CHAPTER 6

DISTRIBUTION OF r AND ITS ROBUSTNESS TO NON-NORMALITY OF THE TRANSFORMATIONS IN SAMPLES FROM FREUND'S BIVARIATE EXPONENTIAL DISTRIBUTION

6.1. Introduction

Gumbel (1960) proposed several bivariate distributions whose marginals are exponential. He did not discuss the appropriateness of the models to particular situations. Freund (1961) presented a different bivariate extension of the exponential distribution, which is designed, in particular for the life testing of two component systems, which can function even after one of the components has failed. Freund's distribution might apply to the study of engine failures in two-engine planes, to the wear of two pens on an executive desk, or to the performance of a person's eyes, kidneys, or other paired organs.

Suppose that an instrument has two components A and B with life times X and Y having density functions (when both the components are under operation)

\[ f_X(x) = \beta_1 e^{-\beta_1 x} \quad ; \quad f_Y(y) = \beta_2 e^{-\beta_2 y} \quad (\beta_1, \beta_2 > 0) \]
\[ x, y > 0 \quad (6.1.1) \]

The random variables X and Y are dependent in such a way that a failure of either component changes the parameter of the
life distribution of the other component. Thus when A fails, the parameter $\beta_2$ for $y$ becomes $\beta'_2$; when B fails the parameter $\beta_1$ for $x$ becomes $\beta'_1$. There is no other dependence. Thus the joint density function of $x$ and $y$ is

$$
 p_{x,y}(x,y) = 
\begin{cases} 
\beta_1 \beta_2 \exp[-\beta'_2 y - (\beta'_2 + \beta'_2) x] & 0 \leq x \leq y \leq \infty \\
\beta_2 \beta'_1 \exp[-\beta'_1 x - (\beta'_1 + \beta'_2) y] & 0 \leq y \leq x \leq \infty 
\end{cases}
$$

(8.1.2)

It is to be noted that in Freund's bivariate model, the dependence between $x$ and $y$ is essentially such that the failure of B component changes the parameter of the exponential life distribution of the A component from $\beta_1$ to $\beta'_1$, while the failure of A component changes the parameter of the exponential life distribution of the B component from $\beta_2$ to $\beta'_2$.

Since the distribution (6.1.2) involves four non-negative parameters $\beta_1, \beta'_1, \beta_2, \beta'_2$. The population correlation coefficient is given by

$$
\rho = \frac{\beta_1 \beta'_2 - \beta'_1 \beta_2}{\sqrt{\beta_1^2 + 2\beta_1 \beta_2 + \beta_2^2} \sqrt{\beta'_2^2 + 2\beta'_1 \beta'_2 + \beta'_2^2}}
$$

(6.1.3)

For the distribution defined in (6.1.2), it can be seen that the population correlation coefficient

$$-1/3 \leq \rho \leq 1
$$

(6.1.4)
and in many applications $\beta' \geq \beta > 0$ and $\beta' \geq \beta > 0$ for various statistical properties and the estimation of the parameters of model (6.1.3) we may refer to Freund(1961) and Johnson and Kotz(1972).

As the Freund's bivariate exponential distribution involves four parameters $\beta_1', \beta_1, \beta_2, \beta_2'$ we have chosen

$$\beta_1 = \beta_2, \quad \beta_1' = \beta_2'$$

and we get

$$\rho = \frac{\beta_1'^2 - \beta_1^2}{(\beta_1'^2 + 3\beta_1^2)}$$

or

$$\beta_1' = \left\{ \sqrt{\frac{1+3\rho}{1-\rho}} \right\} \beta_1$$

(6.1.5)

Thus given $\beta_1$ and $\rho$ we can find $\beta_1'$ from (6.1.5).

In this Chapter, we have considered the model defined in (6.1.2) with the restriction that $\beta_1 = \beta_2, \beta_1' = \beta_2'$ to study the distribution of sample correlation coefficient $r$. Mardia measures of non-normality have been considered to gauge the departure from normality through skewness and kurtosis factors. Various values of $\rho$ and $\beta_1$ are selected to study the behaviour of the distribution of transformed $r$ to assess the robustness to non-normality.

6.2. Moments of Freund's distribution

It can be seen that the joint moment generating function of $(s, t)$th moment for (6.1.2) in case of $\beta_1 = \beta_2$ and $\beta_1' = \beta_2'$ is
\[ m(s, t) = \frac{\beta_1}{(2\beta_1 - s - t)} \left\{ (1-s/\beta_1')^{-1} (1-t/\beta_1')^{-1} \right\} \]  \hspace{1cm} (6.2.1)

from which we get

\[ \nu_{10} = \nu_{01} = \frac{\beta_1 + \beta_1'}{2\beta_1 \beta_1'} \]  \hspace{1cm} (6.2.2)

\[ \nu_{11} = \frac{\beta_1 \beta_1' + \beta_1'^2}{2\beta_1^2 \beta_1'^2} \]  \hspace{1cm} (6.2.3)

\[ \nu_{12} = \frac{1}{2\beta_1} \left[ \frac{1}{\beta_1'^2} + \frac{1}{2\beta_1'} \left( \frac{1}{\beta_1'} + 6 \right) + \frac{1}{\beta_1'^2} \right] \]  \hspace{1cm} (6.2.4)

\[ \nu_{20} = \frac{1}{\beta_1'^2} + \frac{1}{2\beta_1'} \left( \frac{4}{\beta_1'} + \frac{1}{\beta_1} \right) \]  \hspace{1cm} (6.2.5)

\[ \nu_{22} = \frac{1}{\beta_1^2 \beta_1'^2} + \frac{3}{2} \frac{1}{\beta_1^2 \beta_1'} + \frac{3}{2} \frac{1}{\beta_1'^4} \]  \hspace{1cm} (6.2.6)

\[ \nu_{13} = \frac{3}{4} \left[ \frac{1}{\beta_1^3 \beta_1'} + \frac{1}{\beta_1^2 \beta_1'^2} + \frac{1}{\beta_1^3 \beta_1'^3} + \frac{1}{\beta_1^4 \beta_1'^2} \right] \]  \hspace{1cm} (6.2.7)

\[ \nu_{23} = \frac{3}{4} \left[ \frac{1}{\beta_1^2 \beta_1'^3} + \frac{2}{\beta_1^3 \beta_1'^2} + \frac{5}{\beta_1^3 \beta_1'^3} \right] \]  \hspace{1cm} (6.2.8)

\[ \nu_{14} = \left[ \frac{6}{\beta_1^2 \beta_1'} + \frac{6}{\beta_1^3 \beta_1'} + \frac{9}{2 \beta_1^3 \beta_1'} + \frac{1}{\beta_1^3 \beta_1'^2} + \frac{15}{4 \beta_1^3 \beta_1'^3} \right] \]  \hspace{1cm} (6.2.9)
\[ \mu_{34} = 9 \left[ \frac{1}{\beta_1^3 \beta_1'} + \frac{2}{\beta_1^2 \beta_1'} + \frac{1}{2 \beta_1' \beta_1^3} + \frac{45}{3 \beta_1^5 \beta_1'} \right] \] (6.2.10)

\[ \mu_{33} = \left[ \frac{1}{\beta_1^4} + \frac{45}{4 \beta_1^3 \beta_1'} + \frac{81}{8 \beta_1^2} + \frac{9}{4 \beta_1'} + \frac{9}{4 \beta_1^3} \right] \] (6.2.11)

\[ \mu_{40} = \left[ \frac{12}{\beta_1^4} + \frac{2}{\beta_1^3 \beta_1'} + \frac{1}{2 \beta_1^2 \beta_1'} + \frac{45}{3 \beta_1^5 \beta_1'} \right] \] (6.2.12)

\[ \mu_{42} = \left[ \frac{6}{\beta_1^4 \beta_1'} + \frac{9}{\beta_1^3 \beta_1'} + \frac{21}{2 \beta_1^2} + \frac{45}{4 \beta_1' \beta_1^3} \right] \] (6.2.13)

From the above moments the corresponding complimentary moments can be obtained by replacing the respective variables in their counterparts. These moments can be used to calculate \( \psi_t \) functions as defined in Chapter 1 of various transformations in case of Freund's exponential distribution.

5.3. Mardia measures of normality

Using the moments obtained in Section 2, Mardia measures \( \beta_1 \) \& \( \beta_2 \) of non-normality defined in (1.3.5) and (1.3.6) have computed to study the non-normality of the distribution.

Table 6.3.1 shows the Mardia measures of skewness and kurtosis for the selected values of \( \rho(\pm 0.25, \pm 0.15, \pm 0.05) \) and \( \beta_1(0.005, 0.02, 0.60, 0.80) \) to hopefully simulate the extremeness of non-normality one would like to study. Assuming the relationship
Table 8.3.1* Values of $\beta_{12}$ and $\beta_{22}$ in Freund BED

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<th>$\rho$ \ $\beta_1$</th>
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<th>0.020</th>
<th>0.60</th>
<th>0.80</th>
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* Each cell contains $\beta_{12}$ and $\beta_{22}$ respectively.
\[ \beta'_1 = \left\{ \frac{\sqrt{1-\rho}}{1-\rho} \right\} \beta_1 \]

the non-normality measures have been obtained.

The examination of table reveals that Mardia measures in case of Freund's distribution under the parametric selection shows high degree of non-normality. The estimation of Mardia's measures of skewness shows that the increasing value of \( \beta'_1 \) tends to make the Freund's distribution asymmetric. As far the parameter \( \rho \) is concerned it can be seen that \(|\rho|\) tends to 0, the distribution is getting closer to normality.
6.4. Transformations of r

All the four transformations defined in (1.3.14) have been considered here to assess their robustness to non-normality. From the first four moments of \( E[h_i(r)] \), \( i = 1,2,3,4 \) and \( j = 1,2,3,4 \), the criteria defined in (1.3.22) have been computed for the selected combinations of \( \rho \) and \( \beta_1 \). Table 6.4.1 to 6.4.4 exhibit the numerical evaluation of \( \delta_i \), \( i = 1,2,3,4 \). Different values of \( n(20,50,100,200) \), correlation parameter \( \rho(\pm0.15, \pm0.10) \) and \( \beta_1(0.005,0.06,0.4) \) have taken to study the robustness to non-normality of \( r \). Given \( \beta_1 \), the value of \( \beta_i \) from the relation 6.1.5. Hence the value of \( \beta_i \) is not shown in tables 6.4.1 to 6.4.4.

Now let us consider each of \( \delta_i \), for the discussion about their performance of the transformation of \( r \) to assess robustness as when the underlying bivariate population is Freund's bivariate distribution.

\( \delta_i (\text{bias}) \): If the distribution of \( h(r) \) tends to the normal counterpart then this measure should invariably approach to zero. The critical examination of the Tables shows that Arcsine transformation is least biased followed by Fisher's and Nair's transformations for any selected combination of \( \rho \) and \( \beta_1 \). The value of \( \beta_1 \) seems to be a vital parameter since the bias
increases as the value of $\beta_1$ is increased. However, the numerical value of the bias is reasonably small for $|\rho| < 0.10$. It is to be noted that in all transformations, increase in $|\rho|$ fluctuates the value of $\delta_1$ indicating that the non-normal mean is affected to a remarkable degree with respect to bias. The large value of $n$ does not subdue the effects of non-normality. As far as the bias is concerned, Arcsine, Fisher's and Nair's transformations perform reasonably well for $|\rho| < 0.10$.

$\delta_2$ (standard deviation): This parameter seems to be less affected in case of Samiuddin's transformation even for small samples and for $|\rho| < 0.10$. On the basis of $\delta_2$, it appears that all transformations are affected to a great extent for variety of values of $\beta_1$ and $\rho$. Increasing value of $|\rho|$ deviates this measure much from normality.
**Table 0.4.1**: Transformations of \( r \) in Freund's BED

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<th>(-0.10)</th>
<th>(0.10)</th>
<th>(0.15)</th>
<th>(-0.15)</th>
<th>(-0.10)</th>
<th>(0.10)</th>
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* Each cell contains in it: \( \delta_1 \), \( \delta_2 \), \( \delta_3 \) and \( \delta_4 \) respectively.
Table: 6.4.2. Transformations of $r$ in Freund's BED

Samiuddin's Transformation

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Table 6.4.4* Transformations of \( r \) in Freund's BED

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$\delta_3$ (skewness): In the normal case skewness should be zero for all the transformations. In present case, the value of $\delta_3$ indicates that the effect of departure from normality is highly pronounced irrespective of any transformation. The values of $\delta_3$ fluctuate violently in case of Fisher & Arcsine compared to Samiuddin & Nair for a variety of values of $\beta_1$, $\rho$ and $n$.

$\delta_4$ (kurtosis): Once again for the test of robustness to non-normality one would expect that this measure is close to zero. For the consideration of the distribution under study the tables show that the robustness to non-normality of transformations of $r$ with respect to kurtosis is highly affected in case of Fisher’s transformation followed by Arcsine transformation. It is observed that for the various combinations of the parameters Samiuddin transformation performs better than the rest as far as this measure is concerned but it does not ensure robustness to non-normality.

To summarize, none of these transformations is good with respect to all these measures. All the transformations of $r$ are not robust in case of Freund bivariate exponential distribution for the parametric value considered in the study. It is fact that this may perhaps be due to the parametric values considered by us rather than the failure of these transformations.