambda + \delta$.

\[
A_{00} = \begin{pmatrix}
0 & 1 & \cdots & s & s+1 & \cdots & S-s & \cdots & S \\
-\beta & \beta \\
\delta & -\Omega & \beta \\
\vdots & \vdots & \ddots & \ddots \\
s & -\Omega & \cdots & \beta \\
s+1 & \delta & -\omega \\
\vdots & \vdots & \ddots & \ddots \\
S & \delta & -\omega \\
\end{pmatrix}
\]

with $\Omega' = \lambda + \beta + \delta + \mu, \omega' = \lambda + \delta + \mu$.

\[
A_1 = \begin{pmatrix}
0 & 1 & \cdots & s & s+1 & \cdots & S-s & \cdots & S \\
-\beta & \beta \\
\delta & -\Omega' & \beta \\
\vdots & \vdots & \ddots & \ddots \\
s & -\Omega' & \cdots & \beta \\
s+1 & \delta & -\omega' \\
\vdots & \vdots & \ddots & \ddots \\
S & \delta & -\omega' \\
\end{pmatrix}
\]

\section*{5.3. Analysis of Model I}

\subsection*{5.3.1. System stability.} Define the generator matrix $A$ as $A = A_0 + A_1 + A_2$. Let $\pi = (\pi_0, \pi_1, \cdots, \pi_S)$ be the steady state probability vector of $A$. Then we have $\pi A = 0$ and $\pi e = 1$. Solving $\pi A = 0$ we get

\[
\pi_0 = \frac{\mu}{\beta + \delta} \pi_1 + \frac{\delta}{\beta + \delta}
\]

\[
\pi_k = \left(\frac{\beta + \delta + \mu}{\mu}\right)^{k-1} \pi_1 \text{ for } k = 2, \ldots, s + 1
\]
\[ \pi_{s+k} = \left( \frac{\delta + \mu}{\mu} \right)^{k-1} \left( \frac{\beta + \delta + \mu}{\mu} \right)^{s} \pi_{1} \text{ for } k = 2, \ldots, S - 2s \]

\[ \pi_{Q+k} = \frac{\delta + \mu}{\mu} \pi_{Q+k-1} - \frac{\beta}{\mu} \pi_{k-1}, \quad \text{where } Q = S - s, \quad k = 1, 2, \ldots, s, \]

Here \( \pi_{0}, \pi_{2}, \ldots, \pi_{S} \) are all expressed in terms of \( \pi_{1} \). From \( \pi e = 1 \) we can find \( \pi_{1} \) and hence \( \pi_{0}, \pi_{2}, \ldots, \pi_{S} \).

**Theorem 5.3.1.** The Markov chain described by the model is stable if and only if \( \lambda < \mu \).

**Proof.** From the well known results (see Neuts [53]) on positive recurrence of \( \bar{Q} \) which states that \( \pi A_{0} e < \pi A_{2} \). Simplifying this we get \( \lambda < \mu \) \( \Box \)

**5.3.2. Steady state analysis.** Let \( X = (x(0), x(1), \ldots) \) be the steady state probability vector of the Markov chain. Since the model considered here is a LIQBD process, its steady state distribution has a matrix-geometric solution to the equations \( X \bar{Q} = 0 \) and \( X e = 1 \). Then \( x(i) \) has the matrix geometric form

\[ x(i) = x(1) R^{i-1} \text{ for } i \geq 2, \quad (5.3.1) \]

where \( R \) is the minimal non-negative solution of the matrix equation \( A_{0} + RA_{1} + R^{2}A_{2} = 0 \). \( X \bar{Q} = 0 \) gives

\[ x(0) A_{00} + x(1) A_{2} = 0 \quad (5.3.2) \]

\[ x(0) A_{0} + x(1) (A_{1} + RA_{2}) = 0. \quad (5.3.3) \]

Solving the above equations we can find vectors \( x(0) \) and \( x(1) \) subject to the normalizing condition \( X e = 1 \).

That is \( x(0) e + x(1) (1 - R)^{-1} = 1 \). Having found \( x(1), x(i), i \geq 2 \) can be found from (5.3.1).
5.4. Performance Measures

Having computed the system state probabilities, we proceed to find out how the system performs. Let \( X = (x(0), x(1), \ldots) \) be the steady-state probability vector of \( \mathcal{Q} \) where \( x(i) = (y_{i0}, y_{i1}, \ldots, y_{iS}) \).

(1) Expected number of customers, EC in the system is given by

\[
EC = \sum_{i=1}^{\infty} ix(i)e
\]

(2) Expected inventory level EI is given by

\[
EI = \sum_{i=0}^{\infty} \sum_{j=1}^{S} jy_{ij}
\]

(3) Expected waiting time in the system EW is given by

\[
EW = \frac{EC}{\lambda}.
\]

(4) Expected re-order rate ER is given by

\[
ER = \mu \sum_{i=1}^{\infty} y_{i, s+1} + \delta \sum_{i=0}^{\infty} \sum_{j=s+1}^{S} y_{ij}
\]

(5) Expected number of inventory ET, lost due to disaster is given by \( ET = \delta EI \).

(6) Expected number of customers EJ not joining the system when the inventory level is zero is given by

\[
EJ = \lambda \sum_{i=0}^{\infty} y_{i0}.
\]

(7) Expected rate of departure ED after completing service is given by

\[
ED = \mu \sum_{i=0}^{\infty} \sum_{j=1}^{S} y_{ij}
\]
5.5. Cost Function

To construct the cost function we define the following costs as

\[ C = \text{fixed ordering cost} \]
\[ C_1 = \text{procurement cost/unit} \]
\[ C_2 = \text{holding cost of inventory/unit/unit time} \]
\[ C_3 = \text{revenue from service/unit/unit time} \]
\[ C_4 = \text{disaster cost/unit} \]
\[ C_5 = \text{shortage cost of inventory/unit/unit time} \]

Then the total expected cost is defined as

\[ \text{ETC} = [C + QC_1]ER + C_2EI - C_3ED + C_4ET + C_5EJ \]

5.6. Mathematical Description of Model II

The only difference of this model from the first model is that customers join the system even when the inventory level is zero. The infinitesimal generator \( \mathcal{Q} \) of the process has the form of (5.2.1) where the blocks \( A_{00}, A_0, A_1, A_2 \) are square matrices of order \( (S + 1) \) and they are given by

\[
A_0 = \lambda I_{S+1}, \quad A_2 = \begin{bmatrix}
0 & 0 \\
\mu I_S & 0
\end{bmatrix},
\]

\[
A_{00} = \begin{pmatrix}
0 & 1 & \ldots & s & s + 1 & \ldots & S - s & \ldots & S \\
-\Delta & \beta \\
\delta & -\Omega & \beta \\
\vdots & \ddots & & \ddots & \ddots & \ddots \\
\delta & -\Omega & \ldots & \ldots & \beta \\
\delta & -\omega \\
\vdots & \ddots & & \ddots & \ddots \\
\delta & -\omega
\end{pmatrix},
\]

with \( \Delta = \lambda + \beta, \Omega = \lambda + \beta + \delta, \omega = \lambda + \delta. \)
where \( \Omega' = \lambda + \beta + \delta + \mu, \omega' = \lambda + \delta + \mu, \Delta = \lambda + \beta. \)

5.7. Analysis of Model II

5.7.1. System stability. Define the generator matrix \( A \) as \( A = A_0 + A_1 + A_2 \). Let \( \pi = (\pi_0, \pi_1, \ldots, \pi_S) \) be the steady state probability vector of \( A \). Then we have \( \pi A = 0 \) and \( \pi e = 1 \). Solving \( \pi A = 0 \) we get

\[
\begin{align*}
\pi_0 &= \frac{\mu}{\beta + \delta} \pi_1 + \frac{\delta}{\beta + \delta}, \\
\pi_k &= \left(\frac{\beta + \delta + \mu}{\mu}\right)^{k-1} \pi_1 \quad \text{for} \quad k = 2, \ldots, s+1, \\
\pi_{s+k} &= \left(\frac{\beta + \delta + \mu}{\mu}\right)^{k-1} \left(\frac{\beta + \delta + \mu}{\mu}\right) \pi_1 \quad \text{for} \quad k = 2, \ldots, S - 2s, \\
\pi_{Q+k} &= \frac{\delta + \mu}{\mu} \pi_{Q+k-1} - \frac{\beta}{\mu} \pi_{k-1}, \quad \text{where} \quad Q = S - s, \quad k = 1, 2, \ldots, s.
\end{align*}
\]

Here \( \pi_0, \pi_2, \ldots, \pi_S \) are all expressed in terms of \( \pi_1 \). From \( \pi e = 1 \) we can find \( \pi_1 \) and hence \( \pi_0, \pi_2, \ldots, \pi_S \).

**Theorem 5.7.1.** The Markov chain is stable if and only if \( \lambda < \mu(1 - \pi_0) \) where

\[
\pi_0 = \frac{\delta + \mu + \delta M}{(\delta + \mu + \beta) + (\delta + \beta)M}.
\]
and

\[ M = \left( \frac{\beta + \delta + \mu}{\beta + \delta} \right)(x^{s-1} - 1) + \frac{\mu}{\delta} x^s (y^{s-2s} - 1) + \frac{\mu}{\delta} \left[ \frac{\beta}{\beta + \delta} (x^s - 1) - \left( \frac{x}{y} \right)^s + 1 \right] \]

with \( x = \frac{\beta + \delta + \mu}{\mu}, \quad y = \frac{\delta + \mu}{\mu} \)

**PROOF.** From the well known results (see Neuts [53]) on positive recurrence of \( \overline{Q} \) which states that \( \pi A_0 e < \pi A_2 e \). Simplifying this we get \( \lambda < \mu(1 - \pi_0) \)

\[ \square \]

### 5.7.2. Steady state analysis.

Let \( X = (x(0), x(1), \ldots) \) be the steady state probability vector of the Markov chain. Here again the model is a LIQBD process, its steady state probability distribution has a matrix-geometric solution to the equations \( X\overline{Q} = 0 \) and \( Xe = 1 \). Then \( x(i) \) has the matrix geometric form

\[ x(i) = x(1) R^{i-1} \text{ for } i \geq 2 \]  \hspace{1cm} (5.7.1)

where \( R \) is the minimal non-negative solution of the matrix equation

\[ A_0 + RA_1 + R^2 A_2 = 0. \quad X\overline{Q} = 0 \text{ gives} \]

\[ x(0)A_{00} + x(1)A_2 = 0 \]  \hspace{1cm} (5.7.2)

\[ x(0)A_0 + x(1)(A_1 + RA_2) = 0 \]  \hspace{1cm} (5.7.3)

Solving the above equations we can find vectors \( x(0) \) and \( x(1) \) subject to the normalizing condition \( xe = 1 \), that is \( x(0)e + x(1) \left( 1 - R \right)^{-1} = 1 \), then \( x(i) \), for \( i \geq 2 \), can be obtained from (5.7.1).

### 5.8. Performance Measures

Having computed the system state probabilities, we proceed to find out how the system performs. Let \( X = (x(0), x(1), \ldots) \) be the steady-state probability vector of \( \overline{Q} \) where \( x(i) = (y_{i0}, y_{i1}, \ldots, y_{iS}) \).
(1) Expected number of customers, EC in the system is given by

\[ EC = \sum_{i=1}^{\infty} ix(i)e \]

(2) Expected inventory level EI is given by

\[ EI = \sum_{i=0}^{\infty} \sum_{j=1}^{S} jy_{ij} \]

(3) Expected waiting time in the system EW is given by

\[ EW = \frac{EC}{\lambda[1 - \sum_{i=0}^{\infty} y_{i0}]} \]

(4) Expected re-order rate ER is given by

\[ ER = \mu \sum_{i=1}^{\infty} y_{i,s+1} + \delta \sum_{i=0}^{\infty} \sum_{j=s+1}^{S} y_{ij} \]

(5) Expected number of inventory, ET lost due to disaster is given by ET = \( \delta \) EI.

(6) Expected number of departures, ED after completing service is given by

\[ ED = \mu \sum_{i=0}^{\infty} \sum_{j=1}^{S} y_{ij} \]

5.9. Cost Function and Numerical Examples

To construct the cost function we define the following costs as

- \( C \) = fixed ordering cost
- \( C_1 \) = procurement cost/unit
- \( C_2 \) = holding cost of inventory/unit/unit time
- \( C_3 \) = revenue from service/unit/unit time
- \( C_4 \) = disaster cost/unit

Then the total expected cost is defined as

\[ ETC = [C + QC_1]ER + C_2 EI - C_3 ED + C_4 ET \]
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### Table 5.1: Variations in maximum inventory level $S$. $\lambda = 1$, $\mu = 1.4$, $\beta = 1$, $\delta = 0.1$, $s = 10$

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### Table 5.2: Variations in reorder level $s$. $\lambda = 1$, $\mu = 1.4$, $\beta = 1$, $\delta = 0.1$, $S = 30$

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<td>1.4205</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 5.3: Variations in service rate $\mu$. $\lambda = 1$, $\beta = 1$, $\delta = 0.1$, $s = 10$, $S = 25$
### Table 5.4. Variations in arrival rate \( \lambda \). \( \mu = 3, \beta = 1, \delta = 0.1, s = 10, S = 25 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>EC</th>
<th>ER</th>
<th>EW</th>
<th>EJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7</td>
<td>1.3076</td>
<td>2.0780</td>
<td>1.8584</td>
<td>2.0267</td>
</tr>
<tr>
<td>1.8</td>
<td>1.5000</td>
<td>2.4432</td>
<td>1.9505</td>
<td>2.1269</td>
</tr>
<tr>
<td>1.9</td>
<td>1.7272</td>
<td>2.8995</td>
<td>2.0423</td>
<td>2.2268</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0000</td>
<td>3.4860</td>
<td>2.1336</td>
<td>2.3266</td>
</tr>
<tr>
<td>2.1</td>
<td>2.3333</td>
<td>4.2672</td>
<td>2.2246</td>
<td>2.4262</td>
</tr>
<tr>
<td>2.2</td>
<td>2.7500</td>
<td>5.3596</td>
<td>2.3152</td>
<td>2.5250</td>
</tr>
<tr>
<td>2.3</td>
<td>3.2857</td>
<td>6.9943</td>
<td>2.4054</td>
<td>2.6206</td>
</tr>
</tbody>
</table>

### Table 5.5. Variations in disaster rate \( \delta \). \( \lambda = 1, \mu = 1.5, \beta = 1, s = 10, S = 25 \)

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>EC</th>
<th>EI</th>
<th>ER</th>
<th>ET</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>2.0000</td>
<td>2.4929</td>
<td>15.7786</td>
<td>15.8006</td>
</tr>
<tr>
<td>.15</td>
<td>1.9999</td>
<td>3.8463</td>
<td>13.6755</td>
<td>13.7484</td>
</tr>
<tr>
<td>.20</td>
<td>1.9999</td>
<td>4.8356</td>
<td>12.8101</td>
<td>12.9005</td>
</tr>
<tr>
<td>.25</td>
<td>2.0000</td>
<td>6.2035</td>
<td>12.0604</td>
<td>12.1556</td>
</tr>
<tr>
<td>.30</td>
<td>2.0000</td>
<td>8.2363</td>
<td>11.4124</td>
<td>11.4814</td>
</tr>
<tr>
<td>.35</td>
<td>2.0000</td>
<td>11.6023</td>
<td>10.8508</td>
<td>10.7863</td>
</tr>
</tbody>
</table>

### Table 5.6. Variations in replenishment rate \( \beta \). \( \mu = 1.4, \lambda = 1, \delta = 0.1, s = 10, S = 25 \)

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>EC</th>
<th>EI</th>
<th>ER</th>
<th>ET</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>2.4999</td>
<td>3.8735</td>
<td>14.8726</td>
<td>14.9154</td>
</tr>
<tr>
<td>1.3</td>
<td>2.4999</td>
<td>3.5717</td>
<td>15.2012</td>
<td>15.2531</td>
</tr>
<tr>
<td>1.5</td>
<td>2.4999</td>
<td>3.3751</td>
<td>15.4498</td>
<td>15.5054</td>
</tr>
<tr>
<td>1.7</td>
<td>2.4999</td>
<td>3.2375</td>
<td>15.6446</td>
<td>15.7010</td>
</tr>
<tr>
<td>1.9</td>
<td>2.4999</td>
<td>3.1363</td>
<td>15.8013</td>
<td>15.8571</td>
</tr>
<tr>
<td>2.1</td>
<td>2.4999</td>
<td>3.0589</td>
<td>15.9300</td>
<td>15.9845</td>
</tr>
<tr>
<td>2.3</td>
<td>2.4999</td>
<td>2.9979</td>
<td>16.0377</td>
<td>16.0905</td>
</tr>
</tbody>
</table>

### 5.9.1. Interpretation of the Numerical results.

1. **Effect of the maximum inventory level \( S \) on various performance measures**

   From table 5.1 we conclude that as \( S \) increases, inventory level and thus the inventory lost due to disaster increase. Due to the availability of more inventory,
reorder rate decreases as the time interval to reach the reorder level increases. The number of customers do not change in model I as customers do not join when the inventory level is zero, while in model II it decreases.

2. **Effect of the reorder level \( s \) on various performance measures**

   Table 5.2 shows that the changes on various performance measures as \( s \) changes is similar to that of \( S \), except the reorder rate. Here when \( s \) increase reorder rate increases as the time interval to reach the reorder level decreases and more orders are placed.

3. **Effect of the service rate \( \mu \) on various performance measures**

   When service rate increases as we expect the number of customers and hence the waiting time of customers decrease in both models. Reorder rate and hence inventory increase in model II as customers join even when inventory is zero, while in Model I both remain the same, as customers do not join when inventory level is zero (see table 5.3).

4. **Effect of the arrival rate \( \lambda \) on various performance measures**

   When arrival rate increases as we expect the number of customers, the waiting time and the reorder rate increase in both models. The number of customers who do not join when the inventory level is zero also increases (see table 5.4).

5. **Effect of the disaster rate \( \delta \) on various performance measures**

   Table 5.5 shows that as \( \delta \) increases the inventory lost due to disaster increases and so the inventory decrease in both models. Number of customers and reorder rate increase in model II as customers join even when the inventory is zero, while in model I, number of customers is same as customers do not join the system when the inventory level is zero. In Model I reorder rate increases first and then decreases as customers do not join when the inventory level zero. (see the formula for ER)

6. **Effect of the replenishment rate \( \beta \) on various performance measures**

   From table 5.6 we can understand that as \( \beta \) increases inventory level increases in both models. The number of customers and hence the waiting time of customers in model II decrease as the replenishment rate of inventory increase, while in model I no change as customers do not join the system when the inventory is zero. As more
inventory is with the system, in model I, the number of customers who do not join when the inventory level is zero (EJ) decreases.

**Figure 5.1.** $\lambda = 1, \mu = 1.4, \beta = 1, \delta = 0.1, s = 10, C = 100, C_1 = 20, C_2 = 1, C_3 = 5, C_4 = 27, C_5 = 5$

**Figure 5.2.** $\lambda = 1, \mu = 1.4, \beta = 1, \delta = 0.1, S = 30, C = 100, C_1 = 20, C_2 = 1, C_3 = 5, C_4 = 27, C_5 = 5$
**Figure 5.3.** $\lambda = 1$, $\beta = 1$, $\delta = 0.1$, $s = 10$, $S = 25$, $C = 100$, $C_1 = 20$, $C_2 = 1$, $C_3 = 5$, $C_4 = 27$, $C_5 = 5$.

**Figure 5.4.** $\mu = 3$, $\beta = 1$, $\delta = 0.1$, $s = 10$, $S = 25$, $C = 100$, $C_1 = 20$, $C_2 = 1$, $C_3 = 5$, $C_4 = 27$, $C_5 = 5$. 
Figure 5.5. $\mu = 1.5$, $\beta = 1$, $s = 10$, $S = 25$, $C = 100$, $C_1 = 20$, $C_2 = 1$, $C_3 = 5$, $C_4 = 27$, $C_5 = 5$

Figure 5.6. $\mu = 1.4$, $\lambda = 1$, $\delta = 0.1$, $s = 10$, $S = 25$, $C = 100$, $C_1 = 20$, $C_2 = 1$, $C_3 = 5$, $C_4 = 27$, $C_5 = 5$. 

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5.9.2. **Interpretation of the graphs.** Figure 5.1 shows the variation of the cost (ETC) with the maximum inventory level $S$. When $S$ increases the cost decreases, it may be due to the decrease in the reorder rate. The cost increases when $s$ increases as the reorder rate increases (see figure 5.2). From figure 5.3, we can understand that service rate does not affect the cost function in model I as the reorder rate and inventory is same, in model II the cost slightly increases. Total expected cost increases as arrival rate increases (see figure 5.4). Figure 5.5 shows that when the disaster rate increases in model II the cost increases as the reorder rate increases. In model I as the reorder rate first increases and then decreases, the cost function also behaves like that. Figure 5.6 shows that when replenishment rate increases the expected cost increases. This may be due to the increase in the reorder rate.

5.10. **Conclusion**

From all the graphs we may conclude that the expected cost of model I is less than model II. So model I is profitable with the given cost function and parameters. That is it is better for the system to not allow the customers to join when the inventory is zero.