Abstract

In this thesis, we present a study of triaxial mass modeling of elliptical galaxies. We present a large ensemble of triaxial models. We develop a methodology to use this ensemble of mass models to study the intrinsic shapes of elliptical galaxies.

In the morphological classification scheme of Hubble, an elliptical galaxy has a simple appearance. It appears as an ellipse on a photographic plate. It was quite natural to assume that the elliptical galaxies are gravitationally bound system of stars which are flattened due to rotation about some axis through the system. This simple picture of ellipticals has undergone drastic changes after the advent of CCD (Charge Couple Device) photometry and powerful image analysis facilities, such as IRAF(Image Reduction and Analysis Facility) around 1975. The principle results are outlined below.

Low rotational velocity rules out the possibility that the elliptical galaxies are axisymmetric systems with the flattening supported by rotations (Bartola and Capaccioli 1975; Illingworth 1977). It is natural to assume that a majority of elliptical galaxies are triaxial systems, with the pressure due to velocity anisotropy supporting their equilibrium shape and structure (de Zeeuw, 1993). A triaxial system has three axes, and the intrinsic shape determination is aimed at estimating their ratios.

Dynamical modeling of elliptical galaxy requires the knowledge of integrals of motion. Spherical and axisymmetric systems have energy and the components of the angular momentum as the integrals of motion. According to Jeans theorem, the distribution function, which is a function of the integrals of motion, determines the dynamical model. The integrals of motion are the functions of $\bar{z}$ and $\bar{v}$, and therefore, the distribution function itself is function of $(\bar{z}, \bar{v})$. Integration of the distribution function over $\bar{v}$ will give density distribution $\rho(\bar{r})$ of the galaxy. While the photometry studies, the profiles of the projected light, the spectrophotometry reveals the velocity of the galaxy along the line of sight, using the Doplers shift of the spectrum.

In the present thesis, we concentrate on mass models, with reasonable density distributions which may represent an elliptical galaxy. This pro-
posed density distributions is projected on the plane of the sky to obtain the two dimensional surface density. Assuming a constant mass to light ratio, we compare the projected density with the observed photometric data. Agreement between the observation and model provides a justification for the proposed mass models.

In case, the potential function corresponding to a mass model is known in an analytical form, it is straightforward to calculate the orbits by integrating the equation of motion. The orbit families can, in turn, be used to calculate the underlying distribution function numerically (Schwarzschild, 1979).

A triaxial mass model can be constructed in the following way. One considers a spherical model with density $\rho$ as a function of scalar $r$: $\rho = \rho(r)$ and replaces $r^2$ by $m^2$, where $m^2$ is the ellipsoidal radius given by

$$m^2 = x^2 + \frac{y^2}{p^2} + \frac{z^2}{q^2},$$

where $(x, y, z)$ are the Cartesian co-ordinate and $(p, q)$ are the axial ratio $(1 > p > q > 0)$.

Stark (1977) and Binney (1985), showed that for a distribution $\rho = \rho(m^2)$, the isophotes of the projected density are concentric and coaxial similar ellipses; if $p$ and $q$ are constants. Real galaxies however have approximate elliptical isophotes with ellipticity variation and position angle twist. We note that the result of Stark (1977) and Binney (1985) are analytical. The message from this result is clear: the density distribution of real galaxies is more general than that of Stark (1977) and Binney (1985).

Two different ways of constructing a density distribution of a real galaxies, which would show variation in ellipticities and position angle in projection, have been proposed in the literature. de Zeeuw and Carollo (1996) proposed a novel approach of producing a realistic triaxial model. They consider a spherical model $f(r)$ and make it triaxial by adding two more terms, each is a suitable radial function multiplied by the spherical harmonics of low order. The density $\rho$ is

$$\rho(r, \theta, \phi) = f(r) - g(r)Y_0^2(\theta) + h(r)Y_2^2(\theta, \phi)$$

where $(r, \theta, \phi)$ are the spherical polar co-ordinates and $g(r), h(r)$ are two suitably chosen radial functions, and $Y_0^2$ and $Y_2^2$ are the spherical harmonics, in standard notation. The potential function corresponding to the density distribution (2) is also presented in de Zeeuw and Carollo (1996).

Another approach of constructing a realistic triaxial model was proposed by Chakraborty (2004). He considers spherical mass model $\rho = \rho(r)$ and
replaces $r^2$ by $M^2$, where

$$M^2 = x^2 + \frac{y^2}{P^2} + \frac{z^2}{Q^2}$$  \hspace{1cm} (3)$$

where in $P$ and $Q$ are axial ratios which are not constants but are varying with $M^2$. Both the models (2) and (3) show isophotal twist and ellipticity variation in projection. These can be compared with observed profiles of ellipticity and position angle of a galaxy. These are presented in detail, in chapter 2.

We have modified the model (2) and (3) further, so that for a chosen set of parameters, which fix up the models, a variety of profiles of ellipticity and position angle are produced. To modify model (2), we have added two more terms, respectively to $g$ and $h$ which are multiplied by a positive constant number $a$. To modify (3), we have redefined the functional forms of $P$ and $Q$ using a constant number $\beta$.

In addition to producing variations in ellipticity and in position angle, a realistic model should also show a cusp at the center. Lauer (1985), using HST (Hubble Space Telescope) data, have shown that the surface brightness keeps on increasing without limit when the radial distance from the center decreases to zero. Further, it is also required that the total integrated light is finite. Dehnen's (1993) $\gamma$-models ($0 \leq \gamma < 3$ is a parameter) fulfill both these requirements. In our discussion of shape estimates, we have considered the triaxial generalization of Dehnen model, both in model (2) and (3). However, Dehnen models cannot be projected for a general $\gamma$, analytically and one has to resort to numerical integration. Here, complications arise because (i) the range of integration extends to infinity and (ii) a singularity in the integrand falls in the range of integration. By a suitable change of variables those complications can be circumvented. (Dehnen 1993, de Zeeuw and Carollo 1996)

A piece of analytical work has some value. In this connection, we are tempted to consider modified Hubble models. Projection of modified Hubble model can be performed analytically. A modified Hubble version of (2) has been investigated by Chakraborty and Thakur (2000) and many properties were calculated analytically. We continue to explore the traxial modified Hubble model in which $r^2$ is replaced by $M^2$, as given in (3) and study the properties which can be calculated analytically, in limiting case when $P$ and $Q$ are slowly varying.

Using such a large ensemble of photometric models, described above, we find the probability of the intrinsic shape of galaxies. We use Baysian Statistics. The main points of the methodology, which is presented by Statler
We calculate the likelihood of obtaining a set of observed data $p^i_{ob}(i = 1, 2, 3, ..., N)$ from a model which is given by

$$L = \prod_{i=1}^{N} (2\pi \sigma_i)^{-1/2} \exp\left[ -\sum_{i=1}^{N} \frac{(p^i_{ob} - p^i_{cal})^2}{2\sigma_i^2} \right]$$  \hspace{1cm} (4)$$

where $\sigma_i$ is the uncertainty in $p^i_{ob}$, and $p^i_{cal}(i = 1, 2, 3, 4, ..., N)$ is the same data as calculated from a model at some viewing angle. The posterior density is the product of likelihood with prior density. The marginal posterior density (the probability) (MPD) is obtained by integrating the posterior density over the viewing angles, which are generally unknown. A plot of MPD vs the shape of parameters are drawn, which describe the shape estimates. It is necessary that the MPD is a sharply peaked function, so that it is relatively insensitive to the unknown prior density. We use a flat prior. This is called the likelihood dominated MPD. We use a large ensemble of models, so that the shape estimates are model independent. Although the basic ingredients of our shape estimates are the same as in Statler 1994a, we make necessary alterations to suit our requirements.

A priori, it is not clear as to which parameter are best constrained. These can be ascertained by numerical experiments by choosing a variety of shape parameters. Alternatively, it can also be determined by studying the correlation plots between the observed parameters when a model with chosen parameters is viewed in all viewing angles. The co-relations are shown in Statler and Fry (1994). Thakur and Chakraborty (2001) have also presented co-relation plots between ellipticity at a large radii and the ellipticity difference when the model with choices of different intrinsic axial ratios are projected in different viewing angles.

We perform numerical experiments to find out set of parameters which are best constrained. We use ellipticities at two chosen points $R_{in}$ and $R_{out}$ respectively of a galaxy and the difference in the position angles at these points of the observed data. We calculate the MPD as outlined above, from a model. Plots of MPD reveal that the flattening ($q_0, q_\infty$) in inner and outer radii, respectively and the absolute value of triaxiality difference are the best constrained shape parameters. This part of our study is presented in chapter 3.

Having decided the parameters which are best constrained, we find the variations in the intrinsic shapes of 6 elliptical galaxies: NGC 1052, NGC 2986, NGC 3379, NGC 4374, NGC 5638 and NGC 7619. We compare the results with the available previous results of intrinsic shapes. This is presented in detail in chapter 4.
We continue to explore triaxial mass models and study a more general version of triaxial modified Hubble model and other non-Starkian models. Here, we find that the projected properties can be calculated analytically. This is presented in chapter 5.

Finally, in chapter 6, we present possible future avenues, as a natural extension of our present work.