Chapter Five

Growth and Distribution

The Cambridge Growth Model and the Role of Government

Introduction

In the previous chapters we examined at Keynesian, neo-classical and endogenous growth models and in the latter two the production function is used with decreasing as well as non-decreasing returns to scale. We noted that the Harrod-Domar and Solow models had little role for public policy. It was only with the beginnings of the new growth theory starting with Arrow (1962) and Arrow & Kurz (1970) that public expenditures and fiscal policy began to be explicitly modeled along with the neo-classical production function. Subsequent writings went on to study the impact of fiscal policy, deficit and public debt on growth of different nations. The role of distribution, however, remains a neglected aspect in neo-classical economics and dominant growth theory is silent on this aspect. Fortunately, at the core of another set of models, namely the Cambridge models, lies the issue of distribution in the Ricardian tradition. In the chapter below we provide an overview of these models with a special emphasis on the role of government in determining growth rate.

This chapter discusses the literature on growth and distribution under various assumptions and growing degree of complication. We begin with a presentation of the Kaldor (1956) model of distribution and examine how the results change when we adjust for accumulation of workers savings (Pasinetti 1962). We provide a brief
discussion on the controversy that arose following Pasinetti’s findings especially from Meade (1963) and Samuelson & Modigliani (1966) who questioned the Pasinetti process and its general validity. After that we examine the robustness of the Cambridge Theorem to the introduction of government (Steedman 1972) and the counter position of Fleck and Domingho (1987 & 1990) who argue that the introduction of government into the Cambridge growth equation leads to its breakdown.

Pasinetti (1989a & b) re-examines the role of the government and finds that the Theorem holds either when the deficit is monetised or under Ricardian Equivalence. However, two later contributions Denicola & Matteuzzi (1990) and Dalziel (1991) prove that the Cambridge Theorem with government holds under a variety of situations and Ricardian Equivalence is not necessary for its validity.

Early Writings

One of the earliest conceptualisations of a link between the rate of profit and the rate of growth is found in the writings of von Neuman (1945), first presented as early as 1932 in a seminar paper and Kaldor (1937). However, a formal distribution-based growth model not within the framework of marginal theory emerged in Kaldor (1956) and later writings from the so-called Cambridge school.

To set up the basic model we assume that two kinds of agents populate the economy. The national income is distributed between the workers (as wages W) and capitalists (as profit P).
1. \[ Y = W + P \]

The aggregate savings in the economy is given by

2. \[ S = S_w + S_c \]
where \( S_w = s_w \cdot W \) and \( S_c = s_c \cdot P \), and \( s_w \neq s_c \).

If long term stable growth is to emerge then it is assumed that the amount of investment (I) necessary to cope with population growth and technological progress is actually taking place. Therefore, there is macroeconomic equilibrium provided by

3. \[ I = S \]

4. \[ And \ I = S_w + S_c \]
\[ = s_w \cdot W + s_c \cdot P = s_w \cdot (Y - P) + s_c \cdot P \]
\[ = s_w \cdot Y + (s_c - s_w) \cdot P \]

Dividing equation 4 through by \( Y \) and re-arranging gives us:

5. \[ \frac{P}{Y} = \left[ \frac{1}{(s_c - s_w)} \right] \frac{I}{Y} - \left[ \frac{s_w}{(s_c - s_w)} \right] \]

Dividing all elements of equation 4 by \( K \) and re-arranging gives us:

6. \[ \frac{P}{K} = \left[ \frac{1}{(s_c - s_w)} \right] \frac{I}{K} - \left[ \frac{s_w}{(s_c - s_w)} \right] \frac{Y}{K} \]

The above equations (5 & 6) give us the distribution and the rate of profit in the system. The model holds only in the (Pasinetti) range of saving propensities as given by:

\[ s_w < I/Y \quad \text{and} \quad s_c > I/Y \]

1. The reasons for this are evident. If the first assumption is violated then profit-income ratio will go to zero and the economy would be chronically under-employed. If the second equation is violated then there would be over-heating of the economy and perpetual inflation would emerge.
In the classical political economy tradition if we assume that the workers do not save anything, i.e., their rate of saving $s_w = 0$, then equation 5 & 6 reduce to:

7. $P/Y = (1/s_c)I/Y$

8. $P/K = (1/s_c)I/K = r$ \hspace{1cm} where $r$ = rate of profit

These are dramatic results and equation 8 has to come to be identified as the Cambridge growth equation: Given, capitalists' saving propensities, growth is determined by the rate of profit in the system. The saving propensities of any other class of agents (in this case workers) has no influence on the rate of growth. The simplicity of this formulation and the impact of its result for economic theory are profound. These results are universal, especially because there are no restrictive assumptions necessary for the technological choices or production function (Kaldor 1956). However, Kaldor (1956) did not arrive at the generalised Cambridge growth result when the workers had positive savings, i.e., $s_w > 0$. This was left to Pasinetti (1962) who observed that even if workers had a positive saving they would not influence the rate of growth in the economy. If workers save in the system, then a portion of the profit would accrue to them in proportion to their saving as a return on capital owned by them. The distribution of incomes then must be distinguished not only as between profit and wages but also between capitalists earnings and workers earnings.

9. $P = P_c + P_w$

Therefore, in equilibrium
10. \[ I = S \]
   \[ = s_w(W + P_w) + s_cP_c \]
   \[ = s_wY + (s_c - s_w)P_c \]

Rearranging and dividing throughout by \( y \) and \( K \) we get as above:

11. \[ \frac{P_c}{Y} = \left[ \frac{1}{(s_c - s_w)} \right] \frac{1}{Y} - \left[ \frac{s_w}{(s_c - s_w)} \right] \]

Dividing through by \( K \) and re-arranging gives us:

12. \[ \frac{P_c}{K} = \left[ \frac{1}{(s_c - s_w)} \right] \frac{1}{K} - \left[ \frac{s_w}{(s_c - s_w)} \right] \frac{Y}{K} \]

The difference between equation 11 & 12 and their earlier cousins 5 & 6 is that the right hand side of the equation match but the left hand do not.

In the Pasinetti (1962) reformulation, in order to get the division of the social product between wages and profits, we need to undertake some changes from the Kaldor (1956) formulation. Since the workers save and earn a proportion of the profit, overtime they will also own a part of the capital.

13. \[ K = K_c + K_w \]

The distribution can now be written as:

14. \[ \frac{P}{Y} = \frac{P_c}{Y} + \frac{P_w}{Y} \]

15. \[ \frac{P}{K} = \frac{P_c}{K} + \frac{P_w}{K} \]

Since \( \frac{P_c}{K} \) is already known from equation 12 above, we only need to find the value of \( \frac{P_w}{K} \) in order to find the rate of profit \( \frac{P}{K} \). However, \( P = rK \) in a situation of dynamic equilibrium, where "\( r \)" is the equilibrium rate of interest. Therefore,

16. \[ \frac{P_w}{K} = rK_w \] and

\[ ^2 \text{This is a standard assumption in the Kaleckian tradition [see Kalecki 1938].} \]
17. \[ P_w/K = r.K_w/K, \]

The unknown expression in the equation 17 is \( K_w/K \). But again in long term equilibrium,

18. \[ K_w/K = S_w/S = s_w \left[ (Y - P)/I \right] = \left[ s_c.s_w/(s_c - s_w) \right] Y/I = s_w/(s_c - s_w) \]

By replacing the values of \( (K_w/K) \) as above into the equation 17 and ultimately equation 15 yields

19. \[ P/K = \left[ 1\left( s_c - s_w \right) \right].I/K - \left[ s_w/(s_c - s_w) \right].Y/K + r.\left[ \left\{ s_c.s_w/(s_c - s_w) \right\} \right] Y/I - s_w/(s_c - s_w) \]

Similarly,

20. \[ P/Y = \left[ 1/(s_c - s_w) \right].I/Y - \left[ s_w/(s_c - s_w) \right] + r.\left[ \left\{ s_c.s_w/(s_c - s_w) \right\} \right] K/I - \left[ s_w/(s_c - s_w) \right].K/Y \]

As Pasinetti rightly points out, before we arrive at a theory of distribution we need a prior theory of interest rates. The only legitimate assumption that can be made here is that the rate of interest in long run with competitive markets must equal the rate of profit (\( P/K \)). So in the above equations if we replace \( r = P/K \) then equations 19 and 20 reduce to:

19A. \[ P/K \left[ 1 - \left\{ s_c.s_w/(s_c - s_w) \right\} \right] Y/I - s_w/(s_c - s_w)] = \left[ 1/(s_c - s_w) \right].I/K - \left[ s_w/(s_c - s_w) \right].Y/K \]

19B. \[ P/K \left[ s_c \left( I- s_w Y \right) / I \right] = (I - s_w Y)/K \] assuming that \( (I - s_w Y) \neq 0 \).

19C. \[ P/K = (1/s_c).(I/K) \]

Similarly,

20A. \[ P/Y = (1/s_c).(I/Y) \]
This is the famous Independence Theorem where the rate of profit in the economy and share of profits in the social output is independent of the workers savings rate even when $s_w Y \neq 0$ [Pasinetti 1962: 272].

**Income source and Class**

The literature that follows seems to suggest that while Kaldor hinged saving behaviour as dependent on income source (wages or profits), Pasinetti hinged it on the class (workers or capitalists). Chiang (1973) shows that Pasinetti's model is really not an advance over Kaldor's model but another special case of a more generalised distribution model. To demonstrate this Chiang (1973) suggests that instead of two saving propensities, there should be three saving propensities as there three classes of incomes: workers' wages, workers' profits and capitalists profits with the proviso that:

\[ 1 \geq s_{pc} \geq s_{pw} \geq s_{ww} \geq 0. \]

Then,

21. \[ S = s_{pc} P_c + s_{pw} P_w + s_{ww} W \]

Let us add and subtract $s_{ww} P$ on the right hand side of 21 and substitute:

a) \[ W = Y - P \]

b) \[ P_w = P - P_c \]

c) Put $S = I$ in equilibrium, to get:

22. \[ S = s_{pc} P_c + s_{pw} P_w - s_{ww} P + s_{ww} (Y - P) + s_{ww} P = I \]

23. \[ I = s_{pc} P_c + s_{pw} (P - P_c) - s_{ww} P + s_{ww} (Y - P) + s_{ww} P \]
24. \[ I = spc \cdot P_e + spw \cdot (P_e - P_c) - sww \cdot P + sww \cdot Y - sww \cdot P + sww \cdot P \]

25. \[ I = (spc - spw) \cdot P_e + (spw - sww) \cdot P + sww \cdot Y - (sww \cdot P - sww \cdot P) \]

26. \[ I = (spc - spw) \cdot P_e + (spw - sww) \cdot P + sww \cdot Y \]

Divide by \( Y \) and \( K \) respectively to get:

27. \[ I/Y = (spc - spw) \cdot P_e/Y + (spw - sww) \cdot P/Y + sww \]

28. \[ I/K = (spc - spw) \cdot P_e/K + (spw - sww) \cdot P/K + sww \cdot Y/K \]

Having set up this construct, Chiang argues that both Pasinetti and Kaldor models are special cases of this generalised formulation of the model.

Case 1: If the saving propensity is income-dependent and not class-dependent, e.g. worker and capitalists save the same amount from incomes earned as profits, which are higher than the incomes earned as wages, then

\[ spc = spw > sww \]

Equations 27 and 28 then reduce to the Kaldor results:

29. \[ P/Y = [1/(sp_e - sw)] \cdot I/Y - [sw/(sp_e - sw)] \]

30. \[ P/K = [1/(sp_e - sw)] \cdot I/K - [sw/(sp_e - sw)] Y/K \]

Case 2: If, however, the saving propensity is class dependent and is inelastic to the source of income, then

\[ spc > spw = sww \]

and equations 27 and 28 reduce to the Pasinetti equations:

\[ P/Y = [1/(spw - sww)] \cdot I/Y - [(spc - spw)/(spw - sww)] \cdot P_e/Y - [sww/(spc - sww)] \]

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4. \[ P/Y = [1/(spw - sww)] \cdot I/Y - [(spc - spw)/(spw - sww)] \cdot P_e/Y - [sww/(spc - sww)] \]
31. \[ P_c/Y = \left[1/(s_c - s_w)\right]I/Y - \left[s_w/(s_c - s_w)\right]. \]

32. \[ P_c/K = \left[1/(s_c - s_w)\right]I/K - \left[s_w/(s_c - s_w)\right]Y/K \]

These lead to his Independence theorems:

33. \[ P/K = (1/s_c)(I/K) \]

34. \[ P/Y = (1/s_c)(I/Y) \]

Case 3: When all the saving propensities are the same: \( s_{pc} = s_{pw} = s_{ww} \), the distribution equations do not have any role to play. In such a situation we are looking at the neo-classical construct of the representative individual. The Kaldor-Pasinetti equations then are distributionally unimportant.

Case 4: This is the situation when Chiang deviates from both Kaldor and Pasinetti in assuming that all three saving propensities differ: \( s_{pc} > s_{pw} > s_{ww} \). However, if the Pasinetti assumptions are retained that (a) there is a single uniform rate of profit in dynamic equilibrium, and (b) proportion of capital of each class is directly proportional to the savings held by the class, then it can be proved that the independence theorems still hold.

The Pasinetti assumptions allow us to write:

\[
P/K = \left[1/(s_{pw} - s_{ww})\right]I/K - \left[(s_{pc} - s_{pw})(s_{pw} - s_{ww})\right]P_c/K - \left[s_{ww}/(s_{pw} - s_{ww})\right]Y/K
\]

\[
I/Y = (s_c - s_w)P_c/Y + s_w
\]

\[
I/K = (s_c - s_w)P_c/K + s_wY/K
\]

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35. \[ P_c = (P/K)K_c = P.(K_c/K), \] where

36. \[ K_c = S_c/S = s_{pc}P_c/I \]

Substituting equation 36 into 35 gives us:

37. \[ P = (1/s_{pc}).I \]

The profit rate and share of profits is:

38. \[ P/Y = (1/s_{pc}).I/Y \]

39. \[ P/K = (1/s_{pc}).I/K \]

Chiang’s formulation adds to the robustness of the Cambridge growth result. It also demonstrates that the Pasinetti and Kaldor contributions belong to a more general schema. Different results emerge from these two authors because of differing assumptions regarding agent behaviour vis-à-vis income-source and not necessarily error in conceptualisation.

**Government in the Cambridge Equation**

The findings of Kaldor (1956) led to different kinds of explorations. It started off with the neo-classical rebuttal of the Cambridge growth theorem with the Dual Theorem formulated by Meade (1963) and Samuelson-Modigliani (1966) where, on the long run growth path, the saving propensity of the capitalists plays no role but only that of the workers does. Meade’s fundamental result is that the distribution of property along the golden age path is independent of production and is solely dependent on worker’s saving propensity which is supposed to be the dual of the Kaldor-Pasinetti process.
The Dual theorem was found to have limited validity as it is crucially dependent on assumptions about technology – smooth “well-behaved” production functions are necessary for the Meade solution (Pasinetti 1966). If these are relaxed then the Dual theorem may not hold, unlike the Pasinetti solution that makes no such assumptions regarding its existence. Samuelson-Modigliani (1966) argued that if workers propensity to save was high enough (i.e. higher than the average savings ratio in the economy) then the Dual of the Pasinetti process would occur wherein the workers savings rate would determine the growth rate in the economy. The two class economy would cease to exist as it would reduce to a single class equilibrium growth path. However, there was very little attempt for a relatively long period to incorporate the role of government in understanding the distribution of income and rate of profit on the long run growth path. An important contribution came from Steedman (1972) who found that the Cambridge growth model holds even in the presence of government participation in the economy while the Meade results were not robust to the incorporation of government.\footnote{Interestingly, Steedman’s paper was examining the general validity of the Pasinetti solution in light of the challenge of Meade (1963) and Samuelson & Modigliani (1966). Meade suggested that instead of the rate of profit being independent of initial conditions, it was the capital output ratio which would be independent of technology. This was the dual which Steedman wanted to challenge and in the process introduced government activity to see which of the results were more robust -- the Pasinetti theorem or the anti-Pasinetti theorem and ruled in favour of the former.}
In our representation, the introduction of the government sector alters the National Income identity equation marginally. The new item that has been introduced here is government expenditure (G). Since,

\[ Y = C + I + G = W + P = W + p_c + p_w, \]

where \( G \) = Government expenditure

Let,

\[ G = (1 - s_G)T \]

where \( s_G \) = Government's saving propensity, \( T = \) Taxes

The tax function of the government allows for two types of direct taxes (income tax on wages and profits) and an indirect consumption tax:

\[ T = t_w \cdot W + t_p \cdot p_w + t_p \cdot p_c + t_p \cdot [(- s_w)(1 - t_w) \cdot W + (- s_c)(1 - t_p) \cdot p_c] + G \]

Substituting equation 42 into 43 for \( G \) we get:

\[ T = \alpha \cdot [t_w \cdot W + t_p \cdot p_w + t_p \cdot p_c + t_p \cdot [(- s_w)(1 - t_w) \cdot W + (- s_c)(1 - t_p) \cdot p_c]] \]

Where \( \alpha = \frac{1 - t_p(1 - s_G)}{1} \).

The saving function of the three category of savers will be:

46 Worker saving: \( S_w = s_w(W + P_w - t_w \cdot W - t_p \cdot p_w) \)

47 Capitalists’ Saving \( S_c = s_c(p_c - t_p \cdot p_c) \)

48 Government saving \( S_G = s_G \cdot T \)

Aggregate saving in the economy then is:

49 \( S = S_w + S_c + S_G \).
Appropriately expanding gives the savings function:

\[
S = \{s_w(1-t_w) + s_G \alpha (1-t_w)(1-s_w)\}.W + \\
\{s_w(1-t_p) + s_G \alpha (1-t_p)(1-s_w)\}.P_w + \\
\{s_c(1-t_p) + s_G \alpha (1-t_p)(1-s_c)\}.P_c
\]

**Balanced Budget case**

In the case of balanced budgets (Steedman model), \( G = T \), where \( s_G = 0 \).

It is interesting to note that indirect taxation has no effect on the level of savings and equation 50 above reduces to

\[
S = s_w(1-t_w).W + s_w(1-t_p).P_w + s_c(1-t_p).P_c
\]

In the long run, equilibrium is maintained if:

\[
S_w/K_w = S_c/K_c = S/K
\]

Using equations 46 & 47 (since in 48 \( S_i = 0 \)) and the fact that \( S = I \), we get

\[
(1-t_p).P/K = (1/s_c).I/K \quad \text{where} \quad t_p = \text{marginal and average tax rate on profits}
\]

\[
(1-t_p).P/Y = (1/s_c).I/Y
\]

These results obtained by Steedman are known in the literature as the "corrected" rate of profit. Interestingly, once again the worker’s propensity to save, tax rate on wages and a purchase tax on consumption has no impact on after tax return to
capital (net profit rate) and share of profits in income. What this implies is that all taxes imposed on profits would in the final instance be shifted to wages, which bears the burden of the tax.

On the equilibrium growth path, capitalists make sufficient savings in order to sustain the investments necessary to maintain full employment. While this matches the broad contour of classical theory of income distribution, there is an important and significant difference. In classical theory, wages are paid at the beginning of each period in order to sustain production in the period, and what remains as surplus at the end of the production period is retained by the producer as profit.

In the Kaldor framework, it is profits which are pre-determined in the system (sufficient to maintain full employment) while wages are the residual. The economy permits the extraction of surplus by the owners of capital in order to meet investment requirements. The residual product thereafter goes towards payment of wages and therefore wages bear the burden of taxes, which is the transfer of the surplus to government [Pasinetti 1989b: 29]. Steedman (1972) on the other hand looked only at the situation when there are balanced budgets. And though it was a step forward it left a gap in our understanding as to what would happen if there

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7. Steedman (1972) used the option of the government making transfers in order to maintain a balanced budget in the event that any surplus existed with the government.

8. Another important conclusion that Steedman arrived at was that an economy with government activity is likely to function on the full-employment frontier than one without one where there might be problems of "over-saving".
was government activity and this led to budget imbalances. After all modern economies rarely display balanced budgets, even in the long run.

**Pasinetti Process and Budget Deficit (or Surplus)**

The early contributions introducing government imbalance are by Fleck & Domenghino (1987 & 1990) who argued that the existence of the Cambridge theorem was sensitive to relaxation of the balanced budget assumption. However, Fleck & Domenghino (1987) failed to recognise the effect of government savings on the distribution of capital and profits. Dalziel (1989) points out that Fleck & Domenghino (1987) erred in the same way that Kaldor (1956) had done. They failed account for the fact that government would own capital (and therefore earn profits) if it did undertake positive savings. Even in an open economy, the modified Pasinetti theorem holds and the government’s saving as well as workers’ saving play no role in the macroeconomic determination.

In Fleck and Domenghino (1990), there is an attempt to resurrect their earlier results but suffer from violation of assumptions of the existence conditions regarding saving propensities of workers (Pasinetti 1989a and Dalziel 1990).

**Cambridge Equation and Ricardian Equivalence**

The role of budget imbalances on steady state growth was also analysed by Pasinetti (1989a & b). He takes the Steedman (1972) result further by allowing for
budget imbalances in the post-Keynesian distribution model. The argument he presents is that if under the balanced budget assumptions, the Cambridge Theorem (rate of profit net of taxes being equal to the ratio of growth rate and capitalists saving) holds, then the same result should emerge for rational individuals in a regime of budget deficits (or surplus). Here he relies on the rational expectations school – the Ricardian Equivalence Theorem, to arrive at his results. Under certain conditions, rational individuals are indifferent to whether the government in the current period raises resources by running budget deficits or by increased taxation [Pasinetti 1989a: 646].

A budget imbalance is represented in the context of the system of equations developed above: $s_G \neq 0$. This assumes that there is a permanent deficit or surplus that is likely to occur in a situation of long run growth.

In equation 50 we have added together the savings of different components of the economy to arrive at the aggregate saving in the entire economy. When disaggregated by source it turns out that the saving propensities are:

55 $s'_{ww} = \{s_w(1 - t_w) + s_G \alpha \{t_w + t_r(1 - s_w)(1 - t_w)\}\}$

56 $s'_{we} = \{s_w(1 - t_p) + s_G \alpha \{t_p + t_r(1 - s_w)(1 - t_p)\}\}$

57 $s'_{ce} = \{s_c(1 - t_p) + s_G \alpha \{t_p + t_r(1 - s_c)(1 - t_p)\}\}$

9. The fundamental rationality issue is whether the government faces an inter-temporal balanced budget constraint or not.
The overall savings function (50) then reduces to:

\[ S = s_{ww} W + s_{wc} P_w + s_{cc} P_e \]

On substitution of the different saving propensities (equations 55-58) in the proportionality equation 52 we once again get:

\[ \frac{P}{K} = \left( \frac{1}{s_{cc}} \right) \frac{I}{K} \]

This is remarkable because now the Cambridge equation holds even in the presence of a budget imbalance, quite contrary to the reservations raised by Fleck et al (1987 & 1990). Pasinetti (1989b) goes on to conclude that when there is a budget deficit, both capitalists and workers experience a loss in their net saving propensities. But the incidence of the deficit acts differently on the two classes of agents in the economy. Wages, being the residual claim on social surplus, bears the burden of deficit while profits in order to match the investment requirements of the economy gets priority in claiming their share in social surplus. Gross profits rises sufficiently to compensate for the increased tax to leave net profit and therefore net savings equal to the exogenously-determined full-employment investment demand.

Pasinetti (1989b) then poses the question as to what must the conditions be for the Cambridge Theorem to be valid under the revised saving propensities as in equation 55-58 above. Since the concern in public economics has been the persistence of budget deficits on a long-term basis, this case is examined more closely. Budget deficit in the above schema implies: \( s_G < 0 \). i.e., the government has negative savings. This could be financed by: monetary expansion, issue of...
bonds or a combination of both.\textsuperscript{10} When the deficit is financed by monetary expansion it leads to inflationary pressures and government meets the deficit by way of “seigniorage” (inflation tax) additional to the direct and indirect taxes already in operation in the economy. The inflation tax can be considered as a proportionate tax on different income categories as done above for the other taxes and only diminishes private saving to the extent of the shortfall in public saving. The saving propensities described above in equation 55-58 are further revised to include the inflation tax and will yield the Cambridge theorem as in Steedman (1972).

When the deficit is financed by government borrowing (bonds), economic agents realise that in addition to the current tax obligations, the interest and debt redemption obligations will lead to future increase in taxation (the fundamental assumption for the Ricardian Equivalence theorem). Each group of agents then assumes that this debt obligation is proportional to the current tax liability and therefore sets aside from the savings pool a certain proportion for increased taxation in future. Once again the saving propensities in equation 55-58 can be suitably modified to include the additional item which will emerge as a proportionate term representing future tax, kept aside in a sinking fund. And this will again yield the Cambridge equation. Pasinetti (1989b) therefore concluded that the Cambridge equation results were robust not only to introduction of government but also to budget imbalances, though the latter was possible only if assumptions of Ricardian Equivalence were added to the model.

\textsuperscript{10} Here we will discuss cases when monetary financing or bond financing is exclusively used.
There is, however, an internal contradiction that arises in the explanation provided by Pasinetti (1989a & b). He assumes that in the long run there is a persistent budget deficit (or surplus). And he combines this with the notion of Ricardian Equivalence. The problem is that Ricardian Equivalence in its re-incarnation [Barro 1974] presumes the existence of a long run balanced budget which individual rational agents are aware of. It is this inter-temporal balanced budget which allows equivalence between deficit and taxation to emerge. The contradiction here is that Ricardian Equivalence and long run deficits are not compatible because one could either have long run balance or deficit not both. Therefore, the combination of Ricardian Equivalence with budget imbalance in the same model is not feasible.

**Cambridge Equation and Non-Equivalence**

The Ricardian Equivalence assumption introduces a host of restrictions regarding rationality which may not hold in the real world. Subsequent contributions take note of this and here we look at two contributions -- Denicolo & Matteuzzi (1990) and Dalziel (1991).

Denicolo & Matteuzzi (1990) argue that the Ricardian Equivalence assumptions are not necessary for the validity of the Pasinetti process as long as the net rate of profits is properly defined. They examine the cases when there is a budget surplus
or deficit and find that the Cambridge theorem is valid without having to depend on the Ricardian Equivalence assumptions of debt-tax neutrality. They assume a uniform tax on the source of the income (Steedman 1972) and saving rate is class dependent rather than source dependent (Kaldor 1956).

\[ S_w = s_w[(1 - t_w)W + (1 - t_p)P_w] \]

\[ S_c = s_c(1 - t_p)P_c \]

The saving function of the government is explicitly introduced.

\[ S_G = T - G + P_G \quad \text{where } T = \text{Tax}, \quad G = \text{Govt. expenditure and } P_G = \text{Govt. profit} \]

If the government has a positive saving it accumulates capital over time and has positive profit earnings thereof. Government expenditure is assumed to be a proportion of the tax collected.\(^{11}\)

\[ G = \gamma T \quad \text{where } \gamma > 0. \]

Under long run equilibrium,

\[ g = l/K = S_w/K_w = S_c/K_c = S_G/K_G \]

It follows from equations 61 and 64 that

\[ g = s_c(1 - t_p)P_c/K_c \]

\[ g = s_c r' \quad \text{where net rate of profit } \Rightarrow r' = (1 - t_p)P_c/K_c \]
Equation 64 also implies that

\[ g = \frac{S_G}{K_G} = \frac{(T - G + r.K_G)}{K_G} \]

\[ g \left[ (1 - s_c)/s_c \right] = \frac{(G - T)}{K_G} \]

This suggests that long run equilibrium growth is associated with a current primary deficit \((G - T > 0)\). In the long run, the rate of profit \(r'\) is necessarily greater than the rate of growth \(g\). 12 Therefore, government’s interest receipts exceed government’s net investment in capital assets. This excess is used to finance the current primary deficit. 13 As discussed above, the case of government with a long run budget deficit, and a positive public debt has been examined by Pasinetti (1989a & b) under the Ricardian Equivalence conditions. However, if Ricardian Equivalence does not hold, then the bonds held by the public are treated as net wealth (Barro 1974). The net assets \(A\) of the workers and capitalists is re-defined as:

\[ A_w = K_w + B_w \]

\[ A_c = K_c + B_c \]

Total bonds (debt) held are:

\[ B = B_w + B_c \]

11. The reasons for this assumption are only to simplify the arithmetic. \(G\) could be assumed to be a proportion of total government earnings and would yield the same long run growth results as now.

12. This follows from \(g = s_c.r'\) where \(0 \leq s_c \leq 1\). Therefore, \(g \leq r'\).

13. Even though there is a primary deficit, the overall budget is in surplus as \(G = \gamma.T\) (see above).
In long term growth under equilibrium,

\[ g = \frac{S_w}{A_w} = \frac{S_c}{A_c} = \frac{\Delta B}{B} \]

Note that the equation 72 is similar to equation 64 except that the denominator in 72 is replaced by asset (capital and bonds) rather than capital alone. Sectoral savings now is:

\[ S_c = s_c\left[(1- t_p)P_c + iB_c\right] \]

where \( i \) = rate of interest paid on bonds

\[ S_w = s_w\left[(1 - t_w)W + (1 - t_p)P_w + iB_w\right] \]

Denicolo & Matteuzzi (1990) define the net rate of profit \( (r') \) as the ratio of capitalists total earnings and their total assets:\(^\text{14}\):

\[ r' = \frac{s_c\left[(1- t_p)P_c + iB_c\right]}{[K_c + B_c]} \]

The Cambridge theorem holds with \( g = r' \cdot s_c \)

In the long run, the non-monetised portion of the budget deficit is bond financed. Therefore,

\[ \Delta B = G + iB - T \]

and from 72 it follows that

\[ g = \frac{\Delta B}{B} = \frac{[G + iB - T]}{B} \]

\(^{14}\) Note that equation 75 would converge to Steedman's revised rate of profit if (a) rate of return on public debt equates rate of profit on private capital: \( r' = i = (1 - t_p)P_c/K_c \) and (b) when capitalists do not hold any bonds, i.e. \( B_c = 0 \).
From equation 75 and 77 we get

\[ g \left[ \frac{(1 - s_c)}{s_c} \right] = \frac{(T - G)}{B} \]

This suggests that a long run equilibrium with budget deficit is associated with a primary surplus \((T - G > 0)\) and is similar in tenor as the conclusion with a budget surplus (equation 68). The long run equilibrium in the Pasinetti process with a budget deficit is associated with a primary surplus and a budget surplus is associated with a primary deficit. For this result the restrictive assumptions of the Ricardian Equivalence theorem need not hold. Even when the deficit is financed by ‘money creation’, i.e., interest obligations of the government are nil \((i = 0)\), the inflation tax acts like proportional tax just like any other tax (Pasinetti 1898b). The net rate of return on capitalists wealth depends only on the natural rate of growth and the capitalists propensity to save is independent of anything else in the economy including fiscal policy parameters, which is what the Cambridge theorem states.

In a more elaborate model developed by Dalziel (1991), it is demonstrated that the Cambridge Theorem holds in the presence of government with budget deficits in the presence as well as in the absence of Ricardian Equivalence assumptions. Following Pasinetti (1989b), Dalziel uses the savings function above with some modifications (Equation 55-58). He assumes that there is a difference in the saving propensities out of wages or profits \((s_i)\) and out of interest from bonds \((\sigma_i)\). There is also an introduction of two additional tax rates:
\( t_e = \) indirect tax on consumption expenditure, and
\( t_k = \) indirect tax on capital expenditure (investment)

The modified sectoral savings function are:

\[
79 \quad s'_{ww} = s_w(1 - t_w) + s_G.\alpha.[l_w + (1 - l_w).(s_w.t_k + (1 - s_w).t_e)]
\]
\[
80 \quad s'_{wc} = s_w(1 - t_p) + s_G.\alpha.[l_p + (1 - l_p).(s_w.t_k + (1 - s_w).t_e)]
\]
\[
81 \quad s'_{cc} = s_c(1 - t_p) + s_G.\alpha.[l_p + (1 - l_p).(s_c.t_k + (1 - s_c).t_e)]
\]

The aggregate savings function (50) then reduces to:

\[
82 \quad S = s'_{ww}.W + s'_{wc}.P_w + s'_{cc}.P_c + \\
\quad (\sigma_w - \sigma_G).\alpha.(1 - t_e).(1 - t_p).R_w + \\
\quad (\sigma_c - \sigma_G).\alpha.(1 - t_e).(1 - t_p).R_c
\]

where \( \sigma_j = \) propensity to save out of interest receipts of class "j", and "j" = capitalists or workers

Interestingly, equation 82 reduces to the Pasinetti savings function (equation 58) if either of the following happens\(^{15}\):

a) \( R_w = R_c = 0 \), implying that there are no assets (bonds) held by capitalists or workers when there is deficit financing \( \Rightarrow \) Monetisation of debt

b) \( \sigma_w = \sigma_c = \sigma_G = \) propensity to save out of interest receipts of all classes is the same.

\(^{15}\) The equation 58 is: \[ S = s'_{ww}.W + s'_{wc}.P_w + s'_{cc}.P_c \]
What happens if there is monetisation of debt? The likely scenario in a full-employment economy is inflation. Though Pasinetti did indicate a form of proportionate tax he did not explicitly derive it. Dalziel (1991) provides a simple proof.

Let, \( \rho = \text{rate of inflation} \). Then, the extent to which workers and capitalists' real savings need to be reduced to attain full-employment is:

\[
(1 + \rho) = (S_w + S_c)/(S_w + S_c + S_G)
\]

This implies that there would be an equivalent decrease in income and capital stock of all agents. The capitalists share in capital stock (in real terms) would then be:

\[
K_c/K = S_c/(1 + \rho)
\]

Re-arranging the terms gives us:

\[
P_c/K_c = [(1 + \rho).I] / [K_c (1 - t_p).s_c]
\]

Equation 87 looks like the familiar Steedman (1972) result except that there is an inflation correction factor providing for a reduction of the nominal values to real terms in the presence of inflation. Once again the Cambridge result emerges -- if the government attempts to increase its share of the social surplus either by way of capital taxation \((t_k > 0)\) or by monetising deficits (inflation tax), the burden of such
ventures leaves capitalists net profits untouched but is borne by the workers’ wages.

The second case when the Savings function (equation 82) in Dalziel (1991) reduces to Pasinetti (equation 58 case (b) above). Suppose that the government were saving a positive portion of the interest accruing on the debt held by the public, then either the full amount of the interest payments (if $\sigma_G = 1$) or part thereof (if $0 < \sigma_G < 1$) would be additionally financed by issue of fresh debt. In the event we assume that Ricardian Equivalence holds, the economic agents would consider this additional debt issue as deferred taxation. They would then increase their savings appropriately to match the size of $\sigma_G$. This implies that $\sigma_G = \sigma_c = \sigma_w$.

The capitalists would then be holding a share of capital equivalent to:

$$K_c/K = s_c P_c/S$$

Rearranging the terms gives us:

$$r' = (1/s_c) g$$

Finally Dalziel (1991) attempts a generalisation of the Cambridge theorem in the absence of monetisation and Equivalence theorem assumptions. Further, he switches to the Kaldor assumptions of a common class-wide savings propensity rather than a source differentiated saving propensity. In the context of the above notations, this amounts to saying the following:

$$\sigma_G = s_G, \quad \sigma_c = s_c, \quad \text{and} \quad \sigma_w = s_w$$

Government (dis)saving then becomes:
The aggregate savings function then reduces to:

\[ S = s' \cdot W + s' \cdot (P_w + R_w) + s' \cdot (P_e + R_e) - s' \cdot \alpha \cdot D \]

The rate of profit in the economy in steady state will be:

\[ r = \frac{P_e}{K_c} = \frac{R_e}{B_c} = \frac{(P_e + R_e)}{(K_c + B_c)} \]

where,

- \( B_c \) = Amount of debt (bonds) held by capitalists and
- \( R_e \) = Interest received from bonds held.

Since, both capital and bonds are regarded as wealth and treated equivalently (or are substitutable as a form of wealth) for the private asset holder, they can be merged or summed. The rate of profit from capital also equals the rate of interest from bonds. Further, the share of the capitalists’ assets in the economy is given by:

\[ \frac{(K_c + B_c)}{K} = S_e/S \]

Using equation 91 and 92 we can derive:

\[ r = \frac{(P_e + R_e)}{(K_e + B_e)} \]

\[ = \frac{((P_e + R_e)/K)}{((K_e + B_e)/K)} \]

\[ = \frac{((P_e + R_e)/K)}{((K_e + B_e)/K)} \]

\[ = \frac{((P_e + R_e)/K)}{[S_e/S]} \]

\[ = \frac{((P_e + R_e)/K)}{[s_e(1 - \lambda)(P_e + R_e)/I]} \]

\[ = \frac{1}{[s_e(1 - \lambda)]} \cdot (I/K) \]

\[ r = (1/[s_e(1 - \lambda)]) \cdot g \]
Equation 94 is the Steedman equation but in this case with a steady state budget deficit and with Ricardian Non-equivalence between debt and taxes.\textsuperscript{16}

Conclusion

This result re-emphasises that the Cambridge Theorem and Pasinetti process is robust to various kinds of assumptions and is not limited to any particular framework. We have seen it develop from a simple model with no government where workers do not save (Kaldor 1956) to one where workers' savings are incorporated and yield them interest earnings (Pasinetti 1962). Its robustness was further tested with the introduction of government activity but under a balanced budget regime (Steedman 1972). The Theorem held up to relaxing the balanced budget constraint by monetisation and even issue of bonds but under conditions of Ricardian Equivalence (Pasinetti 1989a &b). Finally, even the assumption of Ricardian Equivalence was relaxed (Denicolo & Matteuzzi 1990 and Dalziel 1991) and the Cambridge Theorem results held under these conditions.

The critiques of the Cambridge Theorem were along two strands: One which relaxed the assumptions of lower saving propensities of workers than capitalists which led Meade (1963) and Samuelson & Modigliani (1966) to the formulation of the Duality Theorem. Once the magnitude of the workers allowed to have a high rate of savings the workers were found in the long run to own all the capital to the complete decimation of the capitalists class, which was the reverse of what was proposed by Kaldor or Pasinetti. But this result depended on various strict

\textsuperscript{16} At the core of this result is that the return to savings as a proportion of the savings of all
assumptions concerning the technology (perfectly substitutable factors of production) and savings propensities.

The second strand of criticism came from Fleck and Domenghino (1987 & 90) as we have discussed earlier in the essay. However, they failed to recognise the effect of government savings (accumulation) on the distribution of capital and profits. Their model violated the existence conditions with respect to saving propensities of workers as did the early critiques of the Pasinetti process. The only restriction that applies to the Pasinetti process is with respect to the saving propensities. It is imperative that workers savings propensity must be less than the capitalists saving propensity which must be more than the ratio of investment to income (Laing 1969). In fact this is described in the literature as the Pasinetti range.

This can be interpreted from a so-called neo-Keynesian position of the I/Y being exogenously given in the economy and the saving propensity adjusting to this rate. It can be alternatively interpreted, as Meade-Samuelson & Modigliani have, as the investment being determined by the level of savings generated by the capitalists. Inherent in the Pasinetti process is an assumption of perfectly elastic supply of capital by the capitalists. If an assumption is made that savings is a function of real income, then it implies a backward-sloping capital supply curve of capitalists. This opens up the case that equilibrium prices in the economy are dependent on the demand and supply conditions making saving propensity of the capitalists and the categories of agents must equate in the steady state long run.
rate of profit a variable determined by the demand and supply conditions in the
economy (Laing 1969: 385).

In fact, this conclusion is close to the work of Kalecki (1977) where he defines the
rate of profit in the economy as being dependent on the level of capacity utilisation
(and therefore the level of demand). This opens up the possibility of examining the
implications of the Pasinetti process under less than full employment conditions
and its impact on growth and distribution. This is what we turn to in our next
chapter. The attempt is to bring in the Kaleckian understanding of capitalist
dynamics where the class struggle over the social surplus is manifested through the
wage-profit distribution which in turn is manipulated by the degree of monopoly
power and levels of unemployment in the economy. Kalecki never himself had
occasion to look at growth dynamics with budget deficits and debt as given
features of the economy but left sufficient ground for expansion along those lines.
Our effort in the following chapter is to work towards a Kaleckian growth model
with debt. We examine the implications of working under less than full
employment with government intervention.