III.1 **Data Reduction:**

The basic data in the present experiment consists of the information regarding the particle densities, at various density detectors, for the EAS recorded with a specific trigger requirement and is on the paper tape as explained earlier (chapter II). This data is processed on the CDC 3600/160A computer installation at TIFR, Bombay, in collaboration with the EAS group of TIFR. The data is transferred on a magnetic tape from the paper tapes and former is then used for processing purposes.

a) **Cross and Singles - Calibrations:**

The number $p_i$ corresponding to the density in $i$th density detector, punched on paper tape, represents the pulse height at the output of the Logarithmic amplifier attached to the density detector. This $p_i$ is related to the particle density $\Delta i$ by the relation

$$\Delta i = A_i \times 10^{p_i/B_i}$$

where $A_i$ and $B_i$ are the constants characteristic of the $i$th detector. Thus in order to be able to determine $\Delta i$ from $p_i$ one must know the constants $A_i$ and $B_i$. For this...
purpose a singles and a cross-calibration is done for each detector. These calibrations were done by TIFR group, and constants made available by them for each detector for the duration of the present experiment have been used in the present analysis. The method followed for singles and cross-calibrations is as follows:

A pair of linear amplifiers are used for this purpose. For singles-calibration the pre-amplifier of the detector under calibration is connected to one of the linear amplifiers. The EAS recording system is triggered by single muons passing through the detector under study. These single muons are selected by means of GM counter telescope. The linear output from the calibration amplifier is punched out and from a number of such triggers the average linear pulse height $\bar{p}_1$ corresponding to passage of a singly charged relativistic particle through the detector is obtained. The procedure is repeated for all density detectors and the value of $\bar{p}_1$ are determined.

For cross-calibration, the preamplifier outputs for a given pair of detectors are connected to the calibration amplifiers and shower records obtained then contain both the logarithmic as well as linear output.
A semi-log plot of these outputs with the logarithmic output along the linear scale and corresponding linear output on the log scale then gives a relation between $p_1$ and $\Delta i$. The linear readings are converted into densities by dividing these readings by the single particle height $P_1^{-1}$.

The cross-calibration done in this way for different detectors along with the singles calibration then yield the values of the constants $Ai$ and $Bi$ for various density detectors.

The details of singles and cross-calibrations are given by Sivaprasad (1970).

III.2 Evaluation of Shower Parameters:

An EAS can be characterised by four parameters $(N, X, Y, s)$ at a given level of observation, where $N$ gives the total number of charged particles in the shower and is known as its size, $X$ and $Y$ are the co-ordinates of the core of the shower and $s$ the age parameter of the shower. The particle densities obtained in various density detectors, are used to obtain these parameters, by fitting the densities to the NKG distribution function (Equation 1.3.4) which is known to give a good fit to the experimental data as discussed in section 1.3. For the present observation level $Y_i = 96$ m.
The fit is achieved through the minimisation of a quantity \( \chi^2 \) defined as

\[
\chi^2 = \sum_{i=1}^{n} W_i \left( \bar{p}_i - \Delta_i \right)^2 \quad \ldots (3.2.1)
\]

\( \Delta_i \) & \( \bar{p}_i \) being respectively the observed and expected densities at ith density detector and \( W_i \) is the weight attached to the observed density. Following Scherb (1959) the weights are taken as

\[
W_i = 1/\bar{p}_i \quad \Delta_i \leq 25 \text{ m}^{-2} \quad \ldots 3.2.2
\]

\[
W_i = 25/\bar{p}_i^2 \quad \Delta_i > 25 \text{ m}^{-2} \quad \ldots 3.2.3
\]

The weights are given to take into account the fluctuations in the observed densities. For densities \( \Delta_i < \Delta \) major source of error is statistical fluctuations in the number of shower particles crossing the ith detector and the distribution for this error is nearly Poisson. For \( \Delta_i > \Delta \) the Poisson fluctuation becomes smaller than the instrumental uncertainty which is assumed to have Guassian distribution with a constant relative standard deviation. Taking a value of 20% for relative deviation, \( \Delta' \) is found to be 25 particles / m^2.

The best fit values of the parameters \( (N,X,Y,s) \) are then those which satisfy the following equations.

\[
\frac{\partial \chi^2}{\partial N} = \frac{\partial \chi^2}{\partial X} = \frac{\partial \chi^2}{\partial Y} = \frac{\partial \chi^2}{\partial s} = 0 \quad \ldots 3.2.4
\]
Thus we have a set of four simultaneous equations which can, in principle, be solved to obtain the values of \((N,X,Y,s)\). However, the equations are non-linear and solutions are not possible in all cases. To solve these equations the method of "Steepest descent" has been used. The parameters \((N,X,Y,s)\) can be considered as co-ordinates of the vector.

\[
\mathbf{e}_j = \mathbf{e}((N,X,Y,s)) \quad \ldots \quad (3.2.5)
\]

Then the surface \(\chi^2 = \chi^2(\mathbf{e}_j)\) will have a valley in the neighbourhood of the best fit values of the parameters so that equation 3.2.4 is satisfied. For achieving minimisation, an initial estimate of the vector is made. Then, quantised vector increments, in the direction of \(-\nabla \chi^2\) are given to the vector \(\mathbf{e}_j\), the magnitude of the increments being adjusted in binary approximation.

A new value of the vector \(\mathbf{e}_{j+1}\) is obtained from the previous value \(\mathbf{e}_j\) using following relation

\[
\mathbf{e}_{j+1} = \mathbf{e}_j - \chi_{j+1} \quad \frac{\nabla \chi^2(\mathbf{e}_j)}{|\nabla \chi^2(\mathbf{e}_j)|} \quad \ldots \quad (3.2.6)
\]

and the process is repeated till a certain criterion is satisfied implying the approach of minimum. The criterion requires three consecutive reductions in \(\chi^2/(n-d)\) by magnitudes < 0.01.
A goodness of fit parameter $\chi^2/(n-d)$ is printed out for each shower. Here $n$ is the number of density detectors used for analysis and $d$ is the number of fitted parameters. The method is similar to one used by Scherb (1959).

For the analysis of present data a fortran programme, based on above method, written by EAS group at TIFR, is used. A fixed value of $s$ ($s = 1.25$) is taken for all showers and the remaining three parameters ($N, X, Y$) are then obtained by $\chi^2$ minimisation. The use of $s = 1.25$ for all showers at the level of observation though not strictly correct, does not however introduce serious errors in the present size range as shown by Sivaprasad (1970). Sivaprasad did an extensive error analysis for the array used in the present experiment using the above mentioned fortran programme and has shown that the effect of using $s = 1.25$ for all showers on an incident spectrum of artificial showers is to leave the slope of the spectrum unchanged. There is however a change in absolute intensity $\sim 25\%$ which is of the same order of magnitude as the statistical accuracy of the data under consideration.

III.3 Initial Estimate of Shower Parameters:

In order to obtain $X_0$, $Y_0$, initial estimate of $X$ & $Y$ respectively a weighted average of the co-ordinates
of four density detectors having first four maxima of densities is taken. Then if $\Delta_i$ $(i = 1,4)$ represent the density maxima and $(X_i, Y_i)$, $(i = 1,4)$ are the co-ordinates of corresponding density detectors we have

$$X_o = \frac{\sum X_i \Delta_i}{\sum \Delta_i} \quad \text{and} \quad Y_o = \frac{\sum Y_i \Delta_i}{\sum \Delta_i} \quad \text{...(3.3.1)}$$

For estimating $N_o$, the initial estimate of $N$, use is made of the fact that equation $\frac{dX_i}{dN} = 0$ for the given distribution gives a cubic equation of the form

$$N^3 + aN + b = 0 \quad \text{...(3.3.2)}$$

which always yields a real solution. Here $a$ and $b$ are functions of core location and age of the shower. Thus knowing $X_o$ and $Y_o$, an estimate of $N_o$ can be obtained by solving equation (3.3.2) analytically.

III.4 Errors in Fitted Parameters:

The parameters, fitted to a given shower, are subjected to errors which are not easy to evaluate by direct methods. Hence an 'error analysis' has been done, using showers generated artificially (to be called artificial shower) in the following manner. For a given shower size $N$ and age parameter $s=1.25$, the shower cores are fixed at random locations within a certain distance.
R from the array centre. Uniformly distributed random numbers between 0 and 1 are used for this purpose. Using the equation (1.3.4) the densities $\Delta i$ in all density detectors are calculated. Over these densities are then superposed fluctuations proportional to two Gaussian distributions, one with a mean of $\sqrt{\Delta i}$ and other with a mean of 20% of $\Delta i$. The shower thus generated is then similar, for all practical purposes, to the showers recorded in the experiment, as it is subjected to the fluctuations similar to the fluctuations in the measured densities of the recorded showers. A number of artificial showers ($\sim 100$ for each size) thus generated are then fed to the computer and the showers are processed like real showers. A comparison of the fitted parameters $(N, X, Y)$ with the original parameters $(N_0, X_0, Y_0)$ then yields an estimate of the errors in these parameters. Fig. III.1 to Fig. III.3 show histograms for $\Delta X = X - X_0$, $\Delta Y = Y - Y_0$ and $\log (N/N_0)$ for three different sizes. The error analysis yields following values of the errors $\Delta N$, $\Delta X$ and $\Delta Y$ in the parameters $N$, $X$ & $Y$ respectively,

$$\Delta N = \pm 0.20 \times N$$
$$\Delta X = \pm 2.50 \, \text{m}$$
$$\Delta Y = \pm 2.50 \, \text{m}$$

Fig. III.4 shows the distribution in $\chi^2 / (n-d)$ for SU 7 trigger showers and artificial showers.
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