Appendix A

Sample manual

We provide here a sample manual for the random sampling experiment. This manual was developed along with Dr. Prashanta Tripathy (IIT Madras) and Dr. Kavita Dorai (IISER Mohali) who were the instructors implementing our experiment in their laboratory course.

A.1 Aim and Introduction

The aim of this experiment is to extract information about a given ac source from the voltage probability distribution obtained using a dc voltmeter and a capacitor. In this experiment we would like to measure instantaneous voltage from an ac source using a dc voltmeter. A large number of measurements will then be used to obtain the probability distribution of the voltage. We can then reconstruct the ac waveform from the voltage probability distribution.

To measure the instantaneous voltage it is not sufficient to connect the dc voltmeter directly to the ac source, because, the dc voltmeter will then measure the average voltage, which is zero in this case. A way to avoid this problem is to connect a capacitor across the ac source. The capacitor will become charged and the instantaneous voltage across it will determine the instantaneous charge on the capacitor plate. When the capacitor is disconnected from the circuit, the charge and hence the voltage on the capacitor remains. This voltage can be measured by the dc voltmeter. This dc voltage measured across the capacitor is then the instantaneous ac voltage of the original source.
Let \( f(t) \) be the instantaneous voltage of an ac source. Let us measure the voltage \( N \) random times in an interval, \( 0 \leq t \leq T \). Let \( n(V) \) be the distribution function, i.e. the number of times the measurement of \( f \) results in a value between \( V \) and \( V + \Delta V \).

Define the probability distribution \( P(V) \) as
\[
P(V) \Delta V = \frac{n(V)}{N}
\]

Let \( t_i \) be the times at which the voltage becomes \( V \): i.e.
\[
f(t_i) = V.
\]
Assume that in the neighbourhood of each \( t_i \) we can this function to get
\[
t = g_i(V).
\]
Then
\[
P(V) = \frac{1}{T} \sum_{i=1}^{M} \text{Abs} \left( \frac{dg_i(V)}{dV} \right).
\]

The wave form can be reconstructed from \( P(V) \) using the formula
\[
g(V) = T \int_{V_0}^{V} dV' P(V').
\]

The probability distribution \( P(V) \) may be obtained from the data numerically (e.g. using a C program).

### A.2 Motivation & Background

This experiment has been designed to motivate you to think about probabilistic measurements (probabilistic errors can never be avoided in experimental observations, and every physicist has to develop an intuition for how such errors contribute to an experimental measurement). The apparatus consists of a simple gadget to study the manipulation of probability distributions and during the course of the experiment, you will discover how sampling an AC source at random...
A.3 Experimental Setup

intervals using a capacitor and a DC voltmeter, can be used to get information about the AC waveform.

Imagine inserting the terminals of a DC voltmeter into the AC mains outlet. We expect it to display zero because the DC meter will respond to the average voltage which in this case is zero. The DC voltmeter is not designed to be sensitive to changes of voltage which occur at the frequency (50-60Hz) of a typical AC source. Therefore, the only information we can get from such a measurement is the average voltage.

How then does one measure the instantaneous AC voltage? We need to store this value for long enough that a DC voltmeter will be able to read it. A way to do this is to connect a capacitor across the AC source. The capacitor will get charged and the instantaneous voltage across it will determine the instantaneous charge on the capacitor plates. When the capacitor is disconnected from the circuit, the charge and hence the voltage on the capacitor remains. This can be seen by joining the two terminals of the capacitor, whereby a spark is produced. This voltage can be measured by a DC voltmeter. The DC voltage measured across the capacitor is hence the instantaneous AC voltage of the original AC source. This DC voltage is the key variable measured in this experiment. The experiment can be repeated and a different voltage will be obtained each time! This randomly sampled voltage data can be used to construct the probability distribution of the voltage.

What information about the AC source is contained in this probability distribution? You will see that the probability distribution depends upon the type of AC waveform used. A triangular wave will give rise to a very different probability distribution as compared to a sine wave. Further, under certain circumstances you can reconstruct the waveform from the voltage probability distribution.

A.3 Experimental Setup

The experimental setup consists of a capacitor, a voltmeter, ac source and a double pole switch. The following steps needs to be followed to perform the experiment:

- Connect the capacitor and the dc voltmeter to an ac source.
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Figure A.1: Circuit Schematic

- Keep the switch in place to ensure that the dc voltmeter shows zero voltage.
- Press the switch to disconnect the ac source.
- Record the maximum voltage on the voltmeter.
- Again press the switch so that the ac source is connected.
- Repeat this process large number of times and record the voltmeter reading in each case.

The voltmeter is connected across the capacitor which is in turn connected to the source through the switch. The switch when pressed disconnects the source from the capacitor. At this instant the capacitor begins to discharge through the voltmeter. Upon pressing the switch a second time, the source is reconnected to the capacitor. The voltmeter remains connected across the capacitor throughout.

A.4 Procedure

The data you generate by performing the random sampling of the AC source will by used by you to study the following:

1. Plot the probability distribution of the mathematical function \( V = V_o \sin \omega t \) and compare it with the distribution observed from your plotted graph.
2. Study the effect of sampled data size on the probability distribution by plotting: (a) 100 observations
   (b) 500 observations
   and comparing the two plots.

3. Study the effect of bin size on the probability distribution by plotting the same set of data with different voltage bin sizes and compare the two plots.

4. Reconstruct the sinusoidal source waveform using the information recovered from the random data.

5. Plot the probability distributions of triangular wave, sawtooth wave and square wave and find out which is closest to a sine wave.

Your first task is to record an experimental dataset that samples the distribution of the voltage developed across the capacitor.

1. Make the connections according to the circuit diagram given in the Figure.

2. Switch on the power.

3. Using the toggle switch $S_1$ connect the capacitor to the AC source.

4. Toggle the switch $S_1$ to disconnect the capacitor from the AC source and connect it to the digital voltmeter. At this stage the voltmeter shows the instantaneous DC voltage across the capacitor. Record the maximum value shown on the meter. The voltage across the capacitor will decay slowly as it discharges through the voltmeter. We are not interested in this decay. If a peak reading voltmeter is connected, note down the final reading on the display. If a simple digital multimeter is connected then note down the maximum which appears on the display. The input impedance of the meter should be $\geq 1M\Omega$.

5. Press the switch again to reconnect the AC source. This completes one measurement cycle. To repeat the observation, at some stage the switch is pressed again and the maximum voltage developed across the capacitor is recorded. The experiment is repeated several times and a list of voltages is generated. This is your basic data set from which you will draw your conclusions. Record your observations directly in your lab notebooks (do not waste time in taking observations on rough sheets).
A.5 Analysis of Results

Plotting your data:
Construct a histogram of your data which represents the voltage probability distribution for different parameter values and can be used to reconstruct the corresponding AC waveform.

For purposes of illustration, let us assume that in a particular set the maximum and minimum voltages measured are 8.4V and -8.4V. Take V along the x-axis. Divide the whole voltage range into a number of bins as follows: Let 1V = 1cm. Using this scale, create bins of width 1V from -8.5V to +8.5V. Put each data point into its respective bin i.e. the reading 4.8V will be marked in the bin between 4.5V and 5.5V. The data will be arranged as on the graph shown. The next stage of the experiment is to carry out a statistical analysis of your data using a computer program (the C program to help you do this is given at the end of this manual; if you wish you may write your own program), to bring out the quantitative statistical aspects of the experimental data.

The final stage of this experiment, is to recreate the source waveform, using a function generator and a non-sinusoidal input waveform (say triangular waveforms and sawtooth waveforms). Record the voltage readings using the same procedure, construct the data histogram, and use it to reconstruct the waveforms.
A.6 Questions to Explore

- Figure out how a DPDT switch works and how it is different from a SPST switch. Can you think of other circuits where DPDT switches are useful.

- Can you think of ways of improving the experiment design - for example, in this experiment you have taken the maximum voltage to be the instantaneous voltage across the capacitor. This is an approximation and true only for an ideal voltmeter. For a real instrument, there is always a finite measurement time over which the voltage across the voltmeter builds up from zero. Can you think of how to accurately measure the voltage across the capacitor (perhaps by using a sensitive ballistic galvanometer to measure the charge accumulated on the capacitor).
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