CHAPTER 7

SUMMARY OF THE RESULTS

The research work of this thesis is an advancement over the existing literature in the area of robustness to non-normality of correlation coefficient $r$ and its transformations. The investigations of some new bivariate non-normal populations like Morgenstern Bivariate exponential, Doubly truncated bivariate normal distribution, Freund's bivariate exponential distribution pertaining to the distribution of $r$ and its transformations have been reported critically. The numerical evaluation for the non-normality of the distribution of $r$ and some of its transformations has been explored on the basis of cumulants of the bivariate populations of our interest, using the methods of Mardia(1974), Cook(1951 b) and Subrahmaniam and Gajjar(1980). Method of polynomial representations to the distribution of $r$ has also been studied using the polynomials like Hermite, Legendre and Laguerre when the samples come from Morgenstern bivariate exponential distribution (MBVED).

The following points highlight the observations of the study.

(i) Fisher's transformation seems to be reasonably robust for $|\rho| \leq 0.05$ or $|\alpha| \leq 0.20$ in case of Morgenstern bivariate exponential distribution. Other transformations are not robust. It has been observed that the crucial factor in behaviour of the
transformation is the bias. The test of significance on population correlation coefficient $\rho$ can be performed, using Fisher's transformation while sampling from MBVED at least for $|\rho| \leq 0.05$.

(ii) In case of mixture of two MBVED's, the study reveals that though the amount of mixture $p$ does not have any significant role in the assessment of non-normality of the distribution, it plays a vital role in the assessment of robustness to non-normality of $r$. Of all transformations, Fisher's transformation seems to be more robust in the range $0 \leq p \leq 0.60$.

(iii) Three polynomials namely Hermite, Legendre and Laguerre have been used for approximating the distributions of $r$ and some transformations of $r$ based on a sample drawn from MBVED. It is shown that the Hermite polynomials perform extremely well in representing the distribution of the statistics considered. Legendre and Laguerre polynomials are weak in all the cases examined.

We have not attempted Jacobi polynomials of type I and type II for the transformations considered in the thesis for lack of computer facilities. It is our belief that the polynomial representation using Jacobi polynomials of type I & II might give a better fit for all the transformations considered in the thesis.

It is also felt that two parameter Laguerre polynomials might give good result for a new transformation $(1+4r)/(1-4r)$. The fact that the new transformation based on Nair suggested in the thesis, namely

$$(16-\rho^2)/[8(1-\rho^2)](r-\rho)/(1-r\rho)+(16+56\rho-31\rho^2+4\rho^3)/[8(4-\rho)(1-\rho^2)]$$
performs quite well for $|\rho| \leq 0.05$ using Legendre polynomials leads us to believe that if Jacobi type II polynomials are used this new transformation may perform much better.

(iv) Studies regarding the DTBVND reveal that Samiuddin transformation is quite robust to non-normality due to two-sided truncation for $|\rho| \leq 0.10$. With respect to bias, samiuddin and Arcsin transformations are reasonably robust. The results are at variance with those obtained by Subrahmaniam and Gajjar(1980) in case of singly truncated bivariate normal distribution, where these were shown to be robust for different levels of truncation. These indicate that the degree of the severity of non-normality renders the transformation of $r$ to be non-robust.

(v) In Freund bivariate exponential distribution, for the parametric assumption $\beta_1 = \beta_2$, $\beta'_1 = \beta'_2$, the distribution has been simplified and considered for the investigation. The results depict that of all transformations under study, none performs well. Of course, from the point of view of bias Arcsin transformation is quite good. This may perhaps be due to the choice of parametric values considered by us rather than the weak of the transformations. This aspect needs further investigation for a wide choice of parametric values of $\beta_1$, $\beta_2$, $\beta'_1$ and $\beta'_2$.

Conclusions: Some of the well-known transformations of $r$ are shown to be robust even if $r$ is based on a random sample drawn from non-normal bivariate population like MBVED and DTBVND.
exhibiting a high degree of non-normality as depicted by Mardia measures of skewness and kurtosis.

In case of MBVED Fisher's transformation is robust for $|\rho| \leq 0.05$ while in case of DTBVND Samiuddin transformation is robust for $|\rho| \leq 0.10$. Both these transformations are seriously affected outside the prescribed range of $\rho$. 