CHAPTER I
INTRODUCTION

Since the development of Wilson’s Economic Order Quantity (EOQ) inventory model in 1915, a number of research papers have appeared analyzing mathematical models of inventory control under various costs and demand structures and restrictive situations and/or conditions. These models are broadly classified under two categories: Deterministic and Probabilistic models. A description and analysis of these models are given by authors of standard text books on inventory control viz. Arrow, Karlin and Scraf [1], Naddor [9], Tersine [18], Wagner [20], etc. An up to date account of literature in inventory management is prepared by Silver [17], who has also hinted on possible areas for further research. In all these models, it has been implicitly assumed that the inventory systems under consideration have sufficient storage capacity to store the on-hand inventory in their Own Warehouse (OW).

However, it is possible that, one may purchase (or produce) more than the capacity W of OW, and keep the excess in Rented Warehouse (RW), which may be located away from OW. Such situations will arise, when the procurement or set up costs are high as compared to the costs associated with using RW for excess units, or when there is a price discount for purchase of large quantities etc. In such situations,
customers' demand can be met either from OW only, or from both OW and RW. Normally, inventory holding costs are higher in RW than in OW. Therefore, it is obvious that the stocks of RW are cleared first in order to bring down the holding costs.

A model of the above type was first considered by Hartley [6], by ignoring the transportation cost from RW to OW. During the last decade, several studies have been made admitting the use of additional storage facilities. These studies differ from the conventional inventory systems, both analytically and operationally.

A pioneer in this area is Sarma [11], who has initiated the work on the inventory models with two levels of storage, in which he includes the transportation cost from RW to OW in the cost function. In his study [11], he assumed that \( W \) units are stored at OW, and rest \( (Q - W) \) units at RW, where \( Q \) denotes the on-hand inventory after replenishment. Initially, demands are satisfied from OW, until the stocks level drops to \( (W - K) \), where \( K \leq W \). The stocks of RW are transferred at a rate of \( K \) units per transhipment to OW until the stocks of RW gets exhausted. The average inventory cost per unit per time unit is obtained as

\[
C(Q, K) = \frac{AD}{Q} + \frac{FQ}{2} + \frac{(F - H)W^2}{2Q} - \frac{(F - H)KW}{2Q} + \frac{(F - H)K}{2} - \frac{(F - H)W}{2} + \frac{(Q - W)D}{QK} \tag{1.1}
\]
where

\[ D = \text{annual demand and is deterministic.} \]
\[ A = \text{ordering cost per order} \]
\[ F = \text{holding cost per unit at EW} \]
\[ H = \text{holding cost per unit at OW, and } F > H. \]
\[ K = \text{unit to be transported in a shipment.} \]
\[ Q_t = \text{transportation cost at a time per shipment.} \]

Apart from that it is also assumed that lead time is zero and shortages are not allowed.

For optimum values of \( K \) and \( Q \), \( \frac{dC(Q,K)}{dK} = 0 \), and
\[ \frac{dC(Q,K)}{dK} = 0 \text{ gives } Q \]
\[ K_0 = \left[ \frac{2DQ}{(F - H)} \right]^{1/2} \]
.............................\(1.2\)

and
\[ Q_0 = \left[ \frac{2AD/F + (F - H)V^2/F - 2W/F\{2DC_t(F - H)\}^{1/2}}{2W/F\{2DC_t(F - H)\}^{1/2}} \right]^{1/2} \]
.............................\(1.3\)

Determined the optimal values of lot-size \( Q = Q_0 \) and \( K = K_0 \) and he terms this method as K-release rule. The conditions for the K-release rule to be optimum are also established [12].

An extension of Sarma's work with a finite replenishment (production) rate has been considered by Murdeshwar and Sathe [8]. They have considered an ordinary release pattern
and the bulk release pattern of the stock from RW to OW. Both Sarma [11,12] and Murdeshwar and Sathe [8] have assumed that the transportation cost is independent of the quantity being transported.

Sarma [13] developed a deterministic order level inventory for deteriorating item with two storage facilities, in which, an order level inventory model with a constant rate of deterioration in each warehouse is developed with fixed scheduling period. The objective was to study the effect of deterioration of goods in both warehouses to determine the optimal order level for each period.

Dave [2] has modified the earlier inventory models [11,12,8] by taking into consideration the transportation cost which, depends on the quantity to be transported as well as the distance between two warehouses. Dave argued that, "Since there are two separate storage facilities - OW and RW, where OW has limited capacity, it will be uneconomical to receive the total delivery at OW first and then to transfer the excess quantity to RW. It must then be requested to the supplier to deliver the ordered amount in two separate, appropriate consignments directly to OW and RW respectively. In doing so, naturally, a higher than normal replenishment cost will apply due to additional holding cost for RW."

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The ECO model with infinite production rate and two storage facilities, the average total cost per time unit is obtained as

\[ C(Q, K) = AD/Q + FD/2 + (F - H)(W - K)W/2D + (F - H)(K/2 - W) + D((a - bP)/K + b)(1 - W/Q) \]

\[ \cdots \cdots \cdots \cdots \cdots (1.4) \]

and optimal values of \( K \) and \( Q \) are

\[ K_o = \left( \frac{2D(a - bP)}{(F - H)} \right)^{1/2} \quad \text{if } Q \neq W \]

\[ \cdots \cdots \cdots \cdots \cdots (1.5) \]

and

\[ Q_o = \left( \frac{(2AD + (F - H)(W - K)W - 2DWF((a - bP)/K + b))}{(F - H)(W - K)W} \right)^{1/2} \]

\[ \cdots \cdots \cdots \cdots \cdots (1.6) \]

where

\[ A = C_3 + C_4 \] : Total replenishment cost per order.

While \( C_3 \) is the normal replenishment cost per order for a system with single warehouse DW, and \( C_4 \) is the additional cost per order due to RW.

Cost of transporting \( K \) units from RW to DW is defined by

\[ U(K) = a + b(K - P), \] where \( a \geq b > 0 \) are constants, \( P \) is the maximum number of units that can be transported under the fixed cost 'a' and for every additional unit after 'P', a variable cost 'b' is to be paid.

For the feasibility of two levels of storage model it requires \( Q > W \) and \( K \leq W \). For which, Dave suggested following steps for optimum solution.
Step I: Find the value $K_1$ of $K$ from (1.5). If $K_1 < W$, then using (1.5) in (1.6), the value of $Q_1$ of $Q$ is given by

$$Q_1 = \left[ (2AD + (F-H)W^2 - 2DWB 
- 2W(2D(a - bP)(F-H)^{1/2}) /F1^{1/2} \right]^{1/2} \quad \quad \quad (1.7)$$

otherwise go to step II.

If $Q_1 > W$, then the optimal solution is $Q = Q_1$, $K = K_1$, and from (1.4) the minimum total cost of the system is

$$C(Q_1, K_1) = EQ_1 + Db - (F - H)(W - K_1) \quad \quad \quad (1.8)$$

otherwise go to step III.

Step II: Take $K_1 = W$, then from (1.6), the value $Q_2$ of $Q$ is

$$Q_2 = [2D (A - U(W))] /F1^{1/2} \quad \quad \quad (1.9)$$

If $Q_2 > W$, then the optimal solution is $Q = Q_2$, $K = W$ and from (1.4), the minimum total cost of the system is

$$C(Q_2, W) = EQ_2 + Dw(W)/W - (F - H)W/2 \quad \quad \quad (1.10)$$

otherwise go to step III.

Step III: Take $Q_2 = W$. In this case, we have the classical EOQ situation with only one level of storage, i.e. only OW, which is filled to its maximum capacity, and the total cost of the system is given by Naddor [9]

$$C(W) = HW/2 + AD/W \quad \quad \quad \quad \quad \quad \quad (1.11)$$
Sarma and Sastry [14] have incorporated the transportation costs in holding costs of warehouses. They discussed a single period inventory model for a system with two levels of storage, where deterministic and probabilistic demands are considered. This model has applications for inventory planning for rice and oil mills, fertilizer distribution, etc., for which a common problem is that of physical storage during peak periods.

In 1989, Sarma and Rao [15], constructed an order level inventory model with two storage systems and power pattern demand. Here also, both deterministic and probabilistic models are constructed for determining the optimal order level inventory.

Following this, Sarma [16] developed an algorithm for K-release rule for the model with two levels of storage facilities. In 1970, Pakkala and Achary [10], developed an inventory model for perishable items and finite rate of replenishment with two storage facilities. This model assumed that the two storage warehouses may have different holding costs. The model also is set up by assuming finite replenishment rate, uniform demand rate and with backlogging of shortages. Sensitivity analysis of the model is carried out considering perturbations in order level, set up cost, shortage cost and deterioration rates.
In a single storage model or two levels of storage models, the criterion of expected cost minimization or profit maximization is universally accepted and received considerable attention. The objective of the thesis is to provide a detailed analysis of EOQ models with two storage facilities under typical situations like:

- An order level inventory models with planned shortages.
- Order quantity under conditions of permissible delay in payment or order quantity when delay in payments of order and shortages are permitted;
- Order quantity under conditions of price change anticipations or price discount with limited time interval;
- Periodic review models for determining lot-size or reorder point or lot-size and reorder point.

OUTLINE OF THE STUDY

The following outline briefly discusses the methodology of this research study. This study is organised as follows :-

Chapter - I is an introductory chapter, which gives a brief review of the literature in the area and the development made in various chapter of the thesis.

In Chapter - II a general order level lot-size inventory model is developed under planned shortages and two storage
facilities. Here optimum value of lot-size \( q_0 \) and order level \( S_0 \) are determined, so as to minimize the total cost. Detailed sensitivity analysis is carried out with help of numerical example.

In Chapter - III an EOG model with two storage facilities is studied under the conditions of permissible delay in payments, with and without shortages. The model studied is an extension of Goyal [3], Shah, Patel, and Shah [18], and Mandal and Phaujdar [7], where they have developed single storage models under permissible delay in payment. The models are analysed under two different possibilities:

i) The period of permissible delay is less than or equal to the reordering time.

ii) The periods of permissible delay is greater than the reordering time.

With these possibilities, the economic ordering quantity and the reordering time are determined for each possibility by minimizing the cost function. The optimum solution(s) are obtained and the sensitivity analysis is performed.

Chapter - IV contains profit maximization problems associated with EOG models under two storage facilities. Here, both price change anticipations and a reduction in price during a given specified period are discussed. The problems also include the transportation cost in the total
costs function, in order to determine a special order quantity and maximum profit in the face of a known price increase or price reduction for a given specified period, under the condition of insufficient storage capacity of OW. The derived models are tested by using partial as well as total sensitivity analysis.

Chapters II to IV deal with deterministic systems, while Chapters V to VII deal with probabilistic systems.

Chapter V contains a periodic review stochastic inventory model with two storage facilities. This model is an extension of Haddor's [9] and Heady and Whitin [5] single storage model with continuous demand. Various cases may arise in such a system. We discuss a particular case of $s < W < s + q$, where $s$ denotes reorder point and $q$ denotes ordering quantity. The model is illustrated by using uniform probability distribution of demand over the review period. Sensitivity analysis is carried out by changing various parameter values.

In Chapter VI the model of Chapter V is altered such that $q$ is fixed, shortages are allowed and optimum value of $s$ is determined. This model is developed under the conditions that for a single storage system optimum $s = s_0$ is such that $s_0 < W < s_0 + q$. The model is illustrated with suitable example.
The model in Chapter VII, combines features of both the previous models. Here both reorder point $s$ and lot-size $q$ are the decision variables. This model is developed under the conditions that $s < W \leq s + q$. The model is illustrated with an example.

Part of the work contained in this thesis has been expected for publication in proceeding of conference and/or journal. A list of papers prepared out of this thesis is included at the end of conclusions chapter.