CHAPTER VII

CROSS-FIELD-CURRENT-DRIVEN ELECTROSTATIC INSTABILITIES IN PLASMAS WITH GENERALIZED DISTRIBUTION FUNCTION

VII.1 Introduction

There are various kinds of plasma instabilities which can arise in beam plasma systems. The return-current instability and its impact on plasma heating was discussed in the last chapter (Chapter VI). Now we shall go over to study another class of beam induced instabilities namely, the cross-field-current driven electrostatic instabilities, which play a special role in certain turbulent heating experiments. Cross-field-current driven electrostatic instabilities have been invoked as the basic mechanism of production of anomalous resistivity observed in a number of turbulent heating experiments (Alexeff et al. 1970, 1971; Babykin et al. 1964). In Chapter I, we described the physical mechanisms by which cross
field currents were produced in a couple of experiments. However, there are other physical situations too, where cross-field currents play an important role. For example, the anomalous resistance observed in a number of collisionless shock experiments (Daughney et al. 1970, Kielhacker and Steuer 1971) is interpreted to be due to such cross field currents within the shock profile.

Cross field currents can give rise to a host of electrostatic instabilities. However, we shall concentrate on only three of them namely, the ion acoustic instability, the modified-two-stream instability and the electron cyclotron drift instability; these have rather larger growth rates (in \( \omega > f_t \)) associated with them. The physical nature of these electrostatic instabilities, allows us to classify them into two classes—dissipative instabilities' and 'reactive instabilities' (Hasegawa 1968, Taylor and Dashmore-Davies 1970). A dissipative instability is one in which energy flows from the plasma to the wave or vice versa, depending on whether the wave carries negative or positive energy. On the other hand, a reactive instability can be considered as due to coupling between two waves carrying energies of opposite signs. Thus, a reactive instability involves more than one waves and there is no transfer of energy between the wave and the plasma. The dissipative instability involves only a small fraction of resonant particles whereas the reactive instability involves all the particles in the plasma. Therefore it is easier to
stabilize a dissipative instability than a reactive one.

The cross-field-current driven ion acoustic instability is a dissipative mode. This mode has been studied by many authors (Gary 1970, Arefiev 1969, Barrett 1972). The dependence of this mode on the external magnetic field has also been extensively studied (Lashmore-Davies 1973). As the angle between the direction of propagation of the mode and the magnetic field direction increases, this mode changes from a dissipative one to a reactive one.

Modified two-stream instability, on the other hand, is a reactive mode which arises as a result of coupling between the lower hybrid mode \( (\omega \sim \omega_{\text{ LH}}) \) (\( \omega_{\text{ LH}} \) being the lower hybrid frequency) and the Doppler shifted electron mode \( (\omega \sim k_y U - (k_z/k) (m_i/m_e)^{1/2} \omega_{\text{ LH}}) \), where \( U \) is the relative drift between the two species of the plasma, \( k_z \) and \( k_y \) are the components of the wave vector parallel and perpendicular to the magnetic field direction respectively). For \( \omega_{\text{ pe}} < \omega_{\text{ LH}} \) , the Doppler shifted electron mode is nothing but the Doppler shifted electron plasma oscillations propagating almost perpendicular to the magnetic field. The modified two-stream instability has been studied quite extensively both in the linear regime (Stepanov 1965, Ashby and Paton 1967, Krall and Liewer 1971, Arefiev 1969) as well as in the nonlinear regime (McBride et al. 1972). This instability derives its name from the fact that, in the fluid limit, the form of the dispersion relation is similar to the one for Buneman two
stream instability (Bunemann 1961). However, the magnetic field introduces important differences. The threshold for this instability is \( U > \alpha_i \) whereas that for the Bunemann two-stream instability is \( U > \alpha_e \) (\( \alpha_e,i \) being the electron and the ion thermal velocities). This instability is a nonresonant instability, the nonlinear saturation of which grossly changes the particle distribution functions. Moreover, the growth rate for this instability is comparable to the real part of the frequency. Another interesting feature that comes out of the computer simulation experiments (McBride et al. 1972) is the comparable electron and ion heating as a result of nonlinear saturation of this instability.

The electron cyclotron drift instability is also a reactive instability which results due to the coupling of an ion acoustic mode and a slow Doppler shifted Bernstein mode. This mode was discussed by Wong (1970) and Gary and Sanderson (1970). A good deal of work on the nonlinear development of this instability and a number of computer simulation experiments have also been reported recently (Forslund et al. 1970, 1971, 1972; Lamp et al. 1972). It is observed from these studies that, this instability gives rise to diffusion across the magnetic field and causes substantial electron and ion heating.

Valuable, as these investigations are, they are all carried out on the assumption that the equilibrium distribution function is well represented by a Maxwellian distribution function. However, we note that a number of cross-field
current heating experiments (Alexeff et al. 1970, 1971, Babykin et al. 1964). Use of magnetic mirror configuration to confine the plasma and for such a system, we know that a Maxwellian is not a realistic distribution function. Therefore, in this chapter, we shall use a 'generalized distribution function' (Dory et al. 1965) to study the effects of loss-cone and temperature anisotropy on the above mentioned instabilities. Cross-field-current driven electrostatic instabilities being often invoked as the basic mechanism in explaining the results of the turbulent heating experiments mentioned above, it is of importance to see if the characteristics of these instabilities are significantly altered by the presence of temperature anisotropy and loss cone effects.

From our analysis, we find that, for \( \lambda = k^2 \rho^2 / 2 \ll 1 \), these effects appear as small corrections. On the other hand important modifications occur for \( \lambda \sim 1 \). Algebraic complexities of the dispersion relation, in the region \( \lambda \sim 1 \), make it difficult to draw analytic conclusions over the entire range of parameter space. For this reason, we have numerically solved the general dispersion relation over a wide range of parameter space. Results of these numerical calculations are presented in Section VII.4.

VII.2 Dispersion Relation

Let us consider the waves for which \( |\omega| \gg \Omega_1 \) and \( k \rho_i \gg 1 \), where \( \omega \) and \( k \) are the characteristic wave
frequency and wave number; $\Omega_i$ and $g_i$ are the ion-cyclotron frequency and the ion gyro-radius. Under these conditions the ions are effectively unmagnetized. We choose the axis of the magnetic mirror along the z-direction.

Since the ions are essentially unmagnetized, they are governed by a Maxwellian distribution, namely

$$f_{O_i} = \frac{1}{\pi^{3/2}\alpha_i^2} \exp \left[ - \frac{\left( \frac{\Omega_i^2 - \Omega^2}{\alpha_i^2} \right)^2}{\frac{\Omega_i^2 + \Omega^2}{\alpha_i^2}} \right], \quad (7.1)$$

where $\alpha_i = (2kT_i/m)^{1/2}$ is the ion thermal velocity; $K$, $T$ and $m$ being the Boltzmann constant, ion temperature and ion mass respectively. $\Omega$ in Eq. (7.1) is the relative streaming velocity between the two species of the plasma which we have taken to be in the y-direction. For electrons we take a generalized distribution of DGH type (Dory et al. 1965, Batl 1974, Lakhina et al. 1974), namely

$$f_{O_e} = \frac{J}{\pi^{3/2}\alpha_{Le}^2} \exp \left[ - \frac{\Omega_e^2}{\alpha_{le}^2} - \frac{\Omega_e^2}{\alpha_{le}^2} \right], \quad (7.2)$$

where $J = 0, 1, 2, \ldots$ is the distribution index, $\alpha_{le}^2 = 2kT_{le}c/m$ and $\alpha_{Le}^2 = 2(J + 1)^{-1} kT_{le}/n$; $T_{le}$ and $m$ being electron perpendicular (parallel) temperature and electron mass respectively. For $J = 0$, $f_{OE}^0$ goes over to a Maxwellian distribution and for $J \to \infty$, it behaves like $\delta(v_1 - J^{1/2}\alpha_{le})$ ($\delta$-being a Dirac delta function). Moreover $f_{OE}^J$ is peaked about $J^{1/2}\alpha_{le}$ and has a half width, $\Delta v_1 \sim J^{-1/2}\alpha_{le}$.

For small electrostatic perturbations, the perturbed
distribution function for the electrons, $f_{ie}$ is governed by the linearized Vlasov equation,

$$\frac{\partial f_{ie}}{\partial t} + \frac{\gamma}{2} \nabla f_{ie} - \frac{e}{m} \left[ \mathbf{V} \times \mathbf{B}_0 \right] \cdot \nabla_{\mathbf{V}} f_{ie} = \frac{e}{m} \mathbf{E}_1 \cdot \nabla_{\mathbf{V}} f_{ie},$$  \hspace{1cm} (7.3)

while that for the ions, $f_{ii}$, is governed by

$$\frac{\partial f_{ii}}{\partial t} + \frac{\gamma}{2} \nabla f_{ii} = -\frac{e}{M} \mathbf{E}_1 \cdot \nabla_{\mathbf{V}} f_{ii},$$  \hspace{1cm} (7.4)

where $\mathbf{B}_0$ is the external magnetic field. These equations are solved by following usual method of characteristics and on using these solutions in the Poisson's equation,

$$\nabla \cdot \mathbf{E}_1 = 4\pi e \left[ \int f_{ie} d\mathbf{V} - \int f_{ii} d\mathbf{V} \right],$$  \hspace{1cm} (7.5)

we get the following dispersion relation for the electrostatic waves:

$$1 + \frac{k_{ie}^2}{2 \omega_{pe}^2} + \frac{\omega}{k_{ii}^2} \sum_{n=-\infty}^{\infty} C_n J_n(\lambda) Z \left( \frac{\omega - n \Omega_e}{k_{ii} \alpha_{ie}} \right)$$

$$+ \sum_{n=-\infty}^{\infty} \left[ \frac{\alpha_{ie}^2}{2 \omega_{pe}^2} \right] D_n J_n(\lambda) - C_n J_n(\lambda) \right] \frac{n \Omega_e}{k_{ii} \alpha_{ie}} Z \left( \frac{\omega - n \Omega_e}{k_{ii} \alpha_{ie}} \right)$$

$$- \frac{T_e}{2T_e} \left( 1 + \frac{\alpha_{ie}^2}{2 \omega_{pe}^2} \right)^{-1} Z' \left( \frac{\omega - k_{ii} \mathbf{u}}{k_{ii} \alpha_{ie}} \right) = 0$$  \hspace{1cm} (7.6)

where $\omega_{pe}$ and $\Omega_e$ are the electron plasma frequency and electron cyclotron frequency respectively; $k_{ii}(\omega)$ is the component of wave vector parallel (perpendicular) to the axial magnetic field and $Z$ is the plasma dispersion function.
(Fried and Conte 1961). In obtaining Eq. (7.6), we have taken the wave propagation in the \( y-z \) plane. The functions \( C_n^J(\lambda) \) and \( D_n^J(\lambda) \) are defined by

\[
C_n^J(\lambda) = \frac{1}{\alpha_{1e}^2} \int_0^\infty d\nu_1 \int_0^\infty J_n(\nu_1) \nu_1^{2J} \exp\left[-\frac{\nu_1^2}{\alpha_{1e}^2}\right]
\]

and

\[
D_n^J(\lambda) = \frac{1}{\alpha_{1e}^2} \int_0^\infty d\nu_1 \int_0^\infty J_n(\nu_1) \left[ \frac{\nu_1^{2J}}{\alpha_{1e}^2} - J \nu_1^{2(J-1)} \right] \exp\left[-\frac{\nu_1^2}{\alpha_{1e}^2}\right],
\]

where \( J_n \) are the Bessel functions of order \( n \). From Eqs. (7.7) and (7.8), we immediately see that \( C_n^0(\lambda) = D_n^0(\lambda) = \exp(-\lambda)I_n(\lambda) \).

We can also easily show that they obey the following recurrence relations:

\[
C_n^{J+1}(\lambda) = C_n^J(\lambda) + \frac{\lambda}{J+1} \frac{d}{d\lambda} C_n^J(\lambda)
\]

and

\[
D_n^{J+1}(\lambda) = \frac{J}{J+1} D_n^J(\lambda) + \frac{\lambda}{J+1} \frac{d}{d\lambda} D_n^J(\lambda),
\]

where \( I_n(\lambda) \) are the Bessel functions with imaginary argument. Moreover from Eqs. (7.7) and (7.8), we note that

\[
\sum_{n=-\infty}^{\infty} D_n^0(\lambda) = 1 \text{ for } J = 0
\]

and

\[
\sum_{n=-\infty}^{\infty} C_n^J(\lambda) = 1 \text{ for all } J \text{'s.}
\]
In writing these results, we have made use of the Bessel identity, \( \sum_{n=-\infty}^{\infty} J_n^2(x) = 1 \). For Maxwellian distribution for electrons, i.e., for \( J = 0 \) and \( \alpha_{\perp e} = \alpha_{\| e} \equiv \alpha_e \), Eq. (7.6) goes over to the usual dispersion relation for the crossfield current driven electrostatic instabilities (Barrett et al. 1972). Since it is not possible to solve Eq. (7.6) for \( \omega \) analytically, we have to solve it numerically. However, before we do that we shall discuss some special cases.

VII. 3A Stability Analysis: Low Frequency Waves

For \( \omega^2 \ll \Omega_e^2 \) and \( n^2 \Omega_e^2 \gg k_{\perp e}^2 \), \( \alpha_{\perp e}^2 (n \gg 1) \)

\[
\sum_{n=-\infty}^{\infty} C_n^J(\lambda) Z(\frac{\omega - \nu \Omega_e}{k_{\perp e} \alpha_{\perp e}}) \approx C_0^J(\lambda) Z(\frac{\omega}{k_{\perp e} \alpha_{\perp e}}),
\]

since \( Z(\nu \Omega_e / k_{\perp e} \alpha_{\perp e}) \approx Z(-\nu \Omega_e / k_{\perp e} \alpha_{\perp e}) \). Consequently Eq. (7.6) can be written as

\[
2 \left\{ 1 + k_{\perp}^2 \frac{\lambda_D^2}{A} \left[ 1 + (J+1)^{-1} \right] \right\}^{-1} - C_0^J(\lambda)^2
\]

\[
= C_0^J(\lambda) Z' \left( \frac{\omega}{k_{\perp e} \alpha_{\perp e}} \right) + \frac{T_e}{T_i} \left[ 1 + (J+1)^{-1} \right] Z' \left( \frac{\omega - k_{\perp} \nu}{k_{\perp e} \alpha_{\perp e}} \right)
\]

\[
- 4 \sum_{n=1}^{\infty} \left\{ C_n^J(\lambda) - (J+1)^{-1} D_n^J(\lambda) \right\} \frac{n \Omega_e}{k_{\perp e} \alpha_{\perp e}} Z \left( \frac{n \Omega_e}{k_{\perp e} \alpha_{\perp e}} \right)
\]

(7.10)

where \( \lambda_D = (k T_e / 4 \pi m e^2)^{1/2} \) is the electron Debye length and

\[
A = \frac{T_e}{T_{\perp e}}. \]
1) Modified two-stream Instability

When ions are cold ($T_i = 0$) and $\omega^2/k_i^2 \alpha_{we}^2 >> 1$, Eq. (7.10) yields

$$1 + k_i^4 \frac{\Omega_{Pe}^2}{\Omega_i^2} E_o^J(\lambda) - \frac{\omega^2 \Omega_{pe} C_o^J(\lambda)}{k_i^2 \lambda} - \frac{\omega^2 \Omega_{pi}}{(\omega - k_i U)^2} = 0,$$

(7.11)

where $E_o^O(\lambda) = \left[ 1 - D_o^O(\lambda) \right]/\lambda$

and $E_o^J(\lambda) = \left[ - D_o^J(\lambda) \right]/\lambda$ for $J \neq 0$.

For $J = 0$, Eq. (7.11) reduces to the dispersion relation for the modified-two-stream instability obtained by McBride et al. (1972) and Lashmore Davies and Martin (1973).

For $\lambda \ll 1$; $C_o^O(\lambda) \approx D_o^O(\lambda) \approx 1$ and hence the distribution index has no effect at all. One expects significant modification only in the region $\lambda \sim 1$. In this case Eq. (7.11) can be solved only under very special circumstances. For a given $k p_i$ (i.e., $\lambda$) near unity, let us choose the orientation of the wave vector such that

$$\left( k_i^2 / k_o^2 \right) C_o^J(\lambda) = m/n$$

(7.13)

In this case Eq. (7.11) can be solved exactly and the solution is given by

$$\omega' = \omega_{lH}^2 \left[ 1 + \frac{k_i^2 |U|^2}{4 \omega_{lH}^2} \pm \left( 1 + \frac{k_i^2 |U|^2}{\omega_{lH}^2} \right)^{1/2} \right]$$

(7.14)

where $\omega' = (\omega - k_i U)/2$ and $\omega_{lH}^2 = (\omega_{pe}^2 + \omega_{pi}^2) \left[ 1 + \frac{\omega_{pe}^2}{\omega_{pi}^2} E_o^O(\lambda) \right]^{-1}$. 

Thus the wave having wave vector and its orientation that satisfy Eq. (7.13) becomes unstable if the relative streaming velocity satisfies the relation

$$R_{\perp} U^2 \ll \frac{\omega^{'2}}{\omega_{LH}^{'2}} \tag{7.15}$$

in which case, the growth rate is given by

$$\gamma = \omega_{LH}^{'2} \left[ \left( 1 + \frac{R_{\perp} U^2}{\omega_{LH}^{'2}} \right)^{1/2} - \left( 1 + \frac{k_{\parallel}^2 U^2}{\omega_{LH}^{'2}} \right) \right]^{1/2}$$

$$\approx k_{\parallel} U/2 \quad \text{if} \quad R_{\perp} U^2 \ll \omega_{LH}^{'2}. \tag{7.16}$$

ii) Ion Acoustic Instability

Let us consider the regime where the ions are cold, so that we can still use an asymptotic form for the function

$$Z'(\frac{\omega - k_{\parallel} U}{k_{\perp} U})$$

but the electrons are hot such that $T_e/T_1 \gg 1$ and $(\omega^2/k_{\parallel}^2 \alpha_{\parallel e}^2) \ll 1$, which would allow us to use a power series expansion for the function $Z(\omega^2/k_{\parallel}^2 \alpha_{\parallel e}^2)$. Though $(\omega^2/k_{\parallel}^2 \alpha_{\parallel e}^2) \ll 1$, we shall assume $(n^2 \Omega_e^2/k_{\parallel}^2 \alpha_{\parallel e}^2) \gg 1$ for $n \geq 1$. On writing $\omega = \omega_r + i \gamma$ and assuming a priori that $\gamma^2 \ll (\omega_r - k_{\parallel} U)^2$, Eq. (7.10) can be solved to give

$$\omega_r = k_{\parallel} U \left[ 1 - \frac{\xi_e}{U} \left( 1 + A (J + 1) \right)^{-1} \left( \Delta_J^{1/2} - \frac{C_e}{U} \right) \right]^{-1}$$

(7.17)

and

$$\gamma = \left( \frac{\pi m}{8 \Omega_e^2} \right)^{3/2} \frac{k}{k_{\parallel}} C_e \delta \left( \frac{\omega_r - k_{\parallel} U}{k_{\parallel} U \Delta_J} \right)^{1/2} \left( \Delta_J^{1/2} - \frac{C_e}{U} \right), \tag{7.18}$$

where
\[ \Delta_J = C_0^J(\lambda) + \lambda(J+1)A^{-1}E_0^J(\lambda) + k^2 \lambda_p^2 \left[ 1 + A(J+1)^{-1} \right]^{-1} \] (7.19)

and \( C_s = (T_e/M)^{1/2} \) is the sound speed. It can be easily verified from Eq. (7.17) and (7.18) that the a priori assumption, \( \gamma^2 \ll (\omega_p^2 - k_1^2)^2 \), is satisfied for sufficiently large angle of orientation for the wave vector such that \( (k_{1||}/k^2)(\mu/m) >> 1 \).

For \( J = 0 \) and \( \alpha_{1||} = \alpha_e = \alpha_0 \), these results go over to the results obtained by Barrett et al. (1972). To see how the growth rate varies with \( J \), we assume that the temperature anisotropy, \( A \), is large, so that we can write \( \Delta_J \approx C_0^J(\lambda) \).

In this case, the ratio between the growth rates for a non-Maxwellian plasma \( (J \neq 0) \) and that for a Maxwellian plasma \( (J = 0) \) goes as

\[
\frac{[\gamma]_{J \neq 0}}{[\gamma]_{J = 0}} = \frac{C_0^0(\lambda) \left[ \left( C_0^J(\lambda) \right)^{1/2} - C_s/\mu \right]}{C_0^J(\lambda) \left[ \left( C_0^J(\lambda) \right)^{1/2} - C_s/\mu \right]} \] (7.20)

Since, in the region \( \lambda \sim 1 \), \( C_0^J(\lambda) \) decreases much faster for \( J \geq 1 \) than for \( J = 0 \), it is clear from Eq. (7.20) that the growth rates for a non-Maxwellian plasma are higher than that for a Maxwellian Plasma.

VII.3b Stability Analysis: High Frequency Waves

1) Electron Cyclotron Instability

Since for \( k_{1||} > 0 \) the electron cyclotron waves are damped, we shall study only the case \( k_{1||} = 0 \). Moreover, if
the finite ion temperature effects are neglected, Eq. (7.6) can be simplified to obtain the dispersion relation for these high frequency \( \omega > \Omega_e \) electron cyclotron waves; this is given by,

\[
1 + \frac{\kappa^2 \alpha_{te}^2}{2 \omega_p^2 e} = \sum_{n=-\infty}^{\infty} C_n^J(\lambda) \left( \frac{\omega}{\omega - n \Omega_e} \right) + \frac{k_{\perp}^2 C_s^2 (j+1) A^{-1}}{(\omega - k_{\perp} U)^2} + \sum_{n=-\infty}^{\infty} \left[ (j+1) A^{-1} D_n^J(\lambda) - C_n^J(\lambda) \right] \frac{n \Omega_e}{\omega - n \Omega_e}.
\]  

(7.21)

For \( \omega \approx |n| \Omega_e \) and \( \gamma^2 < (\omega - k_{\perp} U)^2 \), the frequency and growth rate for the \( n \)th harmonic cyclotron wave are given by

\[
\omega_r - k_{\perp} U = \pm k_{\perp} C_s \left\{ 1 + A(j+1)^{-1} \right\}^{-1/2} \\
\left[ 1 + k^2 C_s^2 \left\{ 1 + A(j+1)^{-1} \right\}^{-1} - C_n^J(\lambda) \right]^{-1/2},
\]  

(7.22)

and

\[
\gamma^2 = -\left| n \right| \Omega_e (j+1) A^{-1} D_n^J(\lambda) (\omega_r - k_{\perp} U) \\
\left[ 1 + k^2 C_s^2 \left\{ 1 + A(j+1)^{-1} \right\}^{-1} - C_n^J(\lambda) \right]^{-1}.
\]  

(7.23)

For \( \gamma^2 > 0 \), depending on the sign of \( D_n^J(\lambda) \), we have to choose the appropriate root for \( \omega_r \) as given by Eq. (7.22). From Eq. (7.23) it is clear that the growth rate directly depends on \( D_n^J(\lambda) \) whose variation is shown in Fig. 7.1. We also observe from this figure that for \( n = 1 \) and \( k_{\perp} C_s^2 > 1 \) though \( D_n^0(\lambda) \) is always positive, \( D_n^J(\lambda) \) for \( J > 1 \), becomes negative after
FIGURE 7.1 \((J + 1) D^J_1(\lambda)\) as a function of \(k \xi_{\theta}\).
some critical value of $k \xi_e$. For $J = 0$, $\alpha_{\perp e} = \alpha \parallel e = \alpha e$ and $\lambda > |n|$, the results given by Eqs. (7.19) and (7.20) go over to those obtained by Wong (1970).

This instability is due to the resonant coupling of a Doppler shifted ion-mode and the electron cyclotron mode. As Wong (1970) has shown that this instability needs the following condition to be satisfied.

$$\frac{\omega_{pe}}{\Omega_e} > k \xi_e > |n|$$

VII.4 Results of Numerical Calculations

In an attempt to study the effect of $J$ on modified two-stream instability and ion acoustic instability over a wide range of parameters, we have solved Eq. (7.6) numerically. As was shown by Bashmore-Davies and Martin (1973), the modified two-stream instability and ion-acoustic instability are often not separable. In fact with the variation of certain parameters (e.g. $k_{\parallel} / k$) the instability changes from one to the other. What we follow in the numerical calculations is also a combination of these two instabilities. For carrying out numerical computations, for convenience's sake, we will introduce the dimensionless variables, namely $x = \omega / \omega_{pe}$, $\bar{c} = (k_{\parallel} / k) (k / m)^{1/2}$, $\eta = \omega_{pe} / \Omega_e$ and $\bar{\xi} = U / \alpha_i$.

The selection of the initial parameters is made in such a way that the maximum growth rate for a Maxwellian plasma ($J = 0$)
occurs for \(k \xi \sim 1\). In Fig. 7.2 variations of frequency and growth rate with \(k \xi\) have been shown for \(J = 0, 1, 2\). It is interesting to note that, while a Maxwellian plasma \((J = 0)\) is stable for \(k \xi > 1.25\), a non-Maxwellian plasma \((J = 1, 2)\) is not only unstable but it also sustains higher growth rates for \(k \xi > 1.25\). The growth rates for \(k \xi > 2\) are not shown because the range \(k \xi < 1\) is the one which is interesting from a practical point of view. In the case of \(J = 1\), the increase in the growth rate for \(k \xi > 1.4\) seems to be because of the fact that in this region the root has changed from modified two stream to ion acoustic mode. Moreover, an examination of Eq. (7.18) shows that the growth rate for the ion acoustic instability goes inversely as \(C_0^\prime(\lambda)\), because \(\Delta_j \sim C_0^\prime\) for large values of \(\lambda\). Since \(C_0^\prime(\lambda)\) decreases with \(k \xi\), there is a corresponding increase in the growth rate, as depicted in Fig. 7.2. From this figure, we also note that for \(J \geq 1\), the growth rates are \(\lambda > 0.2 \Omega_p\) which for the parameters used correspond to \(\gamma > \Omega_i\) and hence we conclude that in the range of \(k\)-space under consideration, the system can sustain fast growing modes.

In Fig. 7.3, we have plotted the variation of frequency and growth rate with the angle of orientation of the wave vector for a fixed \(k(k \xi = 1.2)\). It is observed that the growth rates are higher for higher \(J\) values when \(\bar{\xi}\) is small, but they are smaller for higher \(J\) values as \(\bar{\xi}\) increases. However, the real part of the frequency decreases with an
**FIGURE 7.2** Variation of frequency ($\omega / \omega_{pi}$) and growth ($\gamma / \omega_{pi}$) rate with wave number $k_{pe}$. The parameters labeling the curve is the distribution index $J$. Other parameters are $\omega_{pe}/\Omega_e = 10.0$, $T_e/T_i = 50.0$, $U/\alpha_i = 12.5$, $\Lambda = 10.0$, $\bar{e} = 0.4$ and $m/H = 1/1836$. 
FIGURE 7.3 Plots of frequency ($\omega_r/\omega_{pi}$) and growth rate ($\gamma/\omega_{pi}$) versus the angle of orientation of the wave vector ($\bar{\theta}$). The parameter labeling the curves is the distribution index $J$. $k^f_{\parallel}$ for these curves is 1.2 and other parameters are same as in Fig.7.2.
increase in $J$. It is well known that, as $(k_{11}/k)$ increases, the electron Landau damping becomes stronger and stronger and finally it stabilizes the wave. The above results seem to indicate that the electron Landau damping play a more vital role for a loss-cone plasma than for a Maxwellian plasma. As was indicated in the beginning of this section, this figure also shows a transition from modified two stream to ion acoustic instability as $\bar{u}$ increases. It is easy to verify that the left hand side of the figure represents modified two stream instability while the right hand side represents ion-acoustic instability.

Fig. 7.4 shows plots of frequency and growth rate versus temperature anisotropy. A significant increase in the growth rate is observed with an increase in anisotropy. This figure also shows that for an anisotropic plasma the growth rates are higher for $J \geq 1$ than those for $J = 0$. From Fig. 7.5 it is apparent that for $\gamma \geq 2$ the magnetic field has a destabilising effect on the system. Fig. 7.6 shows that the threshold for $T_0/T_i$ for instability decreases as $J$ increases and the maximum growth rate (maximized over $T_0/T_i$) is higher for higher $J$ values.

In order to see the variation with the streaming velocity we determined the maximum growth rate, which occurs at $k = k^*$, for a number of streaming velocities. These results are shown in Table 7.1. For reasons stated earlier, we have restricted ourselves to values of $k \gamma_0$ only upto 3.0.
FIGURE 7.4 Variation of frequency ($\omega_r/\omega_{pi}$) and growth rate ($\gamma/\omega_{pi}$) with anisotropy parameter $A(=T_\perp/e/T_{||}e)$. The parameter labeling the curves is the distribution index $J$. For these curves $k\sigma_e = 1.2$ and $\bar{\sigma} = 0.8$. All other parameters are same as in Fig. 7.2. The growth rates for $J = 2$ are not shown, because with the scales used in this figure they overlap with those for $J = 1$. 
FIGURE 7.5 Variation of frequency ($\omega_r/\omega_{pi}$) and growth rate ($\gamma/\omega_{pi}$) with $\omega_{pe}/\Omega_e$. The parameter labeling the curves is the distribution index J. For these curves $A = 10.0$ and all other parameters are same as in Fig. 7.3. The growth rates for $J = 2$ are not shown, because with the scales used in this figure they overlap with those for $J = 1$. 
FIGURE 7.6 Plots showing the variation of frequency and growth rate ($\gamma/\omega_p$) with the temperature ratio ($T_e/T_1$). The parameter labeling the curves is the distribution index $J$. Other parameters are $\omega_{pe}/\Omega_e = 10.0$, $A = 10.0$, $k f_0 = 1.2$, $U/\alpha_1 = 12.5$ and $e = 0.8$. 
<table>
<thead>
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<th>$U/a_l$</th>
<th>$k^*_{fe}$</th>
<th>$\gamma^{*}/\omega_{pi}$</th>
<th>$\omega_{ce}/\omega_{pi}$</th>
<th>$\omega^*/\omega_{pi}$</th>
</tr>
</thead>
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<tr>
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</tbody>
</table>

**Table 7.1: Maximum Growth Rates**

The maximum growth rates $(\gamma^{*}/\omega_{pi})$ and corresponding frequency $(\omega^*/\omega_{pi})$ are shown for a number of streaming velocities. The constant parameters are $\Omega_{ce}/\omega_{pi} = 10.0$, $\Omega_{ce}/\omega_{pi} = 10.0$, and $\theta = 0.8$.

A dash indicates the absence of maximum growth.
We observe that $\gamma^*$ as well as $k^*$ increase as the streaming velocity decreases. Another interesting thing to be noted is that, there are certain values of streaming velocities (for example $U/\omega_i \leq 12.0$, for the set of parameters considered in Table 7.1) for which the growth rate maximizes for $k_{Fe}^* \sim 1.0$ in the case of $J = 0$ while it goes on increasing even up to $k_{Fe}^* = 3.0$ in the case of $J = 1$.

VII.5 Conclusions

We have shown that the loss-cone and temperature anisotropy in the electron velocity distribution has profound effects on the modified two-stream instability and ion acoustic instability when $\lambda = k^2 \xi_e^2/\omega_i^2 \sim 1$. We also find that, non-Maxwellian plasmas with distribution under $J \geq 1$, can sustain fast growing waves (in $\omega > \Omega_i$) even in regions of $k$-space which is stable for a Maxwellian plasma. Moreover, when $\lambda \sim 1$ the temperature anisotropy is found to have a destabilizing effect on the modified two-stream instability and on the ion acoustic instability.

In this chapter we have not attempted to ascertain quantitatively the modifications in the non-linear theory of these instabilities due to the modifications introduced by the loss-cone and the anisotropy effects; however, we would qualitatively discuss the evolution of the distribution
function due to these nonlinearities. The result of numerical simulation experiments performed by McBride et al. (1972) indicate that the nonlinear stabilization of the modified two-stream instability takes place due to particle trapping in the potential well. They have also shown that the electrons predominantly get heated along the direction of the magnetic field while the ions predominantly get heated along the direction perpendicular to the magnetic field, such that the final parallel temperature of the electrons is comparable to the perpendicular ion temperature. In our case the consequence of such an effect will be that, the predominant parallel heating of the electrons due to modified two-stream instability will eventually isotropize the distribution function even if the equilibrium distribution function for the electrons was sufficiently anisotropic to start with.
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