CHAPTER-5

IF PRE SEMI CLOSED SETS AND IF PRE SEMI SEPARATION AXIOMS

5.1 INTRODUCTION

IF semi pre open set was defined by Young Bae Jun and Seok Zun Song in [35] as generalization of IF open sets. IF semi pre closure (IF spcl), IF semi pre interior (IF spint) and their different properties were studied by Biljan Krsteska and Erdal Ekici in [36]. IF generalized closed (IFGC) set was introduced by S.S.Thakur and Rekha Chaturvedi in [55] as generalization of IF open sets. Different properties of IF generalized closed sets were also studied by R.Dhavaseelan, E. Roja and M. K. Uma in [22]. IF generalized semi pre closed (IFGSPC) sets were introduced by R. Santhi and D. Jayanti in [47].

In this chapter the concept of IF pre semi closed set is introduced. The IF pre semi separation axioms with the help of IF pre semi closed sets are also introduced and different properties of these concepts are studied in IF topological space.
5.2 IF PRE SEMI CLOSED SETS

In this section the concept of IF pre semi closed set in IF topological space is introduced. Relationships between IF pre semi closed sets, IF semi pre closed sets, IF generalized closed sets and IF generalized semi pre closed sets are established and different properties of IF pre semi closed sets are studied in this section.

**Definition 5.2.1** An IF set $A$ in an IF topological space $(X, \tau)$ is called an IF pre semi closed set in $X$ if $\text{spcl}(A) \subseteq O$ whenever $A \subseteq O$ and $O$ is IF generalized open set in $X$.

**Example 5.2.2** Let $X = \{a, b\}$ and $U = < x, (a/.9, b/.2), (a/.1, b/.8) >$. Then the family $\tau = \{0\sim, 1\sim, U\}$ is an IF topology on $X$. Since every IF open set is IF g-open so IF sets $0\sim, 1\sim$ and $U$ are IF g-open sets in $X$. Then the IF set $A = < x, (a/.7, b/.2), (a/.3, b/.8) >$ is an IF pre semi-closed set in $X$, for if $A \subseteq O$ and $O$ is IF g-open set in $X$ then $O = 1\sim$ and $\text{spcl}(A) \subseteq O$.

**Theorem 5.2.3** Every IF semi pre closed set in an IF topological space $(X, \tau)$ is IF pre semi closed set.

**Proof** Let $A$ be an IF semi pre closed set in an IF topological space $(X, \tau)$ and $A \subseteq O$, where $O$ is IF generalized open set in $X$. Since $\text{spcl}(A) = A$, $\text{spcl}(A) = A \subseteq O$, and hence $A$ is IF pre semi closed set.

But the converse may not be true as shown in the next example.
Example 5.2.4  Let $X = \{a, b\}$ and $U = < x, (a/7, b/3), (a/3, b/7) >$. Then the family $\tau = \{0_\sim, 1_\sim, U\}$ is an IF topology on $X$. Since every IF open set is IF g-open so $0_\sim$, $1_\sim$ and $U$ are IF g-open sets in $X$. Then the IF set $A = < x, (a/8, b/3), (a/2, b/7) >$ is an IF pre semi closed set in $X$, for if $A \subseteq O$ and $O$ is IF g-open set in $X$, then $O = 1_\sim$ and spcl($A$) $\subseteq O$. But $A$ is not an IF semi pre closed set, for int($A$) = $U$, so cl(int($A$)) = $1_\sim \supset A$.

Remark 5.2.5 The IF pre semi closed ness is independent from IF generalized closed ness as shown in the following two examples.

Example 5.2.6  In example 5.2.4 $A$ is an IF pre semi closed set in $X$. But not an IF generalized closed set, for $A \subseteq U$ and $U$ is IF open set in $X$, but int(cl($A$)) = $1_\sim \supset A$.

Example 5.2.7  Let $X = \{a, b\}$ and $U = < x, (a/1, b/0), (a/0, b/5) >$ be an IF set in $X$. Then the family $\tau = \{0_\sim, 1_\sim, U\}$ is an IF topology on $X$. Then the IF set $A = < x, (a/1, b/2), (a/0, b/3) >$ is an IF generalized closed set in $X$ but not an IF pre semi closed set in $X$, for if $A \subseteq A$ and $A$ is an IF generalized open set in $X$, but int(cl(int($A$))) = $1_\sim \supset A$ and spcl($A$) = $1_\sim \supset A$.

Theorem 5.2.8  Every IF pre semi closed set in $(X, \tau)$ is an IF generalized semi pre closed set.
**Proof** Let $A$ be an IF pre semi closed set in $(X, \tau)$ and $A \subseteq O$, where $O$ is an IF open set in $X$, then $\text{spcl}(A) \subseteq O$ and $O$ is IF generalized open set in $X$. Hence $A$ is an IF generalized semi pre closed set in $X$.

But the converse may not true as shown in the following example.

**Example 5.2.9** In example 5.2.7 $A$ is an IF generalized semi pre closed set closed set in $X$ but not an IF pre semi closed set.

**Remark 5.2.10** The relationships between IF semi pre closed sets, IF pre semi closed sets and IF generalized semi pre closed sets are shown below:

\[
\begin{array}{c}
\text{IFSPC set} \\
\text{IFPSC set} \\
\text{IFGSPC set}
\end{array}
\]

However the converses are not true in general.

**Theorem 5.2.11** An IF set $A$ in $(X, \tau)$ is IF pre semi closed if and only if $\neg A \sqsubset F \Rightarrow \neg \text{spcl}(A) \sqsubset F$ for every IF generalized closed set $F$ of $X$.

**Proof: Necessity** Let $F$ be an IF generalized closed set of $X$ and $\neg A \sqsubset F$. Then by proposition 1.2.25 $A \subseteq F^c$ and $F^c$ is IF generalized open in $X$. Now since $A$ is IF pre-semi closed, $\text{spcl}(A) \subseteq F^c$. Hence $\neg \text{spcl}(A) \sqsubset F$.

**Sufficiency** Let $O$ be an IF generalized open set of $X$ such that $A \subseteq O$ that is $A \subseteq (O^c)^c$. Hence $\neg A \sqsubset O^c$ and $O^c$ is IF generalized closed set in $X$. Hence by hypothesis $\neg \text{spcl}(A) \sqsubset O^c$.Therefore $\text{spcl}(A) \subseteq (O^c)^c$. That is $\text{spcl}(A) \subseteq O$. Hence $A$ is IF pre-semi closed in $X$. 

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**Diagram:**

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IFSPC set \rightarrow \text{IFPSC set}
\downarrow
\text{IFGSPC set}
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Lemma 5.2.12 Let $A$ be an IFS in $(X,\tau)$. Then $A \cup \text{int}(\text{cl}(\text{int}(A))) \subseteq \text{spcl}(A)$.

Theorem 5.2.13 Let $A$ be an IF generalized open and IF pre semi closed set in an IF topological space $(X,\tau)$. Then $A$ is an IF semi pre closed set.

Proof Since $A$ is IF generalized open and IF pre semi closed, it follows that $A \cup \text{int}(\text{cl}(\text{int}(A))) \subseteq \text{spcl}(A) \subseteq A$. Hence $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ and $A$ is IF semi pre closed.

Theorem 5.2.14 Let $A$ be an IF set in $(X,\tau)$. Then the followings are equivalent:

(a) $A$ is IF regular open.
(b) $A$ is IF open and IF pre-semi closed.
(c) $A$ is IF open and IF generalized semi pre closed.

Proof (a) $\Rightarrow$ (b) Let $A$ be an IF regular open set in $(X,\tau)$. Then $A$ is both IF open and IF semi closed. Now since every IF semi closed set is IF semi pre closed set, hence by theorem 5.2.3 $A$ is an IF pre semi closed set.

(b) $\Rightarrow$ (c) Let $A$ be IF open and IF pre semi closed. Then by theorem 5.2.8 $A$ is IF generalized semi pre closed.

(c) $\Rightarrow$ (a) Let $A$ be IF open and generalized semi pre closed set in $X$. Then $A \subseteq A$ and $A$ is an IF open set in $X$. So $A \cup \text{int}(\text{cl}(\text{int}(A))) \subseteq \text{spcl}(A) \subseteq A$. Hence $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$. Since $A$ is an IF open set, $\text{int}(\text{cl}(A)) \subseteq A = \text{int}(A) \subseteq \text{int}(\text{cl}(A))$. Hence $A$ is IF regular open.
**Lemma 5.2.15** Let $A$ be an IF set in $(X, \tau)$. Then $\text{spcl}(\text{spcl}(A)) = \text{spcl}(A)$.

**Theorem 5.2.16** Let $A$ be an IF pre semi closed set in $(X, \tau)$. If $B$ is an IF set in $X$ such that $A \subseteq B \subseteq \text{spcl}(A)$ then $B$ is also IF pre-semi closed.

**Proof** Let $B$ be an IF set in an IF topological space $(X, \tau)$ such that $B \subseteq O$ and $O$ is an IF generalized open set in $X$. Then $A \subseteq O$, since $A$ is IF pre-semi closed, hence by lemma 5.2.15 $\text{spcl}(B) \subseteq \text{spcl}(\text{spcl}(A)) = \text{spcl}(A) \subseteq O$. Hence, $B$ is IF generalized semi pre closed in $X$.

**Definition 5.2.17** An IF set $A$ in an IF topological space $(X, \tau)$ is called an IF pre semi open if and only if its complement $A^c$ is IF pre semi closed.

**Theorem 5.2.18** An IF set $A$ in $(X, \tau)$ is IF pre semi open if $F \subseteq \text{spint}(A)$ whenever $F \subseteq A$ and $F$ is IF generalized closed set in $X$.

**Proof** Let IF set $A$ in an IF topological space $(X, \tau)$ is IF pre-semi open and $F$ is an IF generalized closed set in $X$ such that $F \subseteq A$. Then $A^c \subseteq F^c$, where $A^c$ is IF pre-semi closed and $F^c$ is an IF generalized open set in $X$. Hence from definition 5.2.1 $\text{spcl}(A^c) \subseteq F^c$. Hence $(F^c)^c \subseteq (\text{spcl}(A^c))^c$. That is $F \subseteq \text{spint}(A^c)^c = \text{spint}(A)$. 
5.3 IF PRE-SEMI-SEPARATION AXIOMS

In this section the concepts of IF pre semi $T_{1/2}$ space, IF semi pre $T_{1/3}$ space and IF pre semi $T_{3/4}$ space in IF topological spaces are introduced with the help of pre semi closed sets. IF semi pre $T_{1/2}$ space was introduced by R. Santhi and D. Jayanti [47]. Relationships between IF pre-semi $T_{1/2}$ space, IF semi-pre $T_{1/3}$ space, IF pre-semi $T_{3/4}$ space and IF semi pre $T_{1/2}$ space are established in this section.

**Definition 5.3.1** An IF topological space $(X, \tau)$ is said to be IF pre semi $T_{1/2}$ space if every IF pre semi closed set in $X$ is IF semi pre closed set in $X$.

**Theorem 5.3.2** Every IF semi pre $T_{1/2}$ space is an IF pre semi $T_{1/2}$ space.

**Proof** Let $(X, \tau)$ be an IF semi pre $T_{1/2}$ space and $A$ be an IF pre semi closed set in $X$. Now by theorem 5.2.8 every IF pre semi closed set is an IF generalized semi pre closed set. Hence $A$ is an IF generalized semi pre closed set in $X$ and consequently $A$ is IF semi pre closed set in $X$. Thus $(X, \tau)$ is IF pre semi $T_{1/2}$ space.

**Definition 5.3.3** An IF topological space $(X, \tau)$ is said to be IF semi pre $T_{1/3}$ space if every IF generalized semi pre closed set in $X$ is IF pre semi closed set in $X$. 
**Theorem 5.3.4** Every IF semi pre $T_{1/2}$ space is an IF semi pre $T_{1/3}$ space.

**Proof** Let $(X,\tau)$ be an IF semi pre $T_{1/2}$ space and $A$ be an IF generalized semi pre closed set in $X$. Then, $A$ is IF semi pre closed set in $X$. Now by theorem 5.2.3, every IF semi pre-closed set in $X$ is IF pre-semi closed. Thus $(X,\tau)$ is IF semi pre $T_{1/3}$ space.

**Definition 5.3.5** An IF topological space $(X,\tau)$ is said to be IF pre semi $T_{3/4}$ space if every IF pre semi closed set in $X$ is IF pre closed set in $X$.

**Theorem 5.3.6** Every IF pre semi $T_{3/4}$ space is an IF pre semi $T_{1/2}$ space.

**Proof** Let $(X,\tau)$ be an IF pre semi $T_{3/4}$ space and $A$ be an IF pre semi closed set in $X$. Then $A$ is IF pre closed set in $X$. Now every IF pre closed set in $X$ is IF semi pre closed set in $X$. Hence $A$ is an IF semi pre closed set in $X$. Thus $(X,\tau)$ is IF pre semi $T_{1/2}$ space.