CHAPTER-4

IF APPROXIMATELY-SEMI OPEN MAPPINGS

4.1 INTRODUCTION

IF generalized closed (IFGC) set was studied by S. S. Thakur and Rekha Chaturvedi in [55]. IF semi generalized closed (IFsgC) set was studied by S. S. Thakur and Jyoti Bajpai in [53] as a generalization of the concept of closed sets with the help of semi-openness. I. M. Hanafy, A. M. Abd El-Aziz and T. M. Salman introduced IF irresolute mappings in [29]. IF sg-irresolute mappings were introduced by R. Santhi and Arun Prakash in [45].

The concept of approximately semi-open (ap-semi open) mapping in fuzzy topological space was introduced by [49].

In this chapter the concepts of IF approximately semi-open (IF ap-semi open) mapping, IF pre semi-open mapping, IF pre semi-closed mapping, IF contra pre semi-open mapping are introduced in IF topological spaces. The relationships between these mappings are established and different properties of these mappings are studied in the section 4.2. Preservation of some IF topological structures under the IF approximately semi-open mappings and its composition with other IF mappings are examined in this section.
4.2 IF APPROXIMATELY-SEMI OPEN MAPPINGS

In this section IF approximately semi-open maps, a new generalization of semi open maps using the concept of IF semi generalized closed sets is considered. Some basic properties of these mappings will be studied. The definition enables us to obtain conditions under which inverse maps preserves semi generalized open sets. The relationship between this map and other mappings will be also established in this section.

**Definition 4.2.1** A mapping \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) is called an IF approximately semi open (IF ap-semi open) mapping if \( \text{scl}(B) \subseteq f(A) \) whenever \( B \) is IF sg-closed subset of \( Y \), \( A \) is IF semi open subset of \( X \) and \( B \subseteq f(A) \).

**Example 4.2.2** Let \( X = \{a, b\} \), \( Y = \{p, q\} \) be two non empty sets and \( A = < x, (a/.3, b/.2), (a/.7, b/.8) > \), \( B = < y, (p/.3, q/.2), (p/.7, q/.8) > \), \( C = < y, (p/.6, q/.7), (p/.4, q/.3) > \). Then the families \( \tau_1 = \{0\sim, 1\sim, A\} \) and \( \tau_2 = \{0\sim, 1\sim, B, C\} \) are IF topologies on \( X \) and \( Y \) respectively. Then the mapping \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) defined by \( f(a) = p \) and \( f(b) = q \), is IF ap- semi open mapping.

**Definition 4.2.3** A mapping \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) is called an IF pre semi open (pre semi closed) mapping if for every IF semi open (semi closed) set \( A \) of \( X \), \( f(A) \) is IF semi open (semi closed) set in \( Y \).
**Theorem 4.2.4** Every IF pre semi open mapping is IF ap-semi open mapping.

**Proof** Let the mapping \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) be an IF pre semi open mapping and for any IF sg-closed subset \( B \) of \( Y \), \( A \) is IF semi open subset of \( X \) such that \( B \subseteq f(A) \). Now since \( f \) is an IF pre semi open mapping so \( f(A) \) is an IF semi open set in \( Y \). Since \( B \) is IF sg-closed subset in \( Y \), so \( \text{scl}(B) \subseteq f(A) \). Hence \( \text{scl}(B) \subseteq f(A) \) whenever \( B \) is IF sg-closed subset of \( Y \), \( A \) is IF semi open subset of \( X \) and \( B \subseteq f(A) \). Hence the mapping \( f \) is an IF ap-semi open.

But the converse is not true in general as it has shown in the next example.

**Example 4.2.5** In Example 4.2.2 let \( D = < x, ( a/.4, b/.5 ), ( a/.5, b/.3 ) > \). Then \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) defined by \( f(a) = p \) and \( f(b) = q \), is IF ap-semi open mapping but not IF pre semi open. Since \( A \subset D \subseteq \text{cl}(A) \), \( D \) is an IF semi open set in \( X \) but \( f(D) = < y, ( p/.4, q/.5 ), ( p/.5, q/.3 ) > \) is not IF semi open set in \( Y \) as \( B \subset f(D) \) but \( f(D) \not\subseteq \text{cl}(B) = C^c = < y, ( p/.4, q/.3 ), ( p/.6, q/.7 ) > \).

**Theorem 4.2.6** If a mapping \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) is IF irresolute and IF ap-semi open then \( f^{-1}(A) \) is IF sg-open whenever \( A \) is IF sg-open subset of \( Y \).

**Proof** Let \( A \in \text{IFSGOS}(Y) \) and \( E \subseteq f^{-1}(A) \), where \( E \in \text{IFSCS}(X) \). Then \( f^{-1}(A^c) \subseteq E^c \) or \( A^c \subseteq f(E^c) \). Since the mapping \( f \) is IF ap-semi open then \( \{\text{sint}(A)\}^c = \text{scl}(A^c) \subseteq f(E^c) \). Hence \( [f^{-1}\{\text{sint}(A)\}]^c \subseteq E^c \) or
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E ⊆ f⁻¹{sint(A)}. Since f is IF irresolute mapping f⁻¹{sint(A)} is IF semi open in X. So E ⊆ f⁻¹{sint(A)} = sint [f⁻¹{sint(A)}] ⊆ sint{f⁻¹(A)} and f⁻¹(A) ∈ IFSGOS(X).

**Theorem 4.2.7** If f : (X,τ₁) → (Y,τ₂) is IF irresolute and IF ap-semi open mapping then f⁻¹(A) is IF sg-closed whenever A is IF sg-closed subset of Y.

**Proof** Assume that A ∈ IFSGCS(Y). Then Aᶜ ∈ IFSGOS(Y). Let E ⊆ f⁻¹(Aᶜ) where E ∈ IFSCS(X). Then f⁻¹((Aᶜ)c) ⊆ Eᶜ or A ⊆ f(Eᶜ). Since f is IF ap-semi open mapping then scl(A) ⊆ f(Eᶜ), where Eᶜ ∈ IFSOS(X). It follows that [f⁻¹ scl(A)] ⊆ Eᶜ. Since f is IF irresolute f⁻¹(sclA) ∈ IFSCS(X). So [scl(f⁻¹scl(A))] = f⁻¹scl(A) = scl(f⁻¹(A)) ⊆ f(Eᶜ) where Eᶜ is IF semi open set. Hence f⁻¹(A) ∈ IFSGCS(X).

**Theorem 4.2.8** If a mapping f : (X,τ₁) → (Y,τ₂) is IF surjective irresolute and IF ap-semi open then f is IF sg-continuous.

**Proof** Let A is IF-open set of Y. Then A is IF sg-open set of Y. Now by theorem 4.2.7 f⁻¹(A) is IF sg-open in X. Hence f is IF sg-continuous.

**Theorem 4.2.9** A mapping f : (X,τ₁) → (Y,τ₂) is IF ap-semi open if f(A) is an IF semi closed set in Y for every IF semi open subset A of X.

**Proof** Let B ⊆ f(A), where A is an IF semi open set of X and B is an IF semi closed set of Y. Therefore scl(B) ⊆ scl(f(A)). Then scl(B) ⊆ f(A). Since f(A) is an IF semi closed set of Y. Thus f is IF ap-semi open mapping.
Definition 4.2.10 A mapping \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) is called an IF ap-semi closed if \( f(A) \subseteq \text{sint}(B) \) whenever \( B \) is IF sg-open subset of \( Y \), \( A \) is IF semi closed subset of \( X \) and \( f(A) \subseteq B \).

Example 4.2.11 The mapping \( f \) in example 4.2.2 is IF ap-semi-closed also.

Theorem 4.2.12 A mapping \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) is IF ap-semi closed if \( f(A) \) is an IF semi open set in \( Y \) for every IF semi closed subset \( A \) of \( X \).

Proof Assume that \( f(A) \subseteq B \), where \( A \) is IF semi closed subset of \( X \) and \( B \) is an IF sg-open subset of \( Y \). Therefore \( \text{sint}(f(A)) \subseteq \text{sint}(B) \). Then \( f(A) \subseteq \text{sint}(B) \), since \( f(A) \) is IF semi open. Thus \( f \) is IF ap-semi closed mapping.

Theorem 4.2.13 Let \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) be a mapping. If the IF semi open and IF semi closed sets of \( Y \) coincide then \( f \) is IF ap-semi open if and only if \( f(A) \in \text{IFSCS}(Y) \) for every IF semi open set \( A \).

Proof Assume that \( f \) be an IF ap-semi open mapping. Let \( A \) be an arbitrary IF sub set of \( Y \) such that \( A \subseteq B \) where \( B \in \text{IFSOS}(Y) \). Since \( A \subseteq B \), so \( \text{scl}(A) \subseteq \text{scl}(B) \). Therefore all IF sub sets of \( Y \) are IF sg-closed (by hypothesis all are IF sg-open). So for any \( C \in \text{IFSOS}(X) \), \( f(C) \) is IF sg-closed in \( Y \). Since \( f \) is IF ap-semi open \( \text{scl}\{f(C)\} \subseteq f(C) \). Hence \( \text{scl}\{f(C)\} = f(C) \), that is \( f(C) \) is IF semi closed in \( Y \).

Conversely, let \( f : (X, \tau_1) \rightarrow (Y, \tau_2) \) be a mapping, IF semi open and IF semi closed sets of \( Y \) coincide and for every IF semi open set \( A \) of \( X \),
f(A) ∈ IFSCS(Y). Now since IF semi open and IF semi closed sets of Y coincide, so f(A) ∈ IFSOS(Y) and f is IF pre semi-open. By theorem 4.2.4 every IF pre semi open mapping is IF ap-semi open. So f is IF ap-semi open.

As an immediate consequence of previous theorem, we have the following:

**Corollary 4.2.14** Let f : (X, τ₁) → (Y, τ₂) be a mapping. If the IF semi open and IF semi closed sets of Y coincide then f is IF ap-semi open if and only if f is IF pre semi-open mapping.

**Definition 4.2.15** A mapping f : (X, τ₁) → (Y, τ₂) is called an IF contra pre semi open mapping if f(A) is IF semi closed set in Y for every IF semi open set A in X.

**Theorem 4.2.16** Every IF contra pre semi open mapping is IF ap-semi open.

**Proof** Assume that f : X → Y is IF contra pre semi open mapping. Let A be an IF semi open sub set of X and B is IF sg-closed sub set of Y such that B ⊆ f(A). Now we know that a mapping is IF contra pre semi-open if f(A) is IF semi-closed set in Y for every IF semi open set A in X. f being contra pre semi-open if f(A) is IF semi-closed set in Y. Therefore \( \text{scl}(B) \subseteq \text{scl}(f(A)) = f(A) \), since f(A) is an IF-semi-closed of Y. Thus f is IF ap-semi-open mapping.

The following examples shows that the concepts of IF contra pre semi open mappings and IF pre semi open mappings are independent.
Example 4.2.17 Let $X = \{a, b\}$, $Y = \{p, q\}$ be two non empty sets and

$A = < x, (a/.4, b/.2), (a/.5, b/.6) >,$

$B = < y, (p/.4, q/.3), (p/.5, q/.2) >,$

$C = < y, (p/.2, q/.3), (p/.5, q/.3) >.$

Then the families $\tau_1 = \{0\sim, 1\sim, A\}$ and $\tau_2 = \{0\sim, 1\sim, B, C\}$ are IF topologies on $X$ and $Y$ respectively. Let the mapping $f : X \to Y$ be defined by $f(a) = p$ and $f(b) = q$. Then $f$ is IF contra pre semi open but not IF pre semi open mapping. Since $A \subset D = < x, (a/.5, b/.4), (a/.5, b/.3) > \subset \text{cl } A = < x, (a/.5, b/.6), (a/.4, b/.2) >$, $D$ is IF semi open in $X$. But $f(D) = < y, (p/.4, q/.5), (p/.5, q/.3) >$ is not semi open in $Y$ as $B \subset f(D) \nsubseteq \text{cl } B = C^c = < y, (p/.4, q/.3), (p/.6, q/.7) >$.

Example 4.2.18 Let $X = \{a, b\}$, $Y = \{p, q\}$ be two non empty sets and

$A = < x, (a/.3, b/.2), (a/.7, b/.8) >,$

$B = < y, (p/.5, q/.3), (p/.5, q/.6) >.$

Then the families $\tau_1 = \{0\sim, 1\sim, A\}$ and $\tau_2 = \{0\sim, 1\sim, B\}$ are IF topologies on $X$ and $Y$ respectively. Let the mapping $f : X \to Y$ be defined by $f(a) = p$ and $f(b) = q$. Then $f$ is IF pre semi open but not IF contra pre semi open mapping. Since $A \subset C = < x, (a/.4, b/.5), (a/.5, b/.3) > \subset \text{cl } A = < x, (a/.7, b/.8), (a/.3, b/.2) >$, $C$ is IF semi open in $X$. But the IF set $f(C) = < y, (p/.4, q/.5), (p/.5, q/.3) >$ is not IF semi-closed set in $Y$. As $f(C) \subset B^c = < y, (p/.5, q/.6), (p/.5, q/.3) >$ but $\text{int } (B^c) = B = \nsubseteq f(C)$. 
Remark 4.2.19  Hence the relationship of IF ap-semi open mappings with IF contra pre semi open mappings, IF pre semi open mappings are as follows:

\[
\begin{array}{c}
\text{IF contra pre semi open mapping} \\
\downarrow \\
\text{IF pre semi open mapping} \\
\downarrow \\
\text{IF ap-semi open map}
\end{array}
\]

However the converses are not true in general.

Theorem 4.2.20  Let \( f : (X, \tau_1) \to (Y, \tau_2) \) be an IF pre semi open mapping and \( g : (Y, \tau_2) \to (Z, \tau_3) \) be an IF ap-semi open mapping. Then the composition \( g \circ f : (X, \tau_1) \to (Z, \tau_3) \) is IF ap-semi open.

Proof  Suppose that \( A \) is an IF semi open sub set in \( X \) and \( B \) be an IF sg-closed set of \( Z \) for which \( B \subseteq (g \circ f)(A) \). Then \( f(A) \) is IF semi open in \( Y \), as \( f \) is IF pre semi open. Since \( g \) is IF ap-semi open, \( \text{scl}(B) \subseteq g\{ f(A) \} \).

Thus \( g \circ f \) is IF ap-semi open.

Theorem 4.2.21  Let \( f : (X, \tau_1) \to (Y, \tau_2) \) be an IF ap-semi open mapping and \( g : (Y, \tau_2) \to (Z, \tau_3) \) be a bijective IF pre semi closed and IF sg - irresolute mapping. Then the composition \( g \circ f : (X, \tau_1) \to (Z, \tau_3) \) is IF ap-semi open.

Proof  Suppose that \( A \) is an arbitrary IF semi open sub set in \( X \) and \( B \) be an IF sg-closed set of \( Z \) for which \( B \subseteq (g \circ f)(A) \). Hence \( g^{-1}(B) \subseteq f(A) \).

Then \( \text{scl}\{g^{-1}(B)\} \subseteq f(A) \), as \( g^{-1}(B) \) is IF sg-closed and \( f \) is IF ap-semi open. So \( \text{scl}(B) \subseteq \text{scl}\{gg^{-1}(B)\} \subseteq g(\text{scl}(g^{-1}(B))) \subseteq g\{ f(A) \} = (g \circ f)(A) \).

Thus the composition \( g \circ f \) is IF ap-semi open.
Theorem 4.2.22 Let $f : (X, \tau_1) \to (Y, \tau_2)$ be an IF ap-semi open mapping and $g : (Y, \tau_2) \to (Z, \tau_3)$ be a mapping such that $g^{-1}$ preserves IF sg-closed sets. Then the composition $g \circ f : (X, \tau_1) \to (Z, \tau_3)$ is IF ap-semi open.

**Proof** Suppose that $A$ is an IF semi open sub set in $X$. Let $B$ be IF sg-closed set of $Z$ and $B \subseteq (g \circ f)(A)$. Hence $g^{-1}(B) \subseteq f(A)$. Since $g^{-1}(B)$ is IF sg-closed and $f$ is IF ap-semi open, hence $scl(g^{-1}(B)) \subseteq f(A)$. Now $(g \circ f)(A) = g \circ (f(A)) \supseteq g \circ (scl(g^{-1}(B))) \supseteq scl(g \cdot g^{-1}(B)) \supseteq scl(B)$. Thus the composition $g \circ f$ is IF ap-semi open.

Theorem 4.2.23 If $f : (X, \tau_1) \to (Y, \tau_2)$ is IF irresolute and IF ap-semi closed then $f^{-1}$ preserves IF sg-closed sets.

**Proof** Suppose that $A$ is an IF semi open sub set in $X$. Let $B$ be an IF sg-closed set of $Z$ and $B \subseteq (g \circ f)(A)$. Hence $g^{-1}(B) \subseteq f(A)$. Since $g^{-1}(B)$ is IF sg-closed and $f$ is IF ap-semi open, hence $scl(g^{-1}(B)) \subseteq f(A)$. Now $(g \circ f)(A) = g \circ (f(A)) \supseteq g \circ scl(g^{-1}(B)) \supseteq scl(g \cdot g^{-1}(B)) \supseteq scl(B)$. Thus $g \circ f$ is IF ap-semi open.

Theorem 4.2.24 Let $f : (X, \tau_1) \to (Y, \tau_2)$ be an IF pre semi closed mapping and $g : (Y, \tau_2) \to (Z, \tau_3)$ be an IF ap-semi closed mapping. Then the composition $g \circ f : (X, \tau_1) \to (Z, \tau_3)$ is IF ap-semi closed.

**Proof** Suppose that $A$ is an IF semi closed sub set in $X$ and $B$ is an IF sg-open set of $Z$ such that $(g \circ f)(A) \subseteq B$. Since $f$ is an IF pre-semi closed mapping, $f(A)$ is an IF semi closed set in $Y$. Now since $g$ is IF ap-semi
closed, \( g\{f(A)\} \subseteq \text{sint}(B) \). Thus the composition \( g \circ f \) is IF ap-semi closed.

**Theorem 4.2.25** Let \( f : (X, \tau_1) \to (Y, \tau_2) \) be an IF ap-semi closed mapping and \( g : (Y, \tau_2) \to (Z, \tau_3) \) be an IF pre semi open mapping. Then the composition \( g \circ f : (X, \tau_1) \to (Z, \tau_3) \) is IF ap-semi closed if \( g^{-1} \) preserves IF sg-open sets.

**Proof** Suppose that \( A \) is an IF semi closed sub set in \( X \) and \( B \) is an IF sg-open set of \( Z \) such that \( (g \circ f)(A) \subseteq B \). Hence \( f(A) \subseteq g^{-1}(B) \). Since \( g^{-1}(B) \) is IF sg-open and \( f \) is IF ap-semi open, so \( f(A) \subseteq \text{sint}(g^{-1}(B)) \). Now \( (g \circ f)(A) = g \circ f(A) \subseteq g \circ \text{sint}(g^{-1}(B)) \subseteq \text{sint}(g \circ g^{-1}(B)) \subseteq \text{sint}(B) \). So the composition \( g \circ f \) is IF ap-semi closed.

**Theorem 4.2.26** If \( f : (X, \tau_1) \to (Y, \tau_2) \) is a mapping from an IF topological space \((X, \tau_1)\) to an IF semi \( T_{1/2} \)-space \((Y, \tau_2)\) then \( f \) is an IF ap-semi open mapping.

**Proof** Suppose that \( B \) is an IF sg-closed subset of \( Y \) and \( B \subseteq f(A) \), where \( A \in \text{IFSOS}(X) \). Since \( Y \) is IF semi \( T_{1/2} \)-space, \( B \) is IF semi closed in \( Y \) that is \( B = \text{scl}(B) \). Hence \( \text{scl}(B) \subseteq f(A) \). Then \( f \) is IF ap-semi open.

**Theorem 4.2.27** If \( f : (X, \tau_1) \to (Y, \tau_2) \) is a mapping from an IF topological space \((X, \tau_1)\) to an IF semi \( T_{1/2} \)-space \((Y, \tau_2)\) then \( f \) is an IF ap-semi closed mapping.

**Proof** Suppose that \( B \) is an IF sg-open subset of \( Y \) and \( A \) is an IF semi closed subset of \( X \) such that \( f(A) \subseteq B \). Since \( B \) be IF sg-open subset of \( Y \),
B^c is IF sg-closed and Y being IF semi T_{1/2}-space, B^c is IF semi closed in Y that is B^c = scl(B^c). Now f(A) ⊆ B gives B^c ⊆ (f(A))^c or scl(B^c) = B^c ⊆ (f(A))^c or f(A) ⊆ B = (scl(B^c))^c = sint (B) Hence f(A) ⊆ B = sint(B). Hence f is IF ap-semi closed.