Chapter 3

INTRINSIC SHAPE: METHODOLOGY

In this chapter and in the next one, we shall describe the application of the mass models to find the intrinsic shapes of elliptical galaxies. In chapter 3, we shall apply analytical models, described in sec 3.3.4 to find out the parameters which are best constrained. These parameters will be called shape parameters. In chapter 4, we shall develop ensembles of triaxial mass models and apply these to find the possible values of the shape parameters for selected elliptical galaxies.

3.1 Earlier shape estimates

As mentioned in chapter 2, the elliptical galaxies are triaxial systems possibly with cusp at their center. The variation in ellipticity and the position angle twist are produced when a triaxial model, which is more general than Stark model, is projected at some viewing angles. The triaxial system has three axes $a \geq b \geq c$ and the aim of the intrinsic shape determination is to find the ratios $p = \frac{b}{a}$, $q = \frac{c}{a}$ and the viewing angles $(\beta', \phi')$.

There are two different approaches. In the first, one attempts to find a distribution of the intrinsic shape from a distribution of the projected property (Ryden 1992). In the second approach are attempts to find shape of each individual galaxy (Binney, 1985, Tenjes et al 1993, Statler 1994b,c).

In this chapter, we shall first describe the work done by various works. In the later part of this chapter, we shall present our investigation which are based on the methodology by Statler (1994b). Although, our shape estimates are based on the methodology of Statler, we present the necessary alterations.
to this methodology to suit our requirements. We also present the works of Tenjes et al. (1993) and of Thakur and Chakraborty (2001) in brief. The models of Tenjes et al. (1993) are an extension of the kinematical model of Binney (1985). The methodology adopted by Thakur and Chakraborty (2001) is similar to Statler method, but using photometric data only.

The work of Ryden (1992) is slightly out of the main theme of our present thesis, but we describe it in short. Use of photometric data alone by Ryden, is criticised by Statler, who rederives Ryden's plot to conclude that photometry alone can not determine shape (Statler, 1994a). We considered this issue seriously, and found that variation in shape can be estimated by photometry alone.

We present our numerical experiments to find the parameters which are best constrained, in this chapter. This information is utilized to constrain the shape of some selected galaxies which is presented in chapter 4.

3.1.1 Ryden's works

Ryden (1992) computed the luminosity weighted axis ratios for a sample 171 elliptical galaxies, which is observed by Djorgovski (1985a). According to her, the elliptical galaxies are triaxial ellipsoids with axis ratios $1 : p : q$ and that the distribution of intrinsic axis ratios $f(p, q)$ has an isotropic Gaussian form, with a peak at the position $p_0, q_0$ and the width $\sigma_0$.

She was more interested in three dimensional luminosity density rather then projected surface brightness. She observed position angle twists and minor axis rotation supporting for the case of triaxiality, and galaxy rotate in axis of symmetry in case of axisymmetry. She used same formula of Binney (1985) for fitting an intrinsic shape distribution. She test the hypothesis that elliptical galaxies are triaxial ellipsoids viewed from randomly chosen angles, and that the axis ratios of ellipsoids are distributed according to the Gaussian function,

$$f(p, q) \propto \exp\left[-\frac{(p - p_0)^2 + (q - q_0)^2}{2\sigma_0^2}\right]$$

with the constraint $1 \geq p \geq q \geq 0$.

The distribution of cluster flattening is obviously very different from the distribution of galaxy flattening.

If elliptical galaxies rotate around their shortest principal axis. This distribution of shape can not explain the observed frequency of galaxies with misalignments of $\sim 90^\circ$ between their rotation axes and apparent minor. Some elliptical galaxies must rotate around their longest principal axes best fitted by a distribution $f(p, q)$ with $\sigma_0 = 1.162, \beta = 0.67$ and $\gamma_0 = 0.45$. 
3.1. EARLIER SHAPE ESTIMATES

3.1.2 Binney’s work

Unlike the work done by Ryden (1992), who used photometry only, Binney (1985), used kinematical models of elliptical galaxies, to show that elliptical galaxies are triaxial, and that the rotational velocities along their apparent major axes must be associated with similar motions along their apparent minor axes. He analyzed the observations for 10 galaxies.

It is assumed by Binney that, the velocities field in any galaxy is made up of an overall figure rotation about either the shortest or the longest principal axis, together with internal streaming around that axis. The internal streaming velocities are assumed to be tangential to the iso-density surfaces.

Binney assumed that the galaxy’s structure is time-independent in a suitably rotating frame of reference. Let this frame rotate with angular velocity \( \dot{\omega} \), and let \( \mathbf{u}(r) \) be the velocity field in the rotating frame. Then the velocity field in observers (inertial) frame is

\[
\mathbf{v}(r) = \mathbf{u}(r) \dot{\omega} \times \mathbf{r}.
\]

(3.2)

The law of conservation implies

\[
\nabla \cdot (\rho \mathbf{v}) = 0,
\]

(3.3)

where the \( \rho \) is the luminosity density. It is also assumed that

\[
\dot{\omega} \cdot \mathbf{u} = 0
\]

(3.4)

i.e, the velocity field is orthogonal to rotation \( \dot{\omega} \).

The most general solution of (3.3) and (3.4) is

\[
\mathbf{u} = \frac{1}{\rho} \nabla \eta \times \dot{\omega}
\]

(3.5)

where \( \mathbf{\hat{\omega}} \) is the unit vector parallel to \( \dot{\omega} \) and \( \eta(x, y, z) \) is an arbitrary function.

Binney considers:

case(i) Tumbling bar where \( \dot{\omega} \) is parallel to \( z \) axis (shortest axis) and

case(ii) spindle galaxy where \( \dot{\omega} \) is parallel to \( x \) axis (longest axis). Binney consider modified Hubble model to discribe \( \rho \) (equation 2.1) and a form of \( \eta \) which is close to the form of \( \rho \).

\[
\eta = L_0 v_0 (a_0 + r^2)^{-1},
\]

(3.6)

where \( L_0, v_0 \) and \( a_0 \) are constants. It is quite interesting to note that the assumed form of \( \eta \) give a velocity field \( \mathbf{u}(r) \), for both case (i) and (ii), which can be projected analytically.

Analysing the projected velocity field of ten selected elliptical galaxy, Binney concluded that a typical elliptical galaxy is probably far from being axisymmetric, but rather has a middle axis length as at equal to the average of lengths of the longest and the shortest axes.
3.1.3 Tenjes’ work

The model of Binney was further modified by Tenjes et al (1993). They assumed that no figure rotation is present, and the kinematic misalignment, i.e. the angle $\psi$ between the shortest axis and the angular momentum is intermediate between 0 and $\frac{\pi}{2}$. Then the velocity field can be written in the form

$$\vec{v}(r) = \frac{-2v_0}{(a^2 + r_0^2)} \frac{1}{2} \left( \frac{y}{p^2} \cos \psi, \frac{z}{q^2} \sin \psi - \cos \psi - \frac{y}{p^2} \sin \psi \right)$$

The co-ordinates $(x', y', z')$ are defined in such a way that $z'$ is oriented to the observer and $x', y'$ be on the plane of the sky. The line of velocity $\vec{v}$ at a point $(x', y')$ on the plane of the sky is given by

$$\nu(x', y') = \frac{1}{\Sigma(x', y')} \int_{-\infty}^{\infty} \rho v_x z' dz'$$

where the surface brightness $\Sigma$ is given by (e.g., Stark 1977; Binney 1985)

$$\Sigma(x', y') = \frac{2L_0}{\sqrt{f}} \int_{-\infty}^{\infty} (z'^2 + a_r^2 + r_0^2)^{-\frac{3}{2}} dz'$$

and $z'$ is unit vector along $z'$

Tenjes et al (1993) analysed the intrinsic shape of there elliptical galaxies NGC 5989, NGC 1947, and NGC 1052. The shape is presented a plot in $(p, q)$ plane (luminosity model used is modified Hubble model with constant axial ratios $p$ and $q$).

3.1.4 Thakur and Chakraborty’s shape estimations

Thakur and Chakraborty (2001) used a family of mass model, which is generalization of a modified Hubble model along with triaxial model (2.1) which are triaxial version of $\gamma$ - models. These are referred to by these as models $A$ and $B$. They obtained triaxial density function $\rho$ with adding two terms, each one of these is a radial function multiplied by a spherical harmonic of low order, to the spherical modified Hubble density. They projected the mass model along a line of sight. The projected surface density $\Sigma$ is calculated analytically and there by, making it possible to investigate some of the projected properties analytically. They calculated the profile of surface density $\Sigma$, the axis-ratio $\frac{b}{a}$ and the position angles $\Theta$ of the major axis as function of radial distance, which has can be compared with photometric data of real galaxies.
The density distribution of the triaxial modified Hubble model has the form (2.46) as given in chapter 2, with

\[ f(r) = \frac{M}{\pi} \frac{1}{(b_0^2 + r^2)^{3/2}}, \]

\[ g(r) = \frac{3M b_1^3 2r^4 + 7b_2^2 r^2}{4\pi b_0^3 (b_0^2 + r^2)^{3/2}}, \]

\[ h(r) = \frac{3M b_3^3 2r^4 + 7b_4^2 r^2}{4\pi b_0^3 (b_0^2 + r^2)^{3/2}}, \]  

(3.10)

where \( b_0 \) is the scale length, and \( b_1, b_2, b_3, b_4 \) and \( M \) are constants. The four ratio \( (\frac{b_1}{b_0}), \ldots, (\frac{b_4}{b_0}) \) can be expressed in terms of the axis ratio of the density distribution at large and small radii where the constant surfaces are approximately ellipsoidal, the value the axis ratios of constant \( \rho \) surfaces at very large and very small radii as \( (p_\infty, q_\infty) \) and \( (p_0, q_0) \) respectively. In above, \( f(r) \) is the modified Hubble density and \( g(r) \) and \( h(r) \) are two radial functions, which were first suggested by Lees and Schwarzschild (1979). The cumulative mass of the model diverges, (cf section 2.1.1) and \( M \) is related to mass, but does not give the total mass.

\( b_0 \) should be found by relating it to effective \( R_e \) which is calculated by least square fit method (Chakraborty and Thakur, 2001). Further, the effective radius for triaxial models also depend on viewing angle, as well as, on axial ratios (de Zeeuw and Carollo, 1996) and for the triaxial modified Hubble model, the change in \( R_e \) with viewing angles and axial ratios is more prominent than for corresponding \( \gamma \) - model (Chakraborty and Thakur, 2001). \( R_e \) should be carefully computed for the application of the models.

It is interesting to note that triaxial modified Hubble model, can be projected analytically \( (r- integrals of f, g \) and \( h \) can be calculated analytically, where the similar \( r- integrals in \gamma \) - model have to be calculated numerically), and an analytical expression of the projected density \( \Sigma \) can be written (Chakraborty and Thakur, 2000). Thus, the ellipticity and the position angles of the elliptical isophotes of the projected density can be calculated analytically.

Thakur and Chakraborty (2001), adopt the methodology of Statler (1996 b) and, restrain themselves to a subclass of models \( A \) and \( B \) in which \( p_0 = p_\infty \) and \( q_0 = q_\infty \). Clearly, these conditions need not hold for real galaxies.

Adopting above models \( A \) and \( B \) with the condition as mentioned, these authors report successful shape estimates of synthetic galaxies, wherein a particular model is treated as a galaxy. Therefore, the shape is known. Numerical experiments, using Statler methodology are found to reproduce
this shape. The shape estimates of prolate and an oblate synthetic galaxies are presented in fig 3.1.

3.2 Statler’s work

Statler (1994 b) uses Bayesian statistics to obtain the shape of galaxies, using methodology as described in Statler 1994 b. Statler concludes that the shape of galaxies can not be determined from photometry alone and kinematical data is required.

Statler is not satisfied with the Ryden’s result. According to Statler, Ryden’s solution is not unique, Statler derived a plot with the same data of Ryden’s which is shown in fig(3.2) with using the modified lucy’s method described by Statler (1994a), which is different from Ryden’s plot.

3.2.1 Statler’s methodology, MPD

Statler (1994 b) uses kinemetical models, as described in Statler (1994 a) and uses methodology based on Bayesian Statistics. We outline below the main points of this methodology. Denoting a set of observed parameters by \( p^a \) \((i = 1, 2, \ldots, N)\) and the same parameters as calculated from a model in a given orientation by \( p^a_{\text{calc}} \) \((i = 1, 2, \ldots, N)\), the likelihood of obtaining the observed data from the model is given by

\[
L(p_{\text{obs}} | p_{\text{calc}}) = \prod_{i=1}^{N} \frac{1}{(2\pi \sigma_i)^{-1/2}} \exp \left[ - \sum_{i=1}^{N} \frac{(p^i_{\text{obs}} - p^i_{\text{calc}})^2}{2\sigma_i^2} \right],
\]

wherein \( \sigma_i \) is the error in \( p^i_{\text{obs}} \). The likelihood is a function of the intrinsic parameters and the viewing angles \((\theta', \phi')\) of the model. The probability (posterior density) \( P \) of obtaining observed data is the product of likelihood and the parent distribution (prior density) \( F \). We consider a flat parent distribution. It is necessary that the likelihood is a sharply peaked function, so that the probability is relatively insensitive to the unknown parent distribution. This is called ‘likelihood-dominated’ posterior density.

We integrate the posterior density over the uninteresting viewing angles to obtain marginal posterior density (MPD). In addition to the shape parameters, a model has other intrinsic parameters. One includes varieties of models (a single ensemble cannot be assumed to depict the true state of the galaxy) and treats these other parameters as ‘uninteresting’, which should be integrated out just like the orientation angles. However, virtually nothing the known about the parent distribution of these parameters. It is assumed
that the likelihood is sufficiently smooth function of these parameters so that the integral can be approximated by unweighted sum over ensembles. The resultant posterior density is complete shape estimate.

Marginal posterior density (MPD) $P$ as a function of the shape parameters may be plotted. Then, the criterion that $P$ is likelihood-dominated can be inspecting the region bounded by a constant $P$ contour, enclosing 68% of the total probability. This region can be interpreted as a $1\sigma$ 'error bar'. Likewise, a region enclosing 95% of the total probability defines a $2\sigma$ 'error bar'. The smaller these areas are, the higher is the domination of likelihood in $P$.

### 3.2.2 HPD

It is clear that the distribution $P$ as a function of appropriate shape parameters $r_i$ constitutes the inference of the galaxy's shape from the data, and not any single number distilled from it. $P$ cannot be expected to be Gaussian and therefore will not in general, be characterizable by a few simple parameters.

The statistical summary of the shape of $P(r_i)$ is desirable, it is to delineate highest posterior density. (HPD) regions, which are regions bounded by iso-probability contours that enclose some fixed fraction of the integrated probability. The 68% HPD region is the region D bounded by a contour $P=\text{constant}$ such that,

$$\int_D P(r_i)dr_i = 0.68. \quad (3.12)$$

Similarly, the 95% HPD region would enclose 0.95 of the total probability. The HPD regions can reasonably be interpreted as $1\sigma$ and $2\sigma$ error bars. We draw the contour, which enclose the region of 68% highest posterior density (HPD). We find that HPD region is small compared to the total space of the shape parameters, satisfying the criteria of a likelihood-dominated shape estimate.

### 3.3 Our shape estimates

Having described the shape estimations of elliptical galaxies by various authors, we now proceed to describe the our shape estimates using photometric models, described in chapter 2.

In our investigation,

(i) we use the methodology based on Bayesian statistics, as developed by Statler (1994 b), with necessary alterations to suit our requirements and

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Figure 3.1: Figures (a) - (d) present shape of test galaxies, estimated by using models B and A. In each figure, the darker shade indicates higher posterior density and a white cross shows the true shape of 'galaxy'. Inner and outer contours represent $1\sigma$ and $2\sigma$ error bar region respectively.
3.3. OUR SHAPE ESTIMATES

(ii) we use photometric models and photometric data on ellipticity and position angles of selected galaxies.

Photometric models are applied successfully by Ryden (1992), and Thakur and Chakraborty (2001) for shape estimates. It was also demonstrated by Pasano (1995) that photometry alone can constrain the shape.

Although, use of photometry alone to estimate shape was criticised by Staller (1994,b also see, section 3.2 and fig 3.3), he himself realized later (Bak and Statler, 2000) that some partial informations of the shape, namely, the flattening can extracted out from photometry. We, in our investigation, have found that the variation of the shape, namely, the flattenings at small and at large radii, and the absolute value of the difference of triaxiality at these radii, can be constrained by using photometry alone.

We shall present our investigation in the remaining part of this chapter, as well as, in chapter 4.

3.3.1 Models

We use the triaxial $\gamma$ - models of deZeeuw and Carollo (1996) (here after $fgh$ models) and $M^2$ models of Chakraborty (2004), which are triaxial generalization of Dehnen’s (1993) $\gamma$ - models. We consider $\gamma = 1.5$, and so, the models have cusp at their center. The $fgh$ model is completely fixed, the values of axial ratios $(p_0, q_0)$ at small radii and $(p_\infty, q_\infty)$ at large radii are chosen. The $M^2$ models as described in chapter 2, require the choice of values of axial ratios at some intermediate radius, in addition to the values at small and large radii. In order to bring the $M^2$ model, at the same footing, as the $fgh$ model, we use a 2-point $M^2$ model, as discussed below.

3.3.2 2-point $M^2$ model

Here, we set the varying axial ratios $P$ and $Q$ by

$$P^{-2} = \frac{b^2 p_0^{-2} + M^2 p_2^{-2}}{b^2 + M^2}$$  \hspace{1cm} (3.13)

and a similar expression for $Q^{-2}$ in terms of $(q_0, q_2)$. $M^2$ is the square of the ellipsoidal radius, defined by

$$M^2 = x^2 + \frac{y^2}{p^2} + \frac{z^2}{Q^2}$$  \hspace{1cm} (3.14)
Putting (3.14) in (3.13), we obtain

\[ M^4 + M^2(b^2 - m_i^2) - b^2m_0^2 = 0 \]  

(3.15)

where

\[ m_{2i}^2 = x^2 + \frac{y^2}{p_{2i}^2} + \frac{z^2}{q_{2i}^2} \]  

(3.16)

\( (i=0,1) \). Equation (3.13) and (3.15) are the 2-point \( M^2 \) analogue of the more general \( M^3 \) model [equations (3.65) and (3.66)] reported in chapter 2.

It is clear that \((p_0, q_0)\) are axial ratios at small radii and \((p_2, q_2)\) are axial ratios at large radii. Solution of equation (3.15) is simple and is

\[ M^2 = \frac{m^2 - b^2 \pm \sqrt{(m^2 - b^2)^2 + 4b^2m_0^2}}{2} \]  

(3.17)

where we have use \( m \) for \( m_2 \). We also use \((p, q)\) to denote \((p_2, q_2)\) as values at large radii. We retain the positive sign in square root in equation (3.17) to as certain that \( M^2 \) is positive. We also note that, in case, \( m = m_0 \), equation (3.17) yields \( M = m \), which is expected. Puting the solution of (3.17) for \( M \) in (2.72), we obtain the required two point \( M^2 \) model.

Projection of 2-point \( M^2 \) model is performed by adopting the procedure as described in chapter 2, making a straight forward changes appropriate to the 2-point model. Projected properties such as ellipticity and position angles are calculated. We find that the variation in ellipticity and in position angle is largely monotonic, over a wide range of selection of parameters \((p_0, q_0, p, q)\) and viewing angles \((\theta', \phi')\).

The monotonic variation in \( \epsilon \) and in \( \Theta_* \) are usually not true for a more general 3-point \( M^2 \) model and a variety of nonmonotonic profiles of projected parameters are generated.

We adopt 2-point \( M^2 \) model because

1. it has same footing as \( fg h \), namely, that both these models are four parameters \((p_0, q_0, p_0, q_0)\) formula.
2. Our shape parameters are found to be axial ratios \((p_0, q_0, p, q)\) and so, four parameters models are more suitable. Note that the shape parameters which are finally selected are \( q_0, q_0, \) and \( |T_d| \) where \( T_d \) is the triaxiality difference \( T_{\infty} - T_0 \). We shall define triaxiality for our models later.

As the models produce monotonic projected parameters, we should select only those galaxies which also exhibit monotonic profiles of \( \epsilon \) and \( \Theta_* \), largely.
3.3 OUR SHAPE ESTIMATES

3.3.3 Shape parameters

It is necessary to find the parameters which are best constrained. These parameters will be used to describe intrinsic shape, and will be called as shape parameters. These are not known, a priori (Statler and Fry 1994) and suitable numerical investigation are needed for this. One method is to find correlations between the projected parameters when a model with a chosen set of intrinsic parameters is projected in all viewing angles. This method is adopted in Statler and Fry, 1994. Thakur and Chakraborty (2001) also find such correlation plots.

We adopt a more direct method. The intrinsic parameters of our models are \((q_0, q_\infty, p_0, p_\infty)\). We take any two of them, calculate MPD by summing over the remaining two parameters, to find those parameters which are best constrained. These parameters can be used as shape parameters.

We present below our results of such numerical experiments.

3.3.4 Analytical models

Consider a \(M^2\) model. When such a model is projected, the line of sight at large \(R\) pierces through the model only at large radii (see, fig 2.1) and therefore, it collects light from the region where the axial ratios are \((p_\infty, q_\infty)\). The model will behave like a Stark model and the values of \((\frac{b}{a})_\infty\) and \(\Theta_c\) will be given by equation (2.40) (2.41) (2.42) with \(p \to p_\infty, q \to q_\infty\). On the other hand, the line of sight at small \(R\) will pass through the central region, as well as the other region of the model. So, it collects light from different regions and the axial ratios \((P, Q)\) are varying in these regions. However, because the center has cusp, the light from the central region dominates over the light from other regions. The result will be that it will again behave like a Stark model with \((p_0, q_0)\) as the axial ratios. Equations (2.40) to (2.42) can again be used for calculating \((\frac{b}{a})_0\) and \((\Theta_0)\). The use of analytic equation to calculate projected parameters saves a considerable computer time.

For the cuspy \(fgh\) models the analytical formulas to calculate the projected parameters are given in deZeeuw and Carollo (1996), which are reproduced below for easy reference. For \(\gamma > 1\), the projected surface density has a cusp at small radii. In this case, the axial ratios \((\frac{b}{a})_0\) at small radii is given by

\[
(\frac{b}{a})_0^{-1} = \frac{d_0 - \Delta_0}{d_0 + \Delta_0}
\]

where

\[
d_0 = 1 + tsin^2\theta cos^2\phi + p_0^2(1 + t sin^2\theta sin^2\phi) + q_0(1 + t cos^2\theta)
\]
Figure 3.2: Intrinsic shape distributed for elliptical galaxies in the \((T, c/a)\) plane derived from photometry alone. Ryden’s (1992) best fit distribution, assumed Gaussian in axis ratio space. Here, \(T\) is the triaxiality and \((c,a)\) are short and long axes lengths.
3.4. NUMERICAL PROCEDURE

\[ \Delta_0^2 = [(1 - q_0^2)h_1 + (p_0^q - q_0^2)h_2]^2 + (1 - p_0^q)^2 h_3^2, \]
\[ \text{where } t = \frac{3 - \gamma}{1 - \gamma}, \quad \text{and } h_1, h_2, h_3 \text{ are given in (2.58)}. \]

At large radii, the axial ratio \( (\frac{b}{a})_{\infty} \) is given by (3.18) with \( \gamma = 4 \) i.e., \( t = \frac{-1}{3} \) and replacing \( (p_0^q, q_0^q) \) by \( (p_{\infty}^q, q_{\infty}^q) \). Position angle \( \Theta_\star \) in \( fgh \) model is already in an analytical form and are given by (2.57)(2.61).

\( M^2 \) and \( fgh \) models under above conditions will be called analytical models because projected properties can be calculated analytically. Using analytical models for shape estimates is not fully justified. We use analytical models here, which give the projected parameters of \( b \) and \( q_\star \) at asymptotic distances. Comparing these with the observed data at finite \( R \) are not justified. The results presented in this chapter, using analytical models, can at the most be regarded only as an approximate. However this provides important informations regarding the parameters which may be used as shape parameters, for the more refined exact treatment of shape estimate which is presented in the next chapter.

The numerical calculation of the shape estimates using an analytic model are fast and a large number of results presented in this chapter forms a prelude to the material of the next chapter.

3.4 Numerical procedure

We now describe our numerical procedure of calculating MPD (section 3.2.3) to find the shape, parameters.

3.4.1 Calculation of MPD

The algorithm is straightforward. We take a pair of the model parameters as the shape parameters. These may be either the axial ratios \( (q_0, q_{\infty}) \) at small and at large radii respectively or \( (T_0, T_{\infty}) \) which are the triaxialities at small and at large radii, respectively.

The \( M^2 \) and \( fgh \) models have four parameters which are \((q_0, q_{\infty}, p_0, p_{\infty})\), or equivalently \((q_0, q_{\infty}, T_0, T_{\infty})\). We shall adopt the later. \( T_0 \) is related to \( q_0 \) and \( p_0 \) and likewise, \( T_{\infty} \) is related to \( q_{\infty} \) and \( p_{\infty} \). For \( M^2 \) models we have

\[ T_0 = \frac{1 - p_0^2}{1 - q_0^2} \]

and

\[ T_{\infty} = \frac{1 - p_{\infty}^2}{1 - q_{\infty}^2} \]
for \( fgh \) models these relations are

\[
T_0 = \frac{1 - p_0^7}{1 - q_0^7}
\]

and

\[
T_{\infty} = \frac{1 - p_{\infty}^4}{1 - q_{\infty}^4}
\]

From these a parameters of a model we pick up 2 parameters as the shape parameters.

The range of axial ratios \((q_0, q_{\infty})\) are considered to go from 0.5 to 1.0 (Elliptical galaxies flatter than 0.5 or 0.4 have not been observed), and the range of triaxiality are considered to go from 0.0 to 0.1. The value zero of the triaxiality describe oblate shape while the value one describes a prolate shape. Triaxiality of 0.5 is referred to as the maximally triaxial shape. Further we shall call the shape as rounder for axial ratio close to 1 and as flatter for axial ratio close to 0.5. The mid value 0.75 of the axial ratio will be regarded as the boundary separating rounder and flatter shapes.

We divide the entire parameter space, into \( n^2 \) rectangular grids of equal size. We calculate the MPD (section 3.2) at the center of each grid by following the procedure outlined in section 3.2. Thus generate \( n^2 = N \) data describing MPD as a function of the observed parameters choses. The sequence of the data has to be maintained, so that it is possible to trace the grid, corresponding to any data. We calculate these in the main program, of the code as developed by us in fortran77.

### 3.4.2 Calculation of \( 1\sigma \) level or HPD

We develop a numerical, code to calculate \( 1\sigma \) level. Here, again the algorithm is straight forward. We make a copy of \( N \) data generated in the main program and bring it to a subroutine. Here we reshuffle the data to arrange it in descending order. Again the algorithm in simple.

step 1. set \( i = 1 \)
step 2. pick up the highest data
step 3. store it in a new array as \( i^{th} \) data
step 4. Make a new set of \((N-1)\) data by leaving out the highest
step 5. increment \( i \) by unity
step 6. go to step 2, if \( i > 2 \), or else return to main.

The above reshuffling algorithm is different from the bubble sorting, but we adopted the above, as it is more transparent.
3.5. \textit{RESULTS OF THE SHAPE ESTIMATES USING ANALYTICAL MODELS.}\

To calculate $1\sigma$ level, we find the sum ($s$) of the data. This is calculated using the original data set. We now start summing the reshuffled set which is in descending order, starting from the top i.e., from $i = 1$ compare it with $s$ and stop summing the reshuffled data at $i = j$ when the cumulative sum just exceeds $s$. The $j^{th}$ data is the $1\sigma$ level, which is returned to the main.

3.4.3 To plot MPD as a function of shape parameter

As the shape estimates presented here are only approximate, we also develop a simple algorithm to plot MPD as a function of shape parameters. We compare each of the $N$ original data in main against the $j^{th}$ data corresponding to $1\sigma$ level, as returned from the subroutine, and reassign the value of the data as 1 if it exceeds $1\sigma$ level and the value 0 it falls short of $1\sigma$. The set of $N$ data is now sequence of 'ones' and 'zeros'. We print this as two dimensional array, as shown in plots (fig 3.3 to fig 3.12). Here, the region enclosed within 'ones' gives $1\sigma$ region or HPD region i.e., the value of the shape parameters which would give MPD within $1\sigma$ error bar. Number 5 (chosen arbitrarily) represents the location of the highest probability.

The important information which is extracted from these plots are the shape parameters which are best constrained. The conditions are that the HPD region should not extend from the lowest to the highest allowed values of any shape parameters. Further, area within $1\sigma$ area should be small (roughly $\frac{1}{3}$ or less of the total parametric space, Statler, 1994b). This will guarantee a likelihood dominated shape which is an important condition of the reliability of the shape estimates by the statistical method adopted.

However, we note that the plots (fig 3.3 to fig 3.12) which give for results of shape estimates are little crude in the following sense. It does not give the relative probabilities at various values of the shape parameters. Within $1\sigma$ area itself, a particular choices of shape parameter may be more probable than the other. This is not exhibited here. In the next chapter we shall present MPD maps in dark gray shade, wherein it is possible to extract out this piece of information.

3.5 \textit{Results of the shape estimates using analytical models.}\

We perform several numerical experiments using the observed data on ellipticity and position angle of NGC 3379. The observed data are taken from Peletier et al (1990) in R-band, and are as follows.
We took ellipticity $c = 0.078$ at $R = R_{in} = 15.7$ arcsec and $c = 0.133$ at $R = R_{out} = 49.3$ arcsec. Position angles are not used directly for the likelihood calculation because it is measured with respect to north in sky while orientation of the model from the direction north can not be fixed. To avoid this, we take position angle difference between above chosen $R_{in}$ and $R_{out}$. For NGC 3379 the difference in position angle between the chosen $R_{in}$ and $R_{out}$ is very small, and we take it to be zero. These choices of ellipticity's and position angles are the same as adopted by Statler 1994 c. It should therefore be reasonable to compare our results with Statler's. This comparison is the subject of chapter 4.

### 3.5.1 Shape $q_0, q_{oo}$

First, we take $(q_0, q_{oo})$ on shape parameters. We consider a model with a chosen intrinsic parameters $(q_0, q_{oo})$. For each choice of $(q_0, q_{oo})$, we choose number of values of $(T_0, T_{oo})$ and project the resultant model ($\gamma = 1.5$) is chosen in some viewing angles $(\theta', \phi')$. We calculate the project properties of ellipticity and position angles by using appropriate analytical formula as mentioned in section 3.3.4. We calculate the likelihood by formula (3.11). We choose error in ellipticities at $R_{in}$ and at $R_{out}$ as 0.01, which is typical error in the observation (See, eg de Carvalho et al 1991, Penereiro et al 1994) and choose an error of 1° in position angle. again as a typical error in observation. This gives the error in position angle difference as $\sqrt{2}$.

We multiply the likelihood by prior density, which we take as 1 (Flat prior) and integrate over the un-interesting parameters $(\theta', \phi')$ and sum over (unweighted sum) over choices of $(T_0, T_{oo})$. These give MPD for each choices of $q_0, q_{oo}$. The result of MPD as a function of shape parameters $q_0, q_{oo}$ is presented in plots 1, 2. We choose $40 \times 40$ values of $q_0, q_{oo}$ spans the entire parameter space of $(q_0, q_{oo})$ and for each $q_0, q_{oo}$, we take $6 \times 6$ values of $(T_0, T_{oo})$ the covering range of $(T_0, T_{oo})$. The clock time for calculating $40 \times 40 = 1600$ MPD data is about 45 second on, a machine of 1.6 GHz.

Plots(1, 2) demonstrate that $q_0$ and $q_{oo}$ are constrained. HPD area cover a small part in the parameters space of parameters $q_0$ and $q_{oo}$. The maximum value of MPD indicated by $s$ is at $q_{0p} = 0.91$ and at $q_{ooP} = 0.83$ we have adopted analytical $fgh$ model for plot 1. The same with analytical $M^2$ model is plot (2), which give $q_{0p} = 0.90$ and $q_{ooP} = 0.84$. 
figure 3.3: Plot of MPD as a function of $(q_0, q_o)$. $q_0$ runs from 0.5 to 1 horizontally, while $q_o$ runs between the same value from bottom to top, using m2 model.
Figure 3.4: Same as Fig 3.3, using M^2 model
3.5. RESULTS OF THE SHAPE ESTIMATES USING ANALYTICAL MODELS

3.5.2 Shape $T_0, T_\infty$

To find shape $T_0, T_\infty$, points of we choose 40 x 40 $T_0, T_\infty$ and for each choice of $T_0, T_\infty$, we sum the MPD over several choices of $q_0$ and $q_\infty$. The procedure and the observational data used are the same as described in section (3.5.1). Using analytic $fgh$, model and analytic $M^2$ model, the results of shape estimate (plot of MPD as a function of shape parameters $T_0, T_\infty$) are plotted in plot 3 with $fgh$, and plot 4 $M^2$ modal.

Unlike the results with shape parameters $q_0$ and $q_\infty$, here we see that $T_0$ and $T_\infty$ are not constrained, 1σ area extends all the way from the lowest to the highest values of both $T_0$ and $T_\infty$.

We would like to rephrase the statement 'photometry alone can not constrain shape' (Statler 1994 a) by 'photometry alone can constrain flattening but not the triaxiality'. We will comment more or this in chapter 4.

3.5.3 Shape $q_\infty, T_\infty$

To explore the debate raised in the last section in more detail, we examine the shape $(q_\infty, T_\infty)$. We calculate MPD for each choice of $(q_\infty, T_\infty)$ summed over values of $(q_0, T_0)$. The result, using analytical $fgh$ and $M^2$ model is presented in plot 5,6.

We find results similar to shape estimates $(q_0, q_\infty)$ and $(T_0, T_\infty)$: $q_\infty$ is constrained but $T_\infty$ is not.

3.5.4 4- dimensional shape presentation:

Our 2- point $M^2$ models, and the $fgh$ models are 4 parameters family. The models have 4 parameters $(q_0, T_0, q_\infty, T_\infty)$. Plot (1), (2) present shape $(q_0, q_\infty)$ when MPD is summed over $(T_0, T_\infty)$, while plots (3), (4) present shape $(T_0, T_\infty)$ when MPD is summed over $(q_0, q_\infty)$. A 4- dimensional plot of MPD as a function of $(q_0, T_0, q_\infty, T_\infty)$ is presented in plot (7)

Here 64 sections and each of constant $(T_0, T_\infty)$ are arrange in a form of a two dimensional array, so that $(T_0$ runs from 0.0 to 1.0 from left to right, while $T_\infty$ runs between the same values from bottom to top of entire plot. Further, in each section $q_0$ runs from left to right from 0.5 to 1.0 and $q_\infty$ runs between the same values from the bottom to top. We have used $fgh$ model to draw this plot.

The result here also the same : $(T_0, T_\infty$ are not constrained (bunches of 'ones' are aligned across the entire plot along the diagonal), while $(q_0, q_\infty)$ are constrained in a small area in each section)
Figure 3.5: Plot of MPD as a function of \((T_0, T_\omega)\). \(T_0\) runs from 0.0 to 1.0 horizontally, while \(T_\omega\) runs between the same value from bottom to top, using fgh model.
figure 3.6: Same as fig 3.5, using m2 model
Figure 3.7: Plot of MPD as function of \((q_c, T_c)\). \(q_c\) runs horizontally, while \(T_c\) runs vertically, using fgh model.
Figure 3.8: Same as Fig 3.7, using m2 models, but here, $T_x$ runs horizontally while $q_x$ runs vertically.
CHAPTER 3. INTRINSIC SHAPE : METHODOLOGY

Plot (8), also exhibits $P$ as a function of 4 parameters $(q_0, T_0, q_\infty, T_\infty)$. The scales, of $(q_0, q_\infty)$ and of $(T_0, T_\infty)$ are interchanged here, with respect to those of the plot (7). Here section of constant $(q_0, q_\infty)$ are arrange in a two dimensional array such that $(q_0, q_\infty)$ run from 0.5 to 1.0 across the plot, while $(T_0, T_\infty)$ run from 0 to 1.0 across each section.

We use $fgh$ models for plot (8) also. Plot (9) and (10) are similar to Plots(7) and (8) but these use $M^2$ models.

3.5.5 Shape parameters:

Plots (3) and (4) indicate that $(T_0, T_\infty)$ are not constrained. However, the HPD, as shown by ‘ones’ lies predominantly along the diagonal $T_0, T_\infty$. This seems to indicate that although $T_0$ and $T_\infty$ individually are not constrained, but their difference $(T_\infty - T_0)$ is constrained. We consider $q_0, q_\infty$ and $T_d = T_\infty - T_0$ as possible shape parameter. We now use an ensemble of full $fgh$ and $M^2$ models and explore the possibility of shape estimates in terms of these shape parameters. This, we shall present in chapter 4.