Chapter Six

Debt and Growth with Involuntary Unemployment

Introduction

In the previous chapter we examined the Kaldor-Pasinetti (Cambridge) growth model which bridges two issues of concern to us – growth and distribution. We noted that the Cambridge growth model was robust in a purely private economy, it was proved to be robust to the introduction of government activity with Ricardian equivalence as well as Non-equivalence.

One area of dispute that arose in the Cambridge-Cambridge debate between Pasinetti, Meade, Samuelson and Modigliani (among others) was the range of savings rates for which the Pasinetti results would be valid. Samuelson and Modigliani (1966) found that if the savings propensity of the workers was high enough then capitalists then a result opposite to the Pasinetti one would emerge – in golden age equilibrium, it is the workers rate of savings that would determine the rate of growth. The Pasinetti assumption they violate is that they allow share of saving in wages to be higher than the share of Investment-income ratio. As Kaldor (1966:311) points out that if one were to violate the basic assumption of high capitalists savings and “s_w < I/Y” -- then no Keynesian macro-economic distribution theory would survive. The relaxation of the Pasinetti range is the founding stone of the neo-classical critique of Pasinetti-Kaldor models. Other than being a theoretical curiosum the conclusion of the Anti-Pasinetti theorem, is implausible with any real life observation.
In contrast to the growth models that use the neo-classical production function the Kaldor-Pasinetti result is remarkable considering that very few restrictive assumptions need to be made for its existence. However, the Pasinetti result holds under full employment conditions. This was not relevant for a developing labour surplus economy and also for some developed countries which have displayed substantial degrees of slack for extended periods. This leaves a gap in the growth theory which makes the effort to seek a relevant growth and distribution theory at less than full employment level worthwhile. What we wish to do is explicitly incorporate the existence of unemployment as a stylised fact; otherwise growth theory would continue to suffer from the esoteric position it is in now.1 You and Dutt (1996) was an early attempt to incorporate the existence of “slack” within the Cambridge growth model and it was found to be robust to these changes. But their analysis is in a continuous time framework. We present below a distribution model in discrete time also with features of involuntary unemployment and unutilised capacity.

One school believes that high deficit leads to crowding-out of private investment and growth is affected by low productivity which is inherent in public expenditures. There is also the belief that interest payments for past debt (which is largely held by the capitalists) comes from tax revenues raised from the workers and therefore adversely affects the distribution of incomes (Michl 1991, You & Dutt 1996). In this chapter we present a distribution model wherein we show that a

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1 To quote You and Dutt (1996: 336): “the connection between debt and income distribution .... has largely escaped serious economic analysis.”
steady state level of debt is compatible with steady state growth in the presence of unemployment and unutilised capacity. This would occur even if public investment did not earn any returns and interest obligations of past debt is also paid for by borrowing, given that under steady state the share of public expenditure with respect to private capital would also remain constant.

In the general solution, two solutions emerge which conform to the Domar (1944) conditions for steady state debt as discussed earlier in this essay. When we invoke the principle of risk it is possible to arrive at a unique level of steady state growth and corresponding to it there would be a stable debt path. In an extension of this model, under certain conditions, a particular solution emerges wherein unique solution emerges even without invoking the principle of risk. We demonstrate with this model that it is possible to have a unique state level of debt either under the principle of risk or as a special case of the general solution even when public investment is “non-profitable” by itself and only has an enabling role for private capital. It is contrary to the argument of conservative economics that countries should discourage public investment and indulge in disinvestment only because it does not earn the rate of financial return equal to that of the private sector. This, they argue, would help in stabilising the fiscal situation in these countries and stop the increase in debt to unmanageable levels. We prove below that even if public investment does not earn any profit, it will reach a steady state level matching which there would be a steady state level of growth for any given rate of interest.

As a first step we introduce the national income equations which at this stage do not involve any behavioural assumptions. At a later point we adopt certain
constitutional rules wherein the government is allowed to raise debt, but only to finance capital expenditures and not consumption expenditures.\(^2\) The logic of this is that very often public policy critics argue that government undertakes “wasteful expenditures”. All ills of the economy are then traced to the growth of public expenditure (including inflation, current account deficits, productivity, low growth rates, etc.). The “cure all” solution for all the ills is said to be trimming of government expenditure – whether it is consumption or investment expenditures.

We will not enter the debate on the merits of government consumption expenditure, but we allow the government to spend by borrowing as long as they have assets to back up the debt. This is a rather conservative Keynesian position but nonetheless an interesting question and it blunts the argument that government is undertaking “wasteful expenditure”. We then go on to set up a stylized macro-model in a Kaleckian framework to incorporate a distribution schema which allows us to examine income gains or losses by different classes of people when there is a change in debt policy. One way to incorporate “unemployment” into the model is the use of the principle of “risk” (Kaldor 1958, Kaldor & Mirrlees 1962) wherein

\(^2\) It is not unknown to have governments being constitutionally bound to run primary deficit only on the capital account, i.e. only if the expenditures are for investment purposes. The logic of such rules is not difficult to find since the debt would then be matched by equivalently valued assets created and therefore, would not lead to bankruptcy given no adverse expectations regarding government stocks. This is also referred to as the Golden Rule of Public Finance and in counties like Germany and the UK, borrowing is allowed to finance only public investment and but not public consumption. The same is also used in the definition of the deficit in the Maatsricht treaty (Ghosh & Mourmouras 2004, Grenier & Semmler 2000).

In China, under the new Budget Law of 1995 the central government was prohibited from borrowing from the central bank and from deficit financing on the revenue account. However, the government was permitted to finance deficits on the capital account by issue of government bonds (Qian 1999).
the entrepreneur seeks a cushion (gap) between the profit she earns and the interest payments she has obligations to pay. Similarly, the principle of increasing risk as enunciated by Kalecki (1937) and Steindl (1945) suggests that as an entrepreneur increases her investment, the risk of losses also rises and therefore she seeks a bigger cushion and will only undertake a higher investment if her expectation of profit is higher. The Principle of Increasing Risk is a stronger condition than the simple “risk premium” requirement of Kaldor (1958) and in our model we use only the Kaldor notion of risk. Profit in the economy could be directly linked to growth since profits will rise only when aggregate demand and production rise in the economy, given capacity utilisation. So the private investor will undertake higher investment if there is higher growth and therefore by implication a higher profit.

The Model

In a macro-economy, the overall public debt in any period is equal to the debt in the previous period and the overall deficit (or surplus) in the current period. This can be stated as below:

\[ D_{t+1} = D_t + B_t \]

where the subscript ‘t’ defines the time period, and

\[ D = \text{Debt} \]

\[ B = \text{Fiscal Deficit (Surplus would be negative deficit)} = \Delta D \]

We can further state that the deficit in the current period is actually the sum of the interest outgoes on previously created debt and the primary deficit in the current
period. We assume that interest rates are given and are not influenced by the debt or deficit levels for our analysis.³

2 \[ B_t = rD_t + B^p_t \] where, \( B^p = \) Primary deficit.

We can now combine the equations 1 and 2 to get the following.

3 \[ D_{t+1} = (1+r)D_t + B^p_t \]

All that equation 3 above is telling us is: debt in any period is equal to the sum of historical debt, the interest liability on it and the primary deficit in the current period. Note that debt would reduce if there is a primary surplus (negative deficit) in the current period.

Now let us suppose that an economy has decided to undertake public investment which is entirely funded by primary deficits. So what is being proposed is that the primary deficit is equivalent in value to public investment being undertaken in the economy.⁴ Therefore,

4 \[ I^G = B^p \]

Putting in the information of equation 4 into equation 3 we get

5 \[ D_{t+1} = (1+r)D_t + I^G_t \]

³ In this analysis we presume that the (real) rate of interest is fixed which is a standard post-Keynesian assumption. One can expand the model to incorporate variability in interest rates but it would detract from the main argument.

⁴ Once again, this is only a bounded case. The public investment could be greater than the level of primary deficit \((I^G \geq B^p)\) but the constitution may not permit consumption expenditures by the government if it has to resort to borrowing to do so. For the time being we will work with the particular case \((I^G = B^p)\) for reasons of convenience.
We now set up a formal stylized model to incorporate distribution in this analysis.
We assume that there are two sectors – the government and the private sector.
Within the private sector there are two classes of agents in an economy. One set of
agents are the workers who earn wages (W) by selling their labour and another set
of agents are the owners of capital (capitalists) who earn profits (P). The national
income identity equation for expenditures can then be set out as:\(^5\)
\[ Y = C^G + C^W + C^P + I^G + I^W + I^p \]

We make some further assumptions at this point. We assume that the government
makes no consumption expenditures and the workers make no investment
expenditures.\(^6\) Equation 6 then reduces to the following:
\[ Y = C^W + C^P + I^G + I^p, \quad \text{since } C^G = 0 \text{ and } I^W = 0. \]

The investments that government makes do not earn any profits, so they may be in
the nature of social investments or infrastructure. By implication all the incomes
earned in the economy go entirely to the private sector and are distributed either as
wages or profits.
\[ Y = W + P \]
Since the workers do not save anything, they consume whatever they earn.\(^7\)

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\(^5\) \(C^G = \) Government consumption \\
\(C^W = \) Workers consumption \\
\(C^P = \) Capitalists consumption \\
\(I^G = \) Public Investment \\
\(I^W = \) Workers investment \\
\(I^p = \) Capitalists Investment

\(^6\) The assumption of government expenditures being zero (\(C^G = 0\)) is only a simplifying step. It
would not in anyway change the results if we had assumed (\(C^G > 0\)). This simplifying assumption is
made only to focus on the problem at hand. The second assumption however is realistic and is a
standard assumption in the Kalecki-Kaldor models (Bhaduri 1986, Dasgupta 1997).

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The capitalists on the other hand have two kinds of earnings. One is their profit earnings from ownership of physical (fixed) capital. And second is their interest earnings from loans to the government—bonds (circulating capital). The capitalists consume a part of their disposable income.

\[ \text{\( Y^p = P + rD \)} \]

\[ \text{\( C^p = \alpha. Y^p = \alpha.(P + rD) \)} \text{ where } \alpha > 0, \text{ and } \alpha \text{ = marginal propensity to consume of the capitalists} \]

From equations 7, 8, and 9 we can state that

\[ \text{\( P + rD = C^p + I^G + I^p + rD \)} \]

This implies that profit equal the sum of expenditures undertaken on consumption by capitalists, private investment and public investment.

\[ \text{\( I^G = (1 - \alpha). (I + \alpha. rD) \)} \text{ where } s = 1 - \alpha \]

"s" can be thought of as the marginal propensity to save of the capitalists.

The wages that workers earn are further assumed to be a given proportion of the aggregate income generated in the economy (national income \( Y \)).

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7 Again, this a standard assumption in the Kaleckian literature (see Bhaduri 1986, Kalecki 1938, 1977).

8 This is again a standard assumption in post-Keynesian literature. It is based on the models with fixed mark-up in pricing. This is in consonance with our assumption of prices being taken to be
This implies that the proportion of the national income that capitalists earn in the form of profits ($\pi$) is also given. The change in aggregate capital stock is a sum of the private and public investment in the economy.

\[ K_t - K_{t-1} = I_t^p + I_t^g = I_t \]

We are now required to make an assumption about the behavioural patterns in investment demand of the government and the capitalists. We presume that investment by either agent is related to the level of capacity utilisation (which is a good proxy for the level of employment and profit generated in the economy). The capacity utilisation would also indicate the future expectations that investors would have about their business prospects. The state too may have a targeted level of capacity utilisation. Since capacity utilisation and labour employment tend to move together it is an indicator of the desire of the state to overcome “slack” and unemployment in the economy.

It would not be out of place here to discuss a little about the nature of state that we have in mind. Unlike some of the literature that naively presumes that the state aims at full employment levels, we are persuaded by a more endogenous form of the state where the dominant coalition (of which the working class is obviously not a part) dictates the economic policies of the state (Bardhan 1984, Kalecki 1943, constant. As Kalecki (1938) has argued the value of $\omega$ would depend on the relative political strength of each class, in his words the “degree of monopoly”. Theories of inflation use the notion of class struggle to explain movements in price (as an indicator of social surplus sharing among different classes), see e.g. Patnaik et al (1976), Patnaik (1988), Rowthorn (1977), Sanyal et al (1989).
Raj 1972). In developing countries like India, the mode of production is believed to be in the transition phase from feudal to a capitalist one. Given peculiar colonial experiences, the bureaucracy has acquired a stake in the state’s functioning and is part of the dominant coalition. The working class whether in the industrial sector or the agrarian sector is not sufficiently organised to challenge the entrenched feudalism in rural areas or the capitalist-bureaucratic coalition in the urban areas. The dominant classes, therefore, are able to manipulate the organs of the state to the extent that unemployment or inflation do not rise beyond critical limits (Patnaik 1984).

The autonomy that bureaucracy in an intermediate regime is believed to acquire is due to inner contradictions between the capitalists and the agrarian elite (Bardhan 1984, Kalecki 1972). The capitalist and feudal conflict over distribution of the social surplus in such regimes can provide the bureaucracy space (autonomy) to act as a “neutral” player. But this neutrality does not necessarily alter the distribution of incomes between the owners of the resources and the sellers of wage labour. The constituents of the dominant coalition might be able to change their respective shares but only amongst themselves without affecting the share between the proprietary classes and the asset-less (Mitra 1979).

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9. The interested reader is referred to Patnaik (1990) where a collection of selected articles on the Mode of Production debate in the Economic and Political Weekly in 1970s and 1980s is available. While some authors believe that the capitalists mode is prevalent even in agrarian India, some others have characterised the situation as semi-feudal semi-capitalist where the process of formation of a disenfranchised proletariat is incomplete.
Presumably, the nature of the state determines the kind of fiscal intervention policy vis-à-vis unemployment that the state would adopt. As Kalecki (1943) points out, there is no reason to believe that full employment would be looked on kindly by the members of the dominant coalition. The dominant coalition would, however, be threatened by social upheaval if it blocked government efforts at “reducing” the level of unemployment in the economy. Therefore, the actual level of capacity utilisation (employment) would be determined by the character of the “legitimisation” crisis that the state faces and the contesting pulls of the conflicting classes for a share of the social surplus. In this formulation we assume that while the workers would like to see the economy being pushed to the full capacity utilisation level, the dominant classes would oppose it since that would reduce their bargaining strength and bid away social surplus (Kalecki 1971).

We state that the level of capacity utilisation is given by ‘\( u_t \)’ (where the sub-script ‘\( t \)’ represents the time period). The term ‘\( \beta \)’ can be conceived of as a parameter which has stock-flow ratio characteristic (Patnaik 1997).

\[
16 \quad u_t = Y_t / K_t \beta
\]

What role does public investment play? Public investment has two objectives: to provide enabling infrastructure to the private sector and to stabilise the economy at a certain level of activity. Additional public investment is forthcoming when there is a difference between the targeted level and the actual level of capacity utilisation. Let ‘\( \bar{u} \)’ be the desired level of capacity utilisation (or economic activity) for the state which provides it the legitimacy to maintain a capitalist form of production. If this level of capacity utilisation is achieved then additional public
expenditure reduces to zero and all agents fulfil their expectations. However, if for some reason the level of economic activity ‘u’ deviates from ‘u’ then public investment kicks in.¹⁰

\[
(1^{G/K})_{t+1} = (1^{G/K})_t - \mu (u_t - \bar{u})
\]

where \( \mu > 0, \ \forall \ u_t < \bar{u}, \ \text{and} \ \mu = 0, \) otherwise.

This implies that the proportion of public investment in the capital stock of the country is dependent on deviation of capacity utilisation in the current period from the desired level. What restrains public investment from pushing the economy to full employment in the long run? Inflation is one of the deterrents (among others that we discuss below) to continuously increasing capacity utilisation. As the capacity utilisation exceeds ‘\( \bar{u} \)’, inflationary pressures are built up and the public authority becomes sensitive.

Private investment on the other hand keeps the economy at a point lower than the level of full employment in order to maintain its bargaining strength vis-à-vis labour. In fact, the degree of slack in the economy is determined by the degree of monopoly, a la Kalecki (1943). The reserve army of labour that keeps wages from rising is maintained, if necessary, by importing labour from other parts of the world even if the government manages to employ all local labour. This has been a standard feature of development in the capitalist world (Patnaik 1997).

\[
(1^{P/K})_{t+1} = (1^{P/K})_t + \rho (u_t - \bar{u})
\]

where \( \rho > 0, \ \forall \ u_t < \bar{u}, \ \text{and} \ \rho = 0, \) otherwise.

¹⁰ This draws on Patnaik (1984) and the mathematical model developed in the Appendix of this article.
The aggregate investment function in the economy (public as well as the private sector), is the sum of the public and private investment functions and we can thereafter combine 17 and 18 to state that:

$$\Rightarrow (I^p/K)_{t+1} + (I^p/K)_{t+1} = (I^p/K)_t + (I^p/K)_t - \mu (u_t - \bar{u}) + \rho (u_t - \bar{u})$$

$$\Rightarrow (I/K)_{t+1} = (I/K)_t - \theta (u_t - \bar{u}) \quad \text{where} \quad \theta = \mu - \rho$$

$$i_{t+1} = i_t - \theta (u_t - \bar{u}) \quad \text{where} \quad i = I/K$$

In steady state, $i_{t+1} = i_t$ since $u_t = \bar{u}$, implying $\theta (u_t - \bar{u}) = 0$

It is being assumed here that the production function of private producers has public capital as one of the inputs and its absence can reduce the productivity of private capital.\^{11} "Infrastructure" is only "enabling" and is available free for private producers. Infrastructure investment is made by the state through purchase of goods produced in the private sector which is financed through borrowing.\^{12}

In some parts of the world where there has been an effort to bring in private investment in enabling infrastructure with the hope of increasing "efficiency" has met with limited success. It has been pointed out that the share of public investment does not add to the revenue at all, and is therefore a kind of a "floor" level for determining desirability of public investment.

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\^{11} This is a standard assumption following up on Arrow & Kurz (1970) except we do not assume any externality effects here. Many endogenous growth models also incorporate public capital in the production function by way of supportive productive activity.

\^{12} It is also not uncommon to hear the argument that the public sector is "inefficient" because the return on public investment is much lower than in the private sector. The low profits from public sector undertakings, is said to impose a burden on the public exchequer and is used by conservative economists to argue in favour of "disinvestment"—selling off public assets. Here we wish to assume that public investment does not add to the revenue at all, and is therefore a kind of a "floor" level for determining desirability of public investment.
investment in GDP, and especially infrastructure, has declined during the last three decades in a number of countries including on average the OECD countries and Latin America, where people fear that long term growth may be adversely affected due to infrastructure gaps [IMF 2004: 5]. There would be less of concern if these declines were due to a maturing private sector moving into invest in infrastructure. However as in the case of Latin America, the truth is that these declines have occurred as a consequence of structural adjustment programmes and the burden of adjustment has fallen on investment especially infrastructure rather than current expenditures (Calderón et al 2002a) and private sector has been unable to fill in this void (Harris 2003).

According to one estimate, the reduction in infrastructure during the 1990s reduced long term growth rates by 3 percent per year in some Latin American countries like Argentina, Bolivia, and Brazil, and by $1\frac{1}{2}-2$ percent in Chile, Mexico, and Peru (Calderón et al 2002b). In the OECD countries, the decline is attributed to the European monetary union restrictions on the level of public deficit and debt that each country can have as per the Maastricht Treaty requirements. What is evident is that public investment is perceived by a large cross section of economists and policy makers as being important in aiding efficiency gains in the economy. In order to enable our steady state result we start off by defining certain terms.

\[ u_t = \frac{Y_t}{K_t}, \beta \]

\[ Y = W + P = \omega Y + P = P / (1 - \omega) = P / \pi \]

where, $\pi = P / Y = 1 - \omega = \text{Share of profits}$

\[ u_t = P / K_t, \beta, (1 - \omega) \]

and since $P = (I_t + \alpha r D_t) / (1 - \omega)$

\[ u_t = (I_t + \alpha r D_t) / s K_t, \beta, (1 - \omega) = (I_t + \alpha r D_t) / s K_t, \beta, \pi \]

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13. Since $u_t = Y_t / K_t, \beta$ (as in 16 above)
Definitions:

1. A steady state debt path is one where \( \frac{D_t}{K_t} = d^* \quad \forall \ 't' \), where the subscript ‘t’ denotes time period.

2. A steady growth path is one where \( \frac{l^*_t}{K^*_t} = \frac{l_t}{K_t} = g \) where ‘g’ is constant \( \forall \ 't' \).

Lemma 1 (Domar Proposition): For \( g > 0 \), \( \forall \ 't' \) there exists a stable debt path only if
\[
\Delta K^P/K^P = \Delta K/K > r, \quad \forall \ 't' \]

Proof: Along a steady state debt path since \( D/K \) is constant, \( \Delta D/D = \Delta K/K \quad \forall \ 't' \)
\[
r + l^*_t/D_t = (\Delta D/D)_t = (\Delta K/K)_t \quad \text{(from equation 5 above)}
\]
Since \( l^*_t \) and \( D_t \) are both positive in equation 18, it must be that
\[
r < (\Delta K/K)_t \]
QED

Lemma 2: A steady state debt path must necessarily be a steady state growth path.

Proof: Along a steady debt path, (see above equation 18)
\[
r + l^*_t/D_t = (\Delta K/K)_t = (l/K)_t
\]
As \( d^* \) is the stable level of debt, it follows that
\[
(l^*/K)_t = d^*.[(l/K)_t - r] \quad \forall \ 't'.
\]
If \( (l/K) \) is not constant then \( u_t \neq \bar{u} \),
Let \( u_t < \bar{u} \)
Then \( (l/K)_{t+1} < (l/K)_t \)
Therefore, from \(20\) above

\[(l^5/K)_{t+1} < (l^5/K)_t\]

Since,

\[
24. \quad u_{t+1} = Y_{t+1}/(K_{t+1} \beta) = (I_{t+1} + \alpha r D_{t+1})/sK_{t+1} \beta (1- \omega)
= (I_{t+1} + \alpha r D_{t+1})/sK_{t+1} \beta \pi
= (I_{t+1}/K_{t+1}) + (\alpha r D_{t+1}/K_{t+1})(1/(s \beta \pi))
= (I_{t+1}/K_{t+1}) + [(\alpha r d_{t+1})/(s \beta \pi)]
\]

It follows from the above that maintenance of a stable debt \((d_{t+1} = d^*)\) would mean:

If, \(u_t < \bar{u}\), then \(u_{t+1} < u_t\)

If, \(u_t > \bar{u}\), then \(u_{t+1} > u_t\)

A non-stationary growth path then must logically witness a steadily increasing (or steadily decreasing) \(I/K\), and with it steadily increasing (or steadily falling) \(u_t\). This is not possible since \(u_t \leq 1\) and cannot fall below a floor level \(u_0\), i.e. \(u_0 \leq u_t \leq 1\). At some time period \(t\), the economy would hit these bounds (when \(u_t \neq 0\)), and get out of a stable debt path. Thus, if the stable debt path is not on a steady growth path then there is some \(t\) at which starting from \(t_0\), the economy will be unable to maintain stable debt.

**Corollary 1:** A stable debt path must necessarily witness steady growth and hence a steady debt-output ratio.

**Lemma 3:** A steady growth path must necessarily be a stable debt path.
Proof: Given that along any steady growth path, \( u_t = \bar{u} \quad \forall 't' \)

For any \('t'\) equation 20 above (which is an identity) may be re-written as

\[
r + (l^f/K)t = (1/K)\]

it follows that with \('r'\), \(l^f/K\) and \(1/K\) being constant \( \forall 't' \) along the steady growth path, \('d_1'\) must also be constant \( \forall 't' \).

QED

**Lemma 4**: Along a steady growth path where \( K \) is growing at the rate \('g'\), both \( K^p \) and \( K^g \) would also grow at \('g'\) (where \( K = K^p + K^g \) and \( K^g = \sum_0^t l^g_i \)).

**Proof**: Since in steady state, \( l^f/K \) and \( l/K \) are constant \( \forall 't' \), it must be that \((\sum_0^t l^g_i)/(K_t = K^g/K \) and \( K^p/K \) are also constant. So if \( K \) grows at \('g'\) then it follows that \( K^p \) and \( K^g \) would also grow at \('g'\).

QED

Let us now introduce the notion of risk into our analysis. We assume that the capitalist would invest as long as the profit rate in the economy, \( P/K \) exceeds the interest rate by a certain positive amount \( \rho \). This is so because in order for an agent to agree to hold an illiquid asset – physical capital, she would want a premium over a liquid asset (e.g. a bond which earns \( 'r' \) returns). It is often assumed by economists in the neo-classical framework that the rate of profit must equate the rate of interest in a perfectly competitive capital market. But it is easy to justify the counter claim that most markets, especially capital markets, are imperfect in developing countries and therefore the rate of profit must exceed the rate of interest for an entrepreneur to undertake investment. Even in a developed
economy with well-functioning capital markets, the entrepreneur's may need a cushion against risk in order to undertake investment (Kaldor 1958, Kaldor & Mirrlees 1962, Kalecki 1937, Steindl 1941 & 1943). A threshold level of risk can be defined as the minimum difference between the rate of profit and the rate of interest that the entrepreneurs need to undertake investment. It is easy to further show that the rate of profit is directly linked to the rate of growth in the economy.

The idea of risk was originally conceived of with profit rates and interest rates in mind and entrepreneur's investment decisions were based on perceived risk to their profit making possibilities. But it would be easy to demonstrate that profit rate is linked to growth rate and the two can be interchangeably used to demonstrate risk in the system. In fact equation 13 (above) and 29 (below) provide a direct link between profit rate \((\pi)\) and growth rate \((g)\). Risk, it can be argued in a macroeconomic sense, is dependent on the state of the economy and \("g-r\"\) is a good indicator of it.

26. \((P/K) - r = \rho\). And correspondingly \(g - r = \epsilon\), where \(\rho\) and \(\epsilon\) indicate risk

Then,

**Theorem 1**: If there is no restriction on \(\epsilon\), other than \(\epsilon > 0\), then there could be two possible positive steady growth rates and associated with each growth rate there would be a particular stable debt ratio.

**Proof**:

27. Since \(\Delta D/D = [I^8 + r.D]/D = [(I^8/K).(K/D) + r] = r + i^8/d = g\)

Given that \(d > 0\), for \(i^8 > 0\), it must be that \((g - r) > 0\) (see Lemma 1 above).\(^{14}\)

\(^{14}\) Note, that \((I^8/K) = i^8\) and \((D/K) = d\)
Since we are investigating steady state situations, we assume that \( i^g \) and \( r \) are both constant. The assumption of the rate of interest being constant is a standard post-Keynesian assumption as we have mentioned elsewhere and \( i^g \) is assumed constant since in steady state the proportion of public investment in capital stock is stable (also follows from Lemma 3 above). Therefore, \( i^g/d = g - r \), is likely to have a rectangular hyperbola curve as below.

We also know that

28. \( P - C^p = 1 \)  

(see equation 13 above)\(^{15} \), it follows

29. \( \pi \beta \bar{u}(1-\alpha) - \alpha rd = I/K = g \)

where, savings out of profits \( => \)

\( \pi \beta \bar{u}(1-\alpha) = \pi(1-\alpha)Y/K = sP/K, \) and

\( \alpha rd = \alpha rD/K = \text{Capitalist} \)

consumption out of income from bonds, both as a proportion of capital stock.

Now,

30. \( \pi \beta \bar{u}(1-\alpha) - (\alpha d + 1)r = g - r \)

When \( g - r = 0 \) then \( d = [\pi \beta \bar{u}(1-\alpha) - r]/\alpha \)

And when \( d = 0 \), then \( g - r = \pi \beta \bar{u}(1-\alpha) - r \)

By combining graphs 1 & 2 we get two possible steady state solutions where the two curves intersect. Equation 27 (and 31 below) describes the trade-off for the state between growth and debt where it meets the budgetary rules of financing.

\(^{15}\) \( P - C^p = 1 \)

\( => \ P - \alpha (P + rD) = 1 \)

Dividing through out by \( K \) and simplifying gives

\( \alpha (1-\alpha)^2 (P/Y)(Y/K) - \alpha rD/K = I/K = g \)

\( \alpha (1-\alpha)^2 \bar{u} - \alpha r.d = g \)
public investment and interest obligations on past debt with new borrowing. On the other hand, equation 29 (and 32 below) represents the capitalists' investment-consumption decisions.

The two roots can be obtained by manipulating equation 31 and 32,

31. \( \frac{i^g}{d} + r = g \)  
   (see Eqn 27 above and illustrated in Figure 1)

32. \( \pi \beta \delta (1-\alpha) - \alpha rd = g \)  
   (see Eqn 28, 29 above and illustrated in Figure 2)

Putting equation 31 in 32 we get

33. \( \frac{i^g}{d} + r = \pi \beta \delta (1-\alpha) - \alpha rd \)

\( \Rightarrow i^g + d.r = \pi \beta \delta (1-\alpha).d - \alpha rd^2 \)

\( \Rightarrow i^g + d.r - \pi \beta \delta (1-\alpha).d + \alpha rd^2 = 0 \)

\( \Rightarrow \alpha rd^2 - [\pi \beta \delta (1-\alpha) - r].d + i^g = 0 \)
34. \[ d = \left\{ \pi \beta \bar{u}(1-\alpha) - r \right\} \pm \sqrt{\left\{ \pi \beta \bar{u}(1-\alpha) - r \right\}^2 - 4 \cdot \alpha \cdot r \cdot i^2} \right\} / (2 \alpha \cdot r) \]

So the two roots would be:

35. \[ d_1 = \left\{ \pi \beta \bar{u}(1-\alpha) - r \right\} + \sqrt{\left\{ \pi \beta \bar{u}(1-\alpha) - r \right\}^2 - 4 \cdot \alpha \cdot r \cdot i^2} \right\} / (2 \alpha \cdot r) \]

36. \[ d_2 = \left\{ \pi \beta \bar{u}(1-\alpha) - r \right\} - \sqrt{\left\{ \pi \beta \bar{u}(1-\alpha) - r \right\}^2 - 4 \cdot \alpha \cdot r \cdot i^2} \right\} / (2 \alpha \cdot r) \]

These are the two roots where debt and growth follow a steady path.

QED

It must be noted here that both the steady state values of debt conform to the Domar (1944) sustainability conditions for steady state debt, which we have discussed earlier in this essay.

The range of values that would yield valid solutions is: \(^{16} \]

\(^{16} \) It is obvious that that there is no real solution to this quadratic if the terms in the bracket are negative, i.e. when \((\pi \beta \bar{u}(1-\alpha) - r)^2 < 4 \cdot \alpha \cdot r \cdot i^2 \)
37. \((\pi \beta(1-\alpha) - r)^2 \geq 4. \alpha r. i^6\)

Let us now examine the solutions in a little more detail:

**Case 1**: If

38. \((\pi \beta(1-\alpha) - r)^2 = 4. \alpha r. i^6\)

Then there is always a unique solution since the root for ‘d’ which is:

39. \(d = [\pi \beta(1-\alpha) - r]/(2 \alpha r)\)

**Case 2**: When

40. \((\pi \beta(1-\alpha) - r)^2 > 4. \alpha r. i^6\), then

It must be the case that \(d_1 > d_2\), since \((\pi \beta(1-\alpha) - r)^2 > 4. \alpha r. i^6\),

In equations 35 and 36, evidently the debt ratio at ‘d_1’ is higher than at ‘d_2’ but the gap between growth and interest rate is lower (see Figure 3). So even though \(d_1\) is a steady state solution it is less desirable in terms of the lower growth possibility there.

**Theorem 2**: In the presence of a minimal level of risk ‘e’, there is only one unique steady growth rate in the economy which is associated to a unique steady state debt ratio.\(^{17}\)

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\(^{17}\) The condition mentioned in Theorem 2 derives from the Principle of risk. Given a fixed rate of interest in the economy, growth rate is above the rate of interest by a minimum amount “e” which is greater than that corresponding to the growth rate associated with the higher level of steady state debt.
Proof: As described in Theorem 1, the two levels of steady state debt values are:

\[ d_1 = \left\{ \pi \beta (1-\alpha) - r \right\} + \sqrt{\left( \pi \beta (1-\alpha) - r \right)^2 - 4 \alpha r \left[ \pi \beta (1-\alpha) - r \right]} / (2 \alpha r) \]  
\[ \text{(equation 35)} \]

\[ d_2 = \left\{ \pi \beta (1-\alpha) - r \right\} - \sqrt{\left( \pi \beta (1-\alpha) - r \right)^2 - 4 \alpha r \left[ \pi \beta (1-\alpha) - r \right]} / (2 \alpha r) \]  
\[ \text{(equation 36)} \]

Corresponding to each of these steady state debt values \( d_1 \) and \( d_2 \), there is a corresponding level of growth - \( g_1 \) and \( g_2 \), respectively, where given that \( d_1 > d_2 \), the \( g_1 > g_2 \).

If the minimal level of risk in the system is described as \( g_0 - r \) > \( g_1 - r \), below which the private investors do not participate, then there is only one feasible steady state debt path described by \( d_2 \) and steady state growth path \( g_2 \).

QED

The economy would never reach the ‘\( d_1 \)’ solution since the ‘low’ growth-interest rate gap would be below the level of risk perceived in the system. As long as agents in economy have a ‘safe’ or ‘high’ growth-interest rate gap which is greater than the prevailing one at \( d_1 \), the economy would have a unique solution at \( d_2 \).

We postulate that investors may have a minimal level of risk which is indicated by the “floor” value of the “g-r”. In the context of the diagram 3 above, let this is given by “\( g_0 - r \)”. As long as “\( g_0 - r \)” is greater than the “\( g_1 - r \)” (the growth-interest gap corresponding to \( d_1 \)), we have a unique steady state equilibrium value of debt, \( d_2 \), to which corresponds a unique level of maximum steady state growth rate \( g_2 \), given ‘\( r \)”.
The other situation when a unique steady state emerges (and is a special case of the general result derived above is) when Curve II is tangential to line I and "g_3-r" > "g_0-r". This situation is discussed above as Case 1 and illustrated in Figure 4 below:

As in equation 40 above, when, \((\pi.\beta(1-\alpha) - r)^2 = 4. \alpha.r\) \(i^2\) (39)

Then the unique level of 'd' is:

\[ d_3 = \frac{\pi.\beta(1-\alpha) - r}{2. \alpha.r} \] (40)

\[ ^{18} \text{For this unique solution, there is no steady state in the system when } "g_3-r" < "g_0-r" \text{ but the parametric equations can be changed to give us a solution.} \]
Case 3: New steady state A

It is feasible to think of a situation where, given the existing curves, I & II, the levels of risk are so high that a feasible steady state does not result, i.e.: when \( g_0 - r > g_2 - r \), \( g_3 - r \) and \( g_1 - r \) (see Figure 5). In fact let us assume the extreme case where the level of risk is so high that: \( g_0 - r > \pi \beta \bar{u}(1-\alpha) - r \), which is the value of \( 'g-r' \) where curve II intersects the horizontal axis.

If the economy had to reach a steady state level of debt and also maintain a steady rate of growth then it would require a rightward shift in the Curve II so that the new intersection between Curve II and the horizontal axis is to the right of \( 'g_0-r' \) once again. This would imply one or more of the following: a higher profit rate \( \pi \), equilibrium capacity utilisation \( \bar{u} \) or lower capitalist consumption \( \alpha \). This would shift the Curve II rightward and the concessions would continue till a new equilibrium is established (see Figure 6).
A new steady state equilibrium would be established at “d₄” corresponding to which the new growth rate would be “g₄”. The new growth rate would be higher than the previous growth rate and the debt level would be lower than the previous growth rate.

Case 4: New Steady State B

We now examine an intermediate case where the risk premium is high but not so high that it resembles Case 3 as discussed above. So $g₃ - r < g₀ - r < \pi\betaui(1-\alpha) - r$. This situation is represented in Figure 7. A solution similar to the one described in Case 3 would be feasible. But in this case an alternative could also work. Even if we do not shift Curve II, we could shift Curve I inwards till a new equilibrium growth rate ($g₅$) is established (see figure 8).
This parametric shift implies that in terms of equation 31 which describes Curve 1, for a given level of "g-r" there is a corresponding lower ratio of "i/g". If a new equilibrium were to be established, a lower level of debt "d_5" with a higher growth "g_s" would occur.

Conclusion

We conclude this chapter by briefly summing up our findings as presented above. Under fairly non-restrictive assumptions we have shown that even when public investment does not earn any profit, and is financed entirely by borrowing, it is possible for the economy to achieve a steady state level of debt and steady state rate of growth. In the presence of a minimal risk (as a boundary condition) perceived by entrepreneurs, it is possible for the economy to arrive at a unique solution. In one particular case, there is a unique solution when the variables are restricted to a certain combination of values. We also examined the possibility of
high risk perception by private investors and how equilibrium could be re-established with appropriate parametric changes (as in Case 3 and 4).

These results contradict the conservative position which claims that an economy where the state finances its expenditures by borrowing would slide into an ever increasing debt path and declining growth. We have demonstrated that this is unlikely to occur. Our model above shows that when the state borrows not only to meet its capital expenditures but also its interest obligations on past debt, the economy would achieve a steady state level of debt and rate of growth.

A criticism that could be levelled against this model is the absence of an explicitly articulated money equilibrium which is a standard feature in most macro models including the Keynesian and New consensus economics (Hicks 1937, Meyer 2001). However, this ‘absence’ is deceptive. A closer look would reveal that we have incorporated a monetary rule rather than a separate money equation which assumes a market clearing arrangement in the financial asset market. As we have explained elsewhere, this is a standard post-Keynesian assumption (Moore 1988). We assume that the economy operates at a given rate of interest which is determined outside this model. It is quite possible that the monetary authority attempts to maintain that rate of interest as a monetary rule. The experience of monetary “targetting” by various central banks seems to indicate that there is good reason to believe this (Fontana & Palacio-Vera 2002). The monetary sector plays an accommodative role to fiscal manoeuvres and adjusts money supply to maintain interest rates. Inflation is not a pure monetary phenomenon and stays controlled since the economy operates at less than full capacity. We have also not examined
issues in stability of the solutions obtained. This is a limitation of the above model. However, we are know from Domar (1944) that as long as the real growth rate is higher than real interest rate, the economy in the long run asymptotically converge to a steady state debt which would be stable. In which case, given the analysis above, we would have a steady state growth rate which would also be stable. But this needs to be tested in the present model before arriving at any conclusive statement. In the next chapter we conclude the essay by summarising our argument.