Chapter - 5

On Fuzzy Pairwise Semi Pre-irresolute Functions

In this chapter we introduce a new class of functions called fuzzy pairwise semi pre- irreseolute functions which is the generalization of fuzzy semi pre-irreseolute functions [5] in fuzzy topological spaces to fuzzy bitopological spaces.

In 1994, J. H. Park and B.H. Park introduced the concept of fuzzy pre-irreseolute functions in [52] and that of fuzzy pre-open sets and fuzzy pre-continuous functions in [67] and then it extended to fuzzy semi-pre-irreseolute function in [5]. The idea of fuzzy semi pre open sets and fuzzy semi pre-continuous function was introduced in 1994 by J. H. Park and et al. In 2004, Y. B. Im [27] introduced fuzzy pairwise pre-irreseolute functions and study their properties. Weaker forms of fuzzy pairwise continuity on fuzzy bitopological spaces as natural generalization of a fuzzy topological spaces have been considered by many workers using the concepts of \((\tau_i, \tau_j)\)-fuzzy semi-open sets and \((\tau_i, \tau_j)\)-fuzzy strongly semi open sets. Sampat Kumar [59] introduced a \((\tau_i, \tau_j)\)-fuzzy pre open set and fuzzy pairwise pre continuous function on fuzzy bitopological spaces.

In this chapter, using the concept of \((\tau_i, \tau_j)\)-fuzzy semi pre-closed set, \((\tau_i, \tau_j)\)-fuzzy semi pre-interior and \((\tau_i, \tau_j)\)-fuzzy semi pre- closure of \(X\) we investigate the properties of a fuzzy pairwise semi pre-irreseolute function on fuzzy bitopological spaces.

**Definition 5.1.** Let \((X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) be a function. Then \(f\) is called a fuzzy pairwise semi pre-irreseolute function if \(f^{-1}(\nu)\) is \((\tau_i, \tau_j)\)-fuzzy semi pre open set of \(X\) for each \((\sigma_i)\)-fuzzy semi pre open set \(\nu\) of \(Y\), or equivalently, if \(f^{-1}(\nu)\) is \((\tau_i, \tau_j)\)-fuzzy semi pre closed set of \(X\) for each \(\sigma_i\)-fuzzy semi pre closed set \(\nu\) of \(Y\).

A function \(f : X \rightarrow Y\) is fuzzy pre- irreseolute if \(f^{-1}(\alpha)\) is a fuzzy pre open subset of \(X\) for each fuzzy pre open subset \(\alpha\) of \(Y\).
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Since every fuzzy pre-open subset is fuzzy semi pre-open which can be extended to fuzzy pairwise semi pre-open. It can be seen that every fuzzy pairwise pre-irresolute function is fuzzy pairwise semi pre irresolute. But the converse is not true in general.

**Example 5.2:** Let \( \alpha_1, \alpha_2, \alpha_3 \) be fuzzy sets on \( I \) defined as follows:

\[
\alpha_1 = \begin{cases} 
0 & 0 \leq x \leq \frac{1}{2} \\
2x-1 & \frac{1}{2} \leq x \leq 1
\end{cases}
\]

\[
\alpha_2 = \begin{cases} 
1 & 0 \leq x \leq \frac{1}{4} \\
4x-1 & \frac{1}{4} \leq x \leq \frac{1}{2} \\
0 & \frac{1}{2} \leq x \leq 1
\end{cases}
\]

\[
\alpha_3 = \begin{cases} 
0 & 0 \leq x \leq \frac{1}{4} \\
-2x+2 & \frac{1}{4} \leq x \leq \frac{1}{2}
\end{cases}
\]

Consider fuzzy topologies

\( \tau_1 = \{0, \alpha_2, 1\} \), \( \tau_2 = \{0, \alpha_3, 1\} \)

Let \( f: (X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2) \) and defined by \( f(x) = x \) for each \( x \in [0, 1] \).

Then it can be easily shown that \(\alpha_3, \alpha_2, \alpha_1\) are \((\tau_i, \tau_j)\)-fspo set.

For \( \alpha_3 \):

\( \tau_2-\text{cl}\tau_2-\text{int}\alpha_3 = \tau_2-\text{cl}\tau_1-\text{int}(\alpha_3) = \tau_2-\text{cl}(\alpha_2) = \alpha_3 \geq \alpha_3 \)

\( \tau_1-\text{cl}\tau_2-\text{int}\alpha_3 = \tau_1-\text{cl}\tau_2-\text{int}(1) = \tau_1-\text{cl}(1) = 1 \geq \alpha_3 \quad \therefore \alpha_3 \) is \((\tau_i, \tau_j)\)-fspo set.

For \( \alpha_1 \):

\( \tau_2-\text{cl}\tau_1-\text{int}\alpha_1 = \tau_2-\text{cl}\tau_1-\text{int}(1) = \tau_2-\text{cl}(1) = 1 \geq \alpha_1 \)

\( \tau_1-\text{cl}\tau_2-\text{int}\alpha_1 = \tau_1-\text{cl}\tau_2-\text{int}(1) = \tau_1-\text{cl}(1) = 1 \geq \alpha_1 \quad \therefore \alpha_1 \) is \((\tau_i, \tau_j)\)-fspo set.

For \( \alpha_2 \):

\( \tau_2-\text{cl}\tau_1-\text{int}\alpha_2 = \tau_2-\text{cl}\tau_1-\text{int}(\alpha_2) = \alpha_1 \geq \alpha_2 \)

\( \tau_1-\text{cl}\tau_2-\text{int}\alpha_2 = \tau_1-\text{cl}\tau_2-\text{int}(1) = \tau_1-\text{cl}(1) = 1 \geq \alpha_2 \quad \therefore \alpha_2 \) is \((\tau_i, \tau_j)\)-fspo set.

\( f^{-1}(\alpha_3)(x) = \alpha_3 f(x) = \alpha_3(x) = \alpha_3 \), where \( \alpha_3 \) is fspo in \((X, \tau_1, \tau_2)\)
Similarly, it can be showed that $\alpha_2, \alpha_1$ are $(\tau_v, \tau_j)$-fspo set.

Which implies $f$ is $(\tau_i, \tau_j)$-fpsp irresolute function.

**Remark:** From the above definition it is clear that the following implication are true:

$$
\tau_i\text{-fo}(\tau_i\text{-fc}) \rightarrow (\tau_i, \tau_j)\text{-fpo }((\tau_i, \tau_j)\text{-fpc}) \rightarrow (\tau_i, \tau_j)\text{-fspo}
$$

$\Rightarrow$ fpsi function $\rightarrow$ fpspi function.

**Theorem 5.3.** Let $(X, \tau_1, \tau_2)$ be a fuzzy bitopological space and $\lambda$ is a fuzzy set of $X$. Then $\lambda$ is $(\tau_i, \tau_j)$-fuzzy semi preopen iff for each fuzzy point $x_{\beta} \in \lambda$ there exists a $(\tau_i, \tau_j)$-fuzzy semi pre open set $\mu$ such that $x_{\beta} \in \mu \leq \lambda$.

**Proof:** Obvious.

**Theorem 5.4.** Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a function from one fbt space to another fbt space. Then the following conditions are equivalent.

(a) $f$ is fuzzy pairwise semi pre-irresolute function.

(b) for every fuzzy point $x_p$ of $X$ and every $(\sigma_i)$-fuzzy semi pre-open set $\beta$ in $Y$ such that $f(x_p) \in \beta$ there is an $(\tau_i, \tau_j)$-fuzzy semi pre-open set $\alpha$ in $X$ such that $x_p \in \alpha$ and $f(\alpha) \leq \beta$.

(c) for every fuzzy point $x_p$ of $X$ and every $\sigma i$-fuzzy semi pre open set $\beta$ such that $f(x_p)\in \beta$ there is an $(\tau_i, \tau_j)$-fspo set $\alpha$ in $X$ such that $x_p \in \alpha$ and $f(\alpha) \leq \beta$.

(d) for every $(\sigma_i)$-fuzzy semi pre-closed set $\beta$. in $Y$, $f^1(\beta)$ is $(\tau_i, \tau_j)$-fuzzy semi pre-closed set $\alpha$ in $X$.

(e) $(\tau_i, \tau_j)$-spcl($f^1(v)$) $\leq$ $f^1((\sigma_i)$-spcl $v$) for each fuzzy set $v$ in $Y$.

(f) $f((\tau_i, \tau_j)$-spcl$\mu)$ $\leq$ $((\sigma_i)$-spcl$)(f(\mu)$ for each fuzzy set $\mu$ in $X$.

**Proof:** (a)$\Rightarrow$(b)

Let $x_p$ be a fuzzy point of $X$ and $\beta$ be a fuzzy semi pre-open set in $Y$ such that $f(x_p) \in \beta$. 

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Put $\alpha = f^{-1}(B)$.

Then by (a) [definition], $\alpha$ is an $(\tau_i, \tau_j)$-fuzzy semi pre-open set in $X$ such that $x_p \in \alpha$ and $f(\alpha) = ff^{-1}(\beta) \leq \beta$.

(b) $\Rightarrow$(a):

Let $x_p$ be a fuzzy point of $X$ and $\beta$ be a fspo set in $Y$. Let $x_p \in f^{-1}(\beta)$. Then $f(x_p) \in \beta$.

Now by (b), there is an $(\tau_i, \tau_j)$-fuzzy pre-open set $\alpha$ in $X$ such that $x_p \in \alpha$ and $f(\alpha) \leq \beta$.

Then $x_p \in \alpha \leq f^{-1}(\beta)$

Hence by theorem 5.3, $f^{-1}(\beta)$ is $(\tau_i, \tau_j)$-fuzzy semi pre-open set $\alpha$ in $X$. It means that $f$ is fuzzy pairwise semi pre irresolute function.

(a) $\Rightarrow$(c): Let $x_p$ be a fuzzy point of $X$ and $\beta$ be a fuzzy semi pre-open set in $Y$ such that $f(x_p) \in \beta$.

Let $\alpha = f^{-1}(\beta)$, then $\alpha$ is $(\tau_i, \tau_j)$-fuzzy semi pre-open set in $X$ [by (a)] and then $x_p \in \alpha$

And $f(\alpha) = ff^{-1}(\beta) \leq \beta$.

(c) $\Rightarrow$(a) Let $\delta$ be an $\sigma_i$-fuzzy semi pre-open set in $Y$ and $x_p \in f^{-1}(\delta)$.

imply $f(x_p) \in \delta$.

Let the fuzzy point $x_p^c = 1 - x_p$, then $f(x_p^c) \in \delta$

Now by (c), there exists an $(\tau_i, \tau_j)$-fpso set $\alpha$ in $X$ such that $x_p^c \in \alpha$ and $f(\alpha) \leq \delta$

Therefore, $x_p^c \in A \Rightarrow x_p^c + \alpha(x) > 1$

$\Rightarrow 1 - x_p + \alpha(x) > 1$

$\Rightarrow \alpha(x) > x_p$

$\Rightarrow x_p \in \alpha$

Thus $x_p \in \alpha \leq f^{-1}(\beta)$

Hence by theorem 5.3, $f^{-1}(\delta)$ is $(\tau_i, \tau_j)$-fspo set $\alpha$ in $X$

Thus $f$ is a fuzzy pairwise semi pre-irresolute function.
(a) $\Leftrightarrow$ (d): Obvious.

(d) $\Rightarrow$ (e): Suppose (d) holds.

Let $\nu$ be a fuzzy set of $Y$.

Then $\nu \leq (\sigma_i$-spcl$\nu)$

And also, $f^{-1}(\nu) \leq f^{-1}(\sigma_i$-spcl$\nu)$,

Here $f$ is fpspi mapping,

$\therefore f^{-1}(\sigma_i$-spcl$\nu)$ is a $(\tau_i, \tau_j)$-fspo set in $X$.

Hence

$$(\tau_i, \tau_j)$-spcl$(f^{-1}(\nu)) \leq (\tau_i, \tau_j)$-spcl$(f^{-1}(\sigma_i$-spcl$\nu)))$$

$$= f^{-1}(\sigma_i$-spcl$\nu))$$

(e) $\Rightarrow$ (f): Let $\mu$ be a fuzzy set of $X$.

$\therefore f(\mu) \leq (\sigma_i$-spcl$\mu)$

and $(\tau_i, \tau_j)$-spcl$\mu \leq (\tau_i, \tau_j)$-spcl$(f^{-1}(f(\mu)))$

$$\leq f^{-1}(f(\mu))$$

$$\leq f^{-1}(\sigma_i$-spcl$\mu)$

$\Rightarrow f(\tau_i, \tau_j)$-spcl$\mu \leq f(f^{-1}(\sigma_i$-spcl$\mu)))$

$$\leq (\sigma_i$-spcl$\mu)$

Hence proved.

(f) $\Rightarrow$ (a): Let $\eta$ be a $(\sigma_i)$-fuzzy semi pre closed set of $Y$.

Then $f(\tau_i, \tau_j)$-spcl$(f^{-1}(\eta)) \leq (\sigma_i)$-spcl$(f^{-1}(\eta))$

$$\leq (\sigma_i)$-spcl$ (\eta))$$

$$= \eta$$

$\Rightarrow (\tau_i, \tau_j)$-spcl$(f^{-1}(\eta)) \leq f^{-1}(\tau_i, \tau_j)$-spcl$$(f^{-1}(\eta))$
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\[ \Rightarrow \quad f^{-1}(\eta) \leq f^{-1}(\eta) \]

\[ \therefore f^{-1}(\eta) \text{ is a } (\tau_i, \tau_j)\text{-fspc set of } X \]

Hence \( f \) is a fuzzy pairwise semi pre irresolute function.

**Theorem 5.5.** If \( f : (X, \rho_1, \rho_2) \rightarrow (Y, \tau_1, \tau_2) \) and \( g: (Y, \tau_1, \tau_2) \rightarrow (Z, \sigma_1, \sigma_2) \) are two fuzzy pairwise semi pre-irresolute functions then \( gof : X \rightarrow Z \) is also fuzzy pairwise semi pre irresolute function.

**Proof:** Obvious. Since \( (g \circ f)(\lambda) = g(f(\lambda)) \) for each fuzzy set \( \lambda \) of \( X \)

And \( (g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu)) \) for each fuzzy set \( \mu \) of \( Z \).

**Theorem 5.6.** A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is a fuzzy pairwise semi pre irresolute iff \( f^{-1}((\sigma_i)\text{-spint}\mu) \leq (\tau_i, \tau_j)\text{-spint}f^{-1}(\mu) \) for each fuzzy set \( \mu \) in \( Y \).

**Proof:** Suppose \( \mu \) is a fuzzy set in \( Y \). Then \( \sigma_i\text{-spint}\mu \leq \mu \).

Since \( f \) is fuzzy pairwise semi pre irresolute,

\[ \therefore f^{-1}((\sigma_i)\text{-spint}\mu) \text{ is } (\tau_i, \tau_j)\text{-fuzzy semi preopen set in } X. \]

Hence \( f^{-1}((\sigma_i)\text{-spint}\mu) = (\tau_i, \tau_j)\text{-spint}(f^{-1}((\sigma_i)\text{-spint}\mu)) \)

\[ \leq (\tau_i, \tau_j)\text{-spint}(f^{-1}(\mu)) \]

Conversely, let \( \mu \) be a \( (\sigma_i)\text{-fuzzy semi preopen set in } Y \)

Then, \( f^{-1}(\mu) = f^{-1}((\sigma_i)\text{-spint}\mu) \)

\[ \leq (\tau_i, \tau_j)\text{-spint}(f^{-1}(\mu)) \]

\( \Rightarrow f^{-1}(\mu) \) is a \( (\tau_i, \tau_j)\text{-fuzzy semi preopen set in } X \). which indicates that \( f \) is fuzzy pairwise semi pre irresolute function.
Theorem 5.7. Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a bijection. Then \( f \) is a fuzzy pairwise semi pre irresolute iff \( (\sigma_i)\text{-spint}_{f(\mu)} \leq f((\tau_i, \tau_j)\text{-spint}_{\mu}) \) for each fuzzy set \( \mu \) in \( X \).

Proof: Let \( \mu \) be a fuzzy set in \( X \) then \( f(\mu) \) in \( Y \).

Then by the previous theorem,

\[
f^{-1}((\sigma_i)\text{-spint}_{\mu}) \leq (\tau_i, \tau_j)\text{-spint}_{f^{-1}(\mu)}
\]

Since \( f \) is bijection,

\[
((\sigma_i)\text{-spint}_{\mu}) \leq f(f^{-1}((\tau_i, \tau_j)\text{-spint}_{f^{-1}(\mu)})
\]

\[
\leq f((\tau_i, \tau_j)\text{-spint}_{f^{-1}(\mu)})
\]

\[
= f((\tau_i, \tau_j)\text{-spint}_{\mu})
\]

Conversely, let \( \alpha \) be a \( (\sigma_i)\)-fuzzy semi preopen set in \( Y \)

Then,

\[
(\sigma_i)\text{-spint}_{f^{-1}(\alpha)}) \leq f((\tau_i, \tau_j)\text{-spint}_{f^{-1}(\alpha)})
\]

As \( f \) is bijection,

\[
(\sigma_i)\text{-spint}_{\alpha) \leq f((\tau_i, \tau_j)\text{-spint}_{f^{-1}(\alpha)}
\]

This implies,

\[
f^{-1}(\alpha) \leq f^{-1}((\tau_i, \tau_j)\text{-spint}_{f^{-1}(\alpha)})
\]

\[
\leq (\tau_i, \tau_j)\text{-spint}_{f^{-1}(\alpha)}
\]

Hence, the statement of previous theorem [5.6]

\( f \) is a fuzzy pairwise semi preopen function.

Theorem 5.8. Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a function from a fbts \( X \) to another fbts \( Y \). Then if the graph function \( g : (X, \tau_1, \tau_2) \rightarrow (X \times Y, \theta_1, \theta_2) \) of \( f \), defined by \( g(x) = (x, f(x)) \) for each \( x \in X \), is fpspi, then \( f \) is also fpspi.
**Proof:** Let \( \alpha \) be a \( \delta_i \)-fpsp set of \( Y \). Then by

\[
(\text{Lemma 2.4 in [1]}) : \text{Let } g : X \to X \times Y \text{ be the graph of a function } f : X \to Y, \text{ then if } \lambda \text{ is a fuzzy set of } X \text{ and } \mu \text{ is a fuzzy set of } Y.
\]

\[
g^{-1}(\lambda \times \mu) = \lambda \land f^{-1}(\mu)
\]

We have \( f^{-1}(v) = 1 \land f^{-1}(v) = g^{-1}(1 \times \alpha) \)

Since \( g \) is fpsp and \( 1 \times \alpha \) is \( \theta_i \)-fpsp set of \( X \times Y \),

\( f^{-1}(\alpha) \) is a \((\tau_i, \tau_j)\)-fpsp set of \( X \).

This implies \( f \) is also fpsp.

**Theorem 5.9.** Let \((X_1, \tau_1, \tau_2), (X_2, \delta_1, \delta_2), (Y_1, \sigma_1, \sigma_2) \) and \((Y_2, \sigma_1, \sigma_2) \) be fbts’s such that \( X_1 \) is product related to \( X_2 \). Then the product \( f_1 \times f_2 : (X_1 \times X_2, \theta_1, \theta_2) \to (Y_1 \times Y_2, \beta_1, \beta_2) \) where \( \theta_i \) (resp \( \beta_k \)) is the fuzzy product topology generated by \( \tau_k \) and \( \delta_k \) (resp \( \omega_k \) and \( \sigma_k \)) (for \( k = 1, 2 \)) of fpsp function, where \( f_i : (X_i, \tau_i, \tau_2) \to (Y_i, \sigma_1, \sigma_2) \) and \( f_2 : (X_2, \delta_1, \delta_2) \to (Y_2, \sigma_1, \sigma_2) \) is fpsp.

**Proof:** Let \( W = \lor_{m,n} (U_m \times V_n) \),

Where \( U_m \)'s are \( \omega_i \)-fo sets of \( Y_1 \) and \( V_n \)'s are \( \sigma_i \)-fo sets of \( Y_2 \), be a \( \beta_i \)-fo sets of \( Y_1 \times Y_2 \).

\[
(\text{By Lemma 2.1 and Lemma 2.3 in [1]}) : \text{Let } f : X \to Y \text{ be a function and } \{\lambda_{\omega}\} \text{ be a family of fuzzy sets of } Y, \text{then } f^{-1}(\lor_{\omega}) = \lor f^{-1}(\lambda_{\omega})
\]

and for function \( f_i : X_i \to Y_i \) and fuzzy sets of \( Y_i \) \( i = 1, 2 \), we have

\[
(f_1 \times f_2)^{-1}(\lambda_1 \times \lambda_2) = f_1^{-1}(\lambda_1) \times f_2^{-1}(\lambda_2)
\]

Now

\[
(f_1 \times f_2)(W) = (f_1 \times f_2)^{-1}(\lor_{m,n} (U_m \times V_n))
\]

\[
= \lor_{m,n} (f_1 \times f_2)^{-1}(U_m \times V_n)
\]

\[
= \lor_{m,n} [f_1^{-1}(U_m) \times f_2^{-1}(V_n)]
\]

Since \( f_1, f_2 \) are fpsp, \( f^{-1}(U_m) \)'s are \( (\tau_i, \tau_j)\)-fpsp sets of \( X_1 \) and \( f_2^{-1}(V_n) \)'s are \( \delta_i \)-fpsp sets of \( X_2 \).
We also know \( \lor \) and \( \land \) are closed under fspo sets and fspo sets are also closed under product topology.

It follows that

\[
(f_1 \times f_2)^{-1}(W) \text{ is a } (\theta_1, \theta_2)\text{-fspo set of } X_1 \times X_2. \text{ Hence } f \text{ is fpsi.}
\]

**Fuzzy pairwise semi pre separated:**

**Definition. 5.10.** Two non-empty subsets \( \lambda \) and \( \mu \) in fbts \((X, \tau_i, \tau_j)\) are said to be fuzzy pairwise semi pre separated if \((\tau_i, \tau_j) - \text{spcl} \lambda \land \mu \quad \text{and} \quad (\tau_i, \tau_j) - \text{spcl} \mu \land \lambda.\)

**Theorem 5.11.** Let two non-empty subsets \( \lambda \) and \( \mu \) are in fbts \((X, \tau_i, \tau_j)\). The following are true.

(i) If \( \lambda \) and \( \mu \) are fuzzy pairwise semi pre-seperated and \( \lambda^c \) and \( \mu^c \) are non empty fuzzy subsets such that \( \lambda^c \leq \lambda \) and \( \mu^c \leq \mu \), then \( \lambda^c \) and \( \mu^c \) are also fuzzy pairwise semi pre-seperated.

(ii) If \( \lambda \land \mu \) and either both are fuzzy pairwise semi pre-open or both are fuzzy pairwise semi pre-closed then \( \lambda \) and \( \mu \) are fuzzy pairwise semi pre-seperated.

(iii) If \( \lambda \land \mu \) and either both are fuzzy pairwise semi pre-open or both are fuzzy pairwise semi pre-closed then \( \lambda \land \mu^c \) and \( \mu \land \lambda^c \) are fuzzy pairwise semi pre-seperated.

**Proof:** (i) and (ii) are easy to proof.

(iii) Let \( \lambda \) and \( \mu \) are fuzzy pairwise semi pre-open. Then \( \lambda^c \) and \( \mu^c \) are fuzzy pairwise semi pre-closed.

Now,

\[
\lambda \land \mu^c \leq \mu^c
\]

\[
\Rightarrow (\tau_i, \tau_j) - \text{spcl} (\lambda \land \mu^c) \leq (\tau_i, \tau_j) - \text{spcl} \mu^c = \mu^c = 1 - \mu
\]

\[
\Rightarrow (\tau_i, \tau_j) - \text{spcl} (\lambda \land \mu^c) \not\subseteq \mu
\]
Hence, \((\tau_i, \tau_j) - \text{spcl}(\lambda \land \mu^c)q (\lambda \land \mu^c)\)

Similarly we can show that \((\tau_i, \tau_j) - \text{spcl}(\mu \land \lambda^c)q (\mu \land \lambda^c)\).

Hence, \(\lambda \land \mu^c\) and \(\mu \land \lambda^c\) are fuzzy pairwise semi-pre-separated.

Let \(\lambda\) and \(\mu\) are fuzzy pairwise semi-pre-closed, then

\[
\lambda = (\tau_i, \tau_j) - \text{spcl}\lambda \quad \text{and} \quad \mu = (\tau_i, \tau_j) - \text{spcl}\mu
\]

\[
\lambda \land \mu^c \leq \mu^c \Rightarrow (\tau_i, \tau_j) - \text{spcl} \quad \mu \land \mu(\lambda \land \mu^c)
\]

And therefore, \((\tau_i, \tau_j) - \text{spcl}(\mu \land \lambda^c)q (\mu \land \lambda^c)\).

Similarly, \((\tau_i, \tau_j) - \text{spcl}(\lambda \land \mu^c)q (\lambda \land \mu^c)\).

\(:. \lambda \land \mu^c\) and \(\mu \land \lambda^c\) are fuzzy pairwise semi-pre-separated.

**Theorem 5.12.** two non-empty subsets \(\lambda\) and \(\mu\) are in fbts \((X, \tau_i, \tau_j)\) are fuzzy pairwise semi-pre-separated iff there exists two fuzzy pairwise semi-pre-open subsets \(\lambda^c\) and \(\mu^c\) such that \(\lambda \leq \lambda^c\) and \(\mu \leq \mu^c\), \(\lambda q \mu^c\) and \(\mu q \lambda^c\).

**Proof:** Let \(\lambda\) and \(\mu\) are fuzzy pairwise semi-pre-separated subsets. Let \(\lambda^c = 1 - (\tau_i, \tau_j) - \text{spcl}\lambda\) and

\[
\mu^c = 1 - (\tau_i, \tau_j) - \text{spcl}\mu.
\]

Then \(\lambda^c\) and \(\mu^c\) are fuzzy pairwise semi-pre-open subsets such that \(\lambda \leq \lambda^c\) and \(\mu \leq \mu^c\)

[since \(\lambda q (\tau_i, \tau_j) - \text{spcl}\mu \Rightarrow \lambda \leq 1 - (\tau_i, \tau_j) - \text{spcl}\mu \Rightarrow \lambda \leq \lambda^c\). Similarly, \(\mu \leq \mu^c\)]

Since, \(\lambda \leq (\tau_i, \tau_j) - \text{spcl}\lambda \Rightarrow \lambda q 1 - (\tau_i, \tau_j) - \text{spcl}\lambda \Rightarrow \lambda q \mu^c\) and also can show that \(\mu q \lambda^c\).

Coversely, let \(\lambda^c\) and \(\mu^c\) are two fuzzy pairwise semi-pre-open subsets such that \(\lambda \leq \lambda^c\) and \(\mu \leq \mu^c\), \(\lambda q \mu^c\) and \(\mu q \lambda^c\).
Since $1-\lambda^c$ and $1-\mu^c$ are fuzzy pairwise semi-pre-closed, we have

$$(\tau_i, \tau_j) - \text{spcl} \lambda \leq 1-\mu^c \leq 1-\mu \text{ and } (\tau_i, \tau_j) - \text{spcl} \mu \leq 1-\lambda^c \leq 1-\lambda.$$ 

Hence, $$(\tau_i, \tau_j) - \text{spcl} \lambda g \mu \text{ and } (\tau_i, \tau_j) - \text{spcl} \mu g \lambda.$$ 

$\therefore \lambda$ and $\mu$ are fuzzy pairwise semi-pre-separated.