STOCHASTIC ANALYSIS OF
A BLOOD BANK SYSTEM MODEL

2.1. INTRODUCTION

A lot of work has been done in the field of reliability theory. But most of it is concerned with the hypothetical models. However, Kumar [72], Kumar and Muthan [74], Singh & Goel [106], Singh et al. [107], Singh & Singh [105], Gupta & Shivakar (57) etc. have analysed real existing system models under different sets of assumptions. We, in the present chapter, analyse a real existing blood bank system model. A blood bank system plays a significant role in saving lives of people. It involves high risk in respect of maintaining the blood within the temperature’s limits between 4°C to 8°C. The blood can be stored in a blood bank for a maximum of 35 days. It is a complex type reparable engineering system which consists of three subsystems namely – Power supply unit (P), Chillers (C₁ & C₂) and Generator (G). The working of these subsystems is as follows–

(a) **Power supply (P):** This is nothing but the electricity supplied by the local Power supply station and it affects due to usual power cuts and any fault in the power supply system.

(b) **Chillers (C₁ & C₂):** These are the main units of the system. The proper functioning of at least one chiller is must for the preservation of blood. The capacity of chillers may vary according to the requirement of the blood bank. The chillers maintain the temperature of the stored blood between 4°C and 8°C. If the temperature of blood goes beyond the limit of temperature, it can affect the activeness of R.B.C. and W.B.C. and the chiller is said to be failed.

(c) **Generator (G):** It is used for the power backup if the power supplied to the system is affected and may also fail during its working. The time taken in activation of generator is assumed to be negligible as the
switching and sensing devices are considered to be perfect & instantaneous.

Using the regenerative point technique, the following measures of system effectiveness are obtained –

(i) Transition probabilities and mean sojourn times in different states.
(ii) Reliability of the system and mean time to system failure (MTSF).
(iii) Pointwise and steady state availabilities of the system during (0, t).
(iv) Expected busy period of repairman during (0, t) and in steady state.
(v) Expected profit incurred by the system during (0, t) and in steady state.

2.2. MODEL DESCRIPTION AND ASSUMPTIONS

The assumptions about the model under study are as under-

(i) The system consists of three subsystems viz. – Power supply unit (P), Chillers \(C_1 \& C_2\) and Generator (G). Initially, the system starts its operation from state \(S_0\) in which both the identical chillers are operative in parallel and power supply is present while the functioning of at least one chiller is sufficient to do the job. The generator is put into cold standby.

(ii) If one chiller fails the system still operates with the other chiller and the failed chiller enters into repair.

(iii) If the electricity is not supplied by the local power supply station, the generator is started automatically with the help of an instantaneous and perfect switching device.

(iv) When any or both the chillers are not in working states but are good then they do not deteriorate i.e. found good whenever required.

(v) A repairman is always available with the system to repair the failed chiller and failed generator.
(vi) When both the chillers and generator are failed and power supply is present then the priority in repair is given to the chiller over the generator to keep the system up as early as possible otherwise the repair discipline is FCFS.

(vii) All failure and repair time distributions are taken as exponential with different parameters.

(viii) The rates of appearance and disappearance of power supply are constant.

(ix) Each repaired unit works as good as new.

2.3. NOTATIONS AND STATES OF THE SYSTEM

Notations

We define the following notations –

\[ \eta \] : constant rate of appearance of power supply.

\[ \theta \] : constant rate of disappearance of power supply.

\[ \alpha_1 \] : constant failure rate of Generator.

\[ \alpha_2 \] : constant failure rate of Chiller.

\[ \beta_1 \] : constant repair rate of the Generator.

\[ \beta_2 \] : constant repair rate of the Chiller.

The other notations used in this chapter are defined in Glossary of general notations mentioned at the starting of the thesis.

Symbols for the states of the system

We define the following symbols for the states of the system –

\[ P_O / P_d \] : power supply is on/down.

\[ G_S / G_g / G_O / G_r / G_w r \] : generator is in standby/good/operative mode/under repair/waiting for repair.
Using these symbols and the assumptions stated earlier, the possible states of the system are as follows–

**Up states**

\[
S_0 = \begin{pmatrix} P & G_S \\ C_O & C_O \end{pmatrix}; \quad S_1 = \begin{pmatrix} P_d & G_O \\ C_O & C_O \end{pmatrix}
\]

\[
S_2 = \begin{pmatrix} P & G_S \\ C_r & C_O \end{pmatrix}; \quad S_3 = \begin{pmatrix} P_d & G_O \\ C_r & C_O \end{pmatrix}
\]

\[
S_5 = \begin{pmatrix} P & G_r \\ C_O & C_O \end{pmatrix}; \quad S_7 = \begin{pmatrix} P & G_r \\ C_{wr} & C_O \end{pmatrix}
\]

**Failed States**

\[
S_4 = \begin{pmatrix} P_d & G_r \\ C_g & C_g \end{pmatrix}; \quad S_6 = \begin{pmatrix} P_d & G_r \\ C_{wr} & C_g \end{pmatrix}
\]

\[
S_8 = \begin{pmatrix} P & G_S \\ C_r & C_{wr} \end{pmatrix}; \quad S_9 = \begin{pmatrix} P_d & G_g \\ C_r & C_{wr} \end{pmatrix}
\]

\[
S_{10} = \begin{pmatrix} P & G_{wr} \\ C_r & C_{wr} \end{pmatrix}.
\]

The transition diagram of the system model, along with all the transitions is shown in Fig. 2.1.

### 2.4. Transition Probabilities and Sojourn Times

(a) Transition Probabilities

\( Q_{ij}(t) \) is defined as the probability that the system transits from state \( S_i \) to state \( S_j \) on or before time \( t \) or the c.d.f. of transition time from regenerative state \( S_i \) to \( S_j \). As an illustration \( Q_{01}(t) \) is obtained as follows-
Suppose the system transits from state $S_0$ to $S_1$ during $(u, u+du)$, $u \leq t$. The probability of this event is $\theta e^{-\theta u} du \cdot e^{-\alpha_1 u}$. Since $u$ varies from 0 to $t$, therefore

$$Q_{01}(t) = \theta \int_0^t e^{-(\theta+2\alpha_2)u} du = \frac{\theta}{(\theta+2\alpha_2)^t \left[ 1 - e^{-(\theta+2\alpha_2)t} \right]}$$

Similarly,

$$Q_{02}(t) = 2\alpha_2 \int_0^t e^{-(\theta+2\alpha_2)u} du = \frac{2\alpha_2}{(\theta+2\alpha_2)^t \left[ 1 - e^{-(\theta+2\alpha_2)t} \right]}$$

$$Q_{10}(t) = \eta \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{\eta}{(\alpha_1+2\alpha_2+\eta)^t \left[ 1 - e^{-(\alpha_1+2\alpha_2+\eta)t} \right]}$$

$$Q_{13}(t) = 2\alpha_2 \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{2\alpha_2}{(\alpha_1+2\alpha_2+\eta)^t \left[ 1 - e^{-(\alpha_1+2\alpha_2+\eta)t} \right]}$$

$$Q_{14}(t) = \alpha_1 \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{\alpha_1}{(\alpha_1+2\alpha_2+\eta)^t \left[ 1 - e^{-(\alpha_1+2\alpha_2+\eta)t} \right]}$$

$$Q_{20}(t) = \beta_2 \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{\beta_2}{(\alpha_1+2\alpha_2+\eta)^t \left[ 1 - e^{-(\alpha_1+2\alpha_2+\eta)t} \right]}$$

$$Q_{23}(t) = \theta \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{\theta}{(\theta+2\alpha_2)^t \left[ 1 - e^{-(\alpha_1+2\alpha_2+\eta)t} \right]}$$

$$Q_{28}(t) = \alpha_2 \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{\alpha_2}{(\alpha_1+2\alpha_2+\eta)^t \left[ 1 - e^{-(\alpha_1+2\alpha_2+\eta)t} \right]}$$

$$Q_{31}(t) = \beta_2 \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{\beta_2}{(\eta+\alpha_1+\alpha_2+\beta_2)^t \left[ 1 - e^{-(\eta+\alpha_1+\alpha_2+\beta_2)t} \right]}$$

$$Q_{32}(t) = \eta \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{\eta}{(\eta+\alpha_1+\alpha_2+\beta_2)^t \left[ 1 - e^{-(\eta+\alpha_1+\alpha_2+\beta_2)t} \right]}$$

$$Q_{36}(t) = \alpha_1 \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{\alpha_1}{(\eta+\alpha_1+\alpha_2+\beta_2)^t \left[ 1 - e^{-(\eta+\alpha_1+\alpha_2+\beta_2)t} \right]}$$

$$Q_{39}(t) = \alpha_2 \int_0^t e^{-(\alpha_1+2\alpha_2+\eta)u} du = \frac{\alpha_2}{(\eta+\alpha_1+\alpha_2+\beta_2)^t \left[ 1 - e^{-(\eta+\alpha_1+\alpha_2+\beta_2)t} \right]}$$
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\[ Q_{41}(t) = Q_{63}(t) = \beta_1 \int_0^t e^{-(\eta + \beta_1)u} \, du = \frac{\beta_1}{(\eta + \beta_1)} [1 - e^{-(\eta + \beta_1)t}] \]

\[ Q_{45}(t) = Q_{67}(t) = \eta \int_0^t e^{-(\eta + \beta_1)u} \, du = \frac{\eta}{(\eta + \beta_1)} [1 - e^{-(\eta + \beta_1)t}] \]

\[ Q_{50}(t) = \beta_1 \int_0^t e^{-(\theta + \beta_1 + 2\alpha_2)u} \, du = \frac{\beta_1}{(\theta + \beta_1 + 2\alpha_2)} [1 - e^{-(\theta + \beta_1 + 2\alpha_2)t}] \]

\[ Q_{54}(t) = \theta \int_0^t e^{-(\theta + \beta_1 + 2\alpha_2)u} \, du = \frac{\theta}{(\theta + \beta_1 + 2\alpha_2)} [1 - e^{-(\theta + \beta_1 + 2\alpha_2)t}] \]

\[ Q_{57}(t) = 2\alpha_2 \int_0^t e^{-(\theta + \beta_1 + 2\alpha_2)u} \, du = \frac{2\alpha_2}{(\theta + \beta_1 + 2\alpha_2)} [1 - e^{-(\theta + \beta_1 + 2\alpha_2)t}] \]

\[ Q_{72}(t) = \beta_1 \int_0^t e^{-(\theta + \beta_1 + \alpha_2)u} \, du = \frac{\beta_1}{(\theta + \beta_1 + \alpha_2)} [1 - e^{-(\theta + \beta_1 + \alpha_2)t}] \]

\[ Q_{76}(t) = \theta \int_0^t e^{-(\theta + \beta_1 + \alpha_2)u} \, du = \frac{\theta}{(\theta + \beta_1 + \alpha_2)} [1 - e^{-(\theta + \beta_1 + \alpha_2)t}] \]

\[ Q_{710}(t) = \alpha_2 \int_0^t e^{-(\theta + \beta_1 + \alpha_2)u} \, du = \frac{\alpha_2}{(\theta + \beta_1 + \alpha_2)} [1 - e^{-(\theta + \beta_1 + \alpha_2)t}] \]

\[ Q_{82}(t) = Q_{93}(t) = Q_{107}(t) = \beta_2 \int_0^t e^{-\beta_2u} \, du = 1 - e^{-\beta_2t} \quad (1-21) \]

(b) Steady – State Transition Probabilities

The steady state transition probabilities are given by

\[ p_{ij} = \lim_{t \to \infty} Q_{ij}(t) \]

Therefore,

\[ p_{01} = \theta \int_0^t e^{-(\theta + 2\alpha_2)t} \, dt = \frac{\theta}{\theta + 2\alpha_2} \]

\[ p_{02} = \frac{2\alpha_2}{\theta + 2\alpha_2}; \quad p_{10} = \frac{\eta}{\eta + \alpha_1 + 2\alpha_2} \]

\[ p_{13} = \frac{2\alpha_2}{\eta + \alpha_1 + 2\alpha_2}; \quad p_{14} = \frac{\alpha_1}{\eta + \alpha_1 + 2\alpha_2} \]
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\[ p_{20} = \frac{\beta_2}{\theta + \alpha_2 + \beta_2}; \quad p_{23} = \frac{\theta}{\theta + \alpha_2 + \beta_2} \]

\[ p_{28} = \frac{\alpha_2}{\theta + \alpha_2 + \beta_2}; \quad p_{31} = \frac{\beta_2}{\eta + \alpha_2 + \beta_2} \]

\[ p_{32} = \frac{\eta}{\eta + \alpha_1 + \alpha_2 + \beta_2}; \quad p_{36} = \frac{\alpha_1}{\eta + \alpha_1 + \alpha_2 + \beta_2} \]

\[ p_{39} = \frac{\alpha_2}{\eta + \alpha_1 + \alpha_2 + \beta_2}; \quad p_{41} = p_{63} = \frac{\beta_1}{\eta + \beta_1} \]

\[ p_{45} = p_{67} = \frac{\eta}{\eta + \beta_1}; \quad p_{50} = \frac{\beta_1}{\theta + \beta_1 + 2\alpha_2} \]

\[ p_{54} = \frac{\theta}{\theta + \beta_1 + 2\alpha_2}; \quad p_{57} = \frac{2\alpha_2}{\theta + \beta_1 + 2\alpha_2} \]

\[ p_{72} = \frac{\beta_1}{\theta + \beta_1 + \alpha_2}; \quad p_{76} = \frac{\theta}{\theta + \beta_1 + \alpha_2} \]

\[ p_{7,10} = \frac{\alpha_2}{\theta + \beta_1 + \alpha_2}; \quad p_{82} = p_{93} = p_{10,7} = 1 \quad (22–42) \]

It can be easily verified that

\[ p_{01} + p_{02} = 1; \quad p_{10} + p_{13} + p_{14} = 1 \]

\[ p_{20} + p_{23} + p_{28} = 1; \quad p_{31} + p_{32} + p_{36} + p_{39} = 1 \]

\[ p_{41} + p_{45} = 1; \quad p_{50} + p_{54} + p_{57} = 1 \]

\[ p_{63} + p_{67} = 1; \quad p_{72} + p_{76} + p_{7,10} = 1 \quad (43–50) \]

(c) Mean sojourn time

Mean sojourn time \( \psi_i \) in state \( S_i \) is defined as the expected time for which the system stays in state \( S_i \) before transiting to any other state. If \( T_i \) is the sojourn time in state \( S_i \), then the mean sojourn time in state \( S_i \) is

\[ \psi_i = \int P(T_i > t) \, dt \]
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Its values for various states are as follows–

\[ \psi_0 = \int e^{-(\theta + 2\alpha_2)t} dt = \frac{1}{(\theta + 2\alpha_2)} \]

Similarly,

\[ \psi_1 = \int e^{-(\eta + \alpha_1 + 2\alpha_2)t} dt = \frac{1}{(\eta + \alpha_1 + 2\alpha_2)} \]

\[ \psi_2 = \int e^{-(\theta + \alpha_2 + \beta_2)t} dt = \frac{1}{(\theta + \alpha_2 + \beta_2)} \]

\[ \psi_3 = \int e^{-(\eta + \alpha_1 + \alpha_2 + \beta_2)t} dt = \frac{1}{(\eta + \alpha_1 + \alpha_2 + \beta_2)} \]

\[ \psi_4 = \psi_6 = \int e^{-(\eta + \beta_1)t} dt = \frac{1}{(\eta + \beta_1)} \]

\[ \psi_5 = \int e^{-(\theta + \beta_1 + 2\alpha_2)t} dt = \frac{1}{(\theta + \beta_1 + 2\alpha_2)} \]

\[ \psi_7 = \int e^{-(\theta + \beta_1 + \alpha_2)t} dt = \frac{1}{(\theta + \beta_1 + \alpha_2)} \]

\[ \psi_8 = \psi_8 = \psi_{10} = \int e^{-\beta_2 t} dt = \frac{1}{\beta_2} \] (51–58)

(d) Now, we define \( m_{ij} \) as the mean sojourn time taken by the system in state \( S_i \) when the system is to transit to regenerative state \( S_j \) i.e.

\[ m_{ij} = \int t dQ_{ij}(t) = \int t q_{ij}(t) dt \]

Therefore,

\[ m_{01} = \theta \int t e^{-(\theta + 2\alpha_2)t} dt = \frac{\theta}{(\theta + 2\alpha_2)^2} \] (59)

Similarly,

\[ m_{02} = \frac{2\alpha_2}{(\theta + 2\alpha_2)^2}, \]

\[ m_{10} = \frac{\eta}{(\eta + \alpha_1 + 2\alpha_2)^2} \]
The following relations among $m_{ij}$'s are observed

$$m_{01} + m_{02} = \psi_0$$
$$m_{10} + m_{13} + m_{14} = \psi_1$$
$$m_{20} + m_{23} + m_{28} = \psi_2$$
$$m_{31} + m_{32} + m_{36} + m_{39} = \psi_3$$
$$m_{41} + m_{45} = \psi_4$$
$$m_{50} + m_{54} + m_{57} = \psi_5$$
$$m_{63} + m_{67} = \psi_6$$
$$m_{72} + m_{76} + m_{7,10} = \psi_7$$
2.5. ANALYSIS OF RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let the random variable \( T_i \) denotes the time to system failure (TSF) when the system starts functioning from state \( S_i \in E \). Then the reliability of the system is given by

\[
R_i(t) = P[T_i > t]
\]

To determine the reliability of the system we regard the failed states \( S_4, S_6, S_8, S_9 \) and \( S_{10} \) of the system as absorbing. By using simple probabilistic arguments, we have the following recursive relations in \( R_i(t) \)’s (\( i = 0, 1, 2, 3 \))

\[
\begin{align*}
R_0(t) &= Z_0(t) + q_{01}(t) \circ R_1(t) + q_{02}(t) \circ R_2(t) \\
R_1(t) &= Z_1(t) + q_{10}(t) \circ R_0(t) + q_{13}(t) \circ R_3(t) \\
R_2(t) &= Z_2(t) + q_{20}(t) \circ R_0(t) + q_{23}(t) \circ R_3(t) \\
R_3(t) &= Z_3(t) + q_{31}(t) \circ R_1(t) + q_{32}(t) \circ R_2(t)
\end{align*}
\]

where,

\[
\begin{align*}
Z_0(t) &= e^{-(\theta + 2\alpha_2)t} \\
Z_1(t) &= e^{-(\eta + \alpha_3 + 2\alpha_2)t} \\
Z_2(t) &= e^{-(\theta + \alpha_2 + \beta_2)t} \\
Z_3(t) &= e^{-(\eta + \alpha_3 + \alpha_2 + \beta_2)t}.
\end{align*}
\]

As an illustration, \( R_0(t) \) is the sum of the following contingencies-

(i) The system remains up in state \( S_0 \) without making any transition to any other state up to time \( t \). The probability of this contingency is \( Z_0(t) \).

(ii) The system first enters the state \( S_1 \) from \( S_0 \) during \((u, u + du), u \leq t \) and then starting from \( S_1 \), it remains up continuously during the remaining time \((t - u)\). The probability of this contingency is
(iii) The system first enters the state $S_2$ from $S_0$ during $(u, u + du)$, $u \leq t$ and then starting from $S_2$, it remains up continuously during the remaining time $(t - u)$. The probability of this contingency is

$$\int_0^t q_{01}(u)du R_1(t-u)=q_{01}(t)R_1(t)$$

Taking Laplace Transform of the relations (1–4), we can write the solution for $R_i^*(s)$ in the matrix form as follows –

$$\begin{bmatrix}
R_0^* \\
R_1^* \\
R_2^* \\
R_3^*
\end{bmatrix} = \begin{bmatrix}
1 & -q_{01}^* & -q_{02}^* & 0 \\
-q_{10}^* & 1 & 0 & -q_{13}^* \\
-q_{20}^* & 0 & 1 & -q_{23}^* \\
0 & -q_{31}^* & -q_{32}^* & 1
\end{bmatrix}^{-1} \begin{bmatrix}
Z_0^* \\
Z_1^* \\
Z_2^* \\
Z_3^*
\end{bmatrix}$$

On solving the above matrix equation for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)}$$  \hspace{1cm} (5)

Where,

$$N_1(s) = (Z_0^* + q_{01}^*Z_1^*)\left(1 - q_{23}^*Z_{32}^*\right) - (Z_{02}^* + q_{02}^*Z_2^*)\left(q_{13}^* + q_{02}^*Z_{23}^*\right)$$

and

$$D_1(s) = \left(1 - q_{01}^*\right)\left(1 - q_{23}^*Z_{32}^*\right) - q_{13}^*q_{31}^*\left(1 - q_{02}^*q_{20}^*\right) - q_{02}^*q_{20}^*$$

For brevity, we have omitted the argument ‘$s$’ from $q_{ij}^*(s)$, $Z_i^*(s)$ and $R_i^*(s)$. Taking the inverse Laplace transform of equation (5), we can get the reliability of the system when it initially starts from state $S_0$.

The mean time to system failure (MTSF) can be obtained by using the formula
\[ E(T_0) = \int R_0(t) \, dt \]
\[ = \lim_{s \to 0} R_0^*(s) \]
\[ = \frac{N_1(0)}{D_1(0)} \quad (6) \]

To determine \( N_1(0) \) and \( D_1(0) \), we first obtain \( Z_1^*(0) \) by using the following result
\[ \lim_{s \to 0} Z_i^*(s) = \int Z_i(t) \, dt \]
Therefore,
\[ Z_i^*(0) = \psi_i (i = 0, 1, 2, 3) \]

Thus, using \( q_{ij}(0) = p_{ij} \), we get
\[ N_1(0) = (\psi_0 + p_{01} \psi_1) (1 - p_{23} p_{32}) - (\psi_0 + p_{02} \psi_2) p_{13} p_{31} + p_{02} \psi_2 + p_{01} p_{13} (p_{32} \psi_2 + \psi_3) + p_{02} p_{23} (p_{31} \psi_1 + \psi_3) \quad (7) \]

and
\[ D_1(0) = (1 - p_{01} p_{10}) (1 - p_{23} p_{32}) - p_{13} p_{31} (1 - p_{02} p_{20}) - p_{02} p_{20} - p_{01} p_{13} p_{32} p_{20} - p_{02} p_{23} p_{31} p_{10} \quad (8) \]

Hence using (7) and (8) in (6), we get \( E(T_0) \), the expected life time of the system when it initially starts from state \( S_0 \).

2.6. **AVAILABILITY ANALYSIS**

By definition, \( A_i(t) \) is defined as the probability that the system is up at epoch \( t \) when it initially starts from state \( S_i \in E \). By simple probabilistic arguments, we have the following recursive relations among \( A_i(t) \)'s \( (i = 0, 1, 2, \ldots, 10) \) –
\[ A_0(t) = Z_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) \]
\[ A_i(t) = Z_i(t) + q_{10}(t) \odot A_0(t) + q_{13}(t) \odot A_3(t) + q_{14}(t) \odot A_4(t) \]
\[ A_2(t) = Z_2(t) + q_{20}(t) \odot A_0(t) + q_{23}(t) \odot A_3(t) + q_{28}(t) \odot A_8(t) \]
\[ A_3(t) = Z_3(t) + q_{31}(t) \odot A_1(t) + q_{32}(t) \odot A_2(t) + q_{36}(t) \odot A_6(t) + q_{39}(t) \odot A_9(t) \]
\[ A_4(t) = q_{41}(t) \odot A_4(t) + q_{45}(t) \odot A_5(t) \]
\[ A_5(t) = Z_5(t) + q_{50}(t) \odot A_0(t) + q_{54}(t) \odot A_4(t) + q_{57}(t) \odot A_7(t) \]
\[ A_6(t) = q_{63}(t) \odot A_3(t) + q_{67}(t) \odot A_7(t) \]
\[ A_7(t) = Z_7(t) + q_{72}(t) \odot A_2(t) + q_{76}(t) \odot A_6(t) + q_{7,10}(t) \odot A_{10}(t) \]
\[ A_8(t) = q_{82}(t) \odot A_2(t) \]
\[ A_9(t) = q_{93}(t) \odot A_3(t) \]
\[ A_{10}(t) = q_{10,7}(t) \odot A_7(t) \]

(1–11)

Where,
\[ Z_5(t) = e^{-(\theta + \beta_1 + 2\alpha_z)t} \quad \text{and} \quad Z_7(t) = e^{-(\theta + \beta_1 + \alpha_z)t}. \]

As an illustration, \( A_0(t) \) is the sum of the following contingencies-

(i) The system remains up in state \( S_0 \) without making any transition to any other state up to time \( t \). The probability of this contingency is \( Z_0(t) \).

(ii) The system transits to the state \( S_1 \) from \( S_0 \) during \((u, u + du), \, u \leq t \) and then remains up in state \( S_1 \) for the remaining time \((t - u)\). The probability of this contingency is
\[ \int_0^t q_{01}(u)du A_1(t-u) = q_{01}(t) \odot A_1(t) \]

(iii) The system transits to the state \( S_2 \) from \( S_0 \) during \((u, u + du), \, u \leq t \) and then remains up in state \( S_2 \) for the remaining time \((t-u)\). The probability of this contingency is
\[ \int_0^t q_{02}(u)du A_1(t-u) = q_{02}(t) \odot A_2(t) \]
Taking Laplace Transform of the relations (1–11), we can write the solution for \( A_1^*(s) \) in the matrix form as follows:

\[
\begin{bmatrix}
A_0^* \\
A_1^* \\
A_2^* \\
A_3^* \\
A_4^* \\
A_5^* \\
A_6^* \\
A_7^* \\
A_8^* \\
A_9^* \\
A_{10}^*
\end{bmatrix}
= 
\begin{bmatrix}
1 & -q_{101}^* & -q_{102}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-q_{10}^* & 1 & 0 & -q_{13}^* & -q_{14}^* & 0 & 0 & 0 & 0 & 0 & 0 \\
-q_{20}^* & 0 & 1 & -q_{23}^* & 0 & 0 & 0 & 0 & -q_{28}^* & 0 & 0 \\
0 & -q_{31}^* & -q_{32}^* & 1 & 0 & 0 & 0 & 0 & 0 & -q_{39}^* & 0 \\
0 & -q_{41}^* & 0 & 0 & 1 & -q_{45}^* & 0 & 0 & 0 & 0 & 0 \\
-q_{50}^* & 0 & 0 & 0 & -q_{54}^* & 1 & 0 & -q_{57}^* & 0 & 0 & 0 \\
0 & 0 & 0 & -q_{63}^* & 0 & 0 & 1 & -q_{67}^* & 0 & 0 & 0 \\
0 & 0 & -q_{72}^* & 0 & 0 & 0 & -q_{76}^* & 1 & 0 & 0 & -q_{7,10}^* \\
0 & 0 & -q_{82}^* & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -q_{93}^* & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -q_{10,7}^* & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
Z_0^* \\
Z_1^* \\
Z_2^* \\
Z_3^* \\
Z_4^* \\
Z_5^* \\
Z_6^* \\
Z_7^* \\
Z_8^* \\
Z_9^* \\
Z_{10}^*
\end{bmatrix}
\]

Solving the above matrix equation for \( A_0^*(s) \), we have

\[
A_0^*(s) = \frac{N_2(s)}{D_2(s)}
\]  \hspace{1cm} (12)

Where,

\[
N_2(s) = \begin{bmatrix}
Z_0^* & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_1^* & 1 & 0 & -q_{13}^* & -q_{14}^* & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_2^* & 0 & 1-q_{28}^* & -q_{23}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_3^* & -q_{31}^* & -q_{32}^* & 1-q_{39}^* & q_{93}^* & 0 & 0 & -q_{36}^* & 0 & 0 & 0 \\
0 & -q_{41}^* & 0 & 0 & 1 & -q_{45}^* & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -q_{54}^* & 1 & 0 & -q_{57}^* & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -q_{63}^* & 0 & 0 & 1 & -q_{67}^* & 0 & 0 & 0 \\
Z_5^* & 0 & -q_{72}^* & 0 & 0 & 0 & -q_{76}^* & 1 & 0 & 0 & -q_{7,10}^* \\
0 & 0 & 0 & -q_{82}^* & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
Z_7^* & 0 & -q_{93}^* & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
Z_8^* & 0 & 0 & 0 & 0 & 0 & -q_{10,7}^* & 0 & 0 & 1 & 0
\end{bmatrix}
\]

and
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To calculate the steady state availability of the system we first compute

\[ Z_5^*(0) = \psi_5 \quad \text{and} \quad Z_7^*(0) = \psi_7. \]

The value of \( Z_i^*(0) \), \( i = 0, 1, 2, 3 \) is already defined.

Also, using the result \( q_{ij}(0) = p_{ij} \), we have

\[
N_2(0) = \begin{bmatrix}
\psi_0 & -p_{01} & -p_{02} & 0 & 0 & 0 & 0 & 0 \\
\psi_1 & 1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\
\psi_2 & 0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\
\psi_3 & -p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
0 & -p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
\psi_5 & 0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\
0 & 0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} \\
\psi_7 & 0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10} \\
\end{bmatrix}
\]

and

\[
D_2(0) = \begin{bmatrix}
1 & -p_{01} & -p_{02} & 0 & 0 & 0 & 0 & 0 \\
-p_{10} & 1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\
-p_{20} & 0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\
0 & -p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
0 & -p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
-p_{50} & 0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\
0 & 0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} \\
0 & 0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10} \\
\end{bmatrix}
\]
Adding all the columns of the above determinant into first column and using relations (probability sum). We observe that all the elements of the first column of $D_2(0)$ become zero. Therefore, $D_2(0) = 0$.

Therefore, the steady state probability that the system will be operative is given by

$$A_0 = \lim_{t \to \infty} A_0(t)$$

$$= \lim_{s \to 0} s A_0^*(s)$$

$$= \lim_{s \to 0} s \frac{N_2(s)}{D_2(s)}$$

$$= \frac{N_2(0)}{D_2(0)}$$

(13)

We have,

$$N_2(0) = U_0 \psi_0 + U_1 \psi_1 + U_2 \psi_2 + U_3 \psi_3 + U_5 \psi_5 + U_7 \psi_7$$

(14)

Where,

$$U_0 = \begin{vmatrix}
1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\
0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\
-p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
-p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\
0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} \\
0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10}
\end{vmatrix}$$

$$U_1 = (-1) \begin{vmatrix}
-p_{01} & -p_{02} & 0 & 0 & 0 & 0 & 0 \\
0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\
-p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
-p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\
0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} \\
0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10}
\end{vmatrix}$$
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\[ U_2 = \begin{pmatrix} -p_{01} & -p_{02} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\ -p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\ -p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\ 0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\ 0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} \\ 0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10} \end{pmatrix} \]

\[ U_3 = (-1) \begin{pmatrix} -p_{01} & -p_{02} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\ 0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\ -p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\ 0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\ 0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} \\ 0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10} \end{pmatrix} \]

\[ U_4 = \begin{pmatrix} -p_{01} & -p_{02} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\ 0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\ -p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\ 0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\ 0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} \\ 0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10} \end{pmatrix} \]

\[ U_5 = (-1) \begin{pmatrix} -p_{01} & -p_{02} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\ 0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\ -p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\ -p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\ 0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} \\ 0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10} \end{pmatrix} \]
To obtain $D'_2(0)$, we collect the coefficients of $m_{ij} = -q_{ij}(0)$ in $D_2(0)$ as follows:

(i) coefficient of $m_{01} = (-1) egin{array}{c}
\begin{bmatrix}
-p_{10} & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\
-p_{20} & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\
0 & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
0 & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
-p_{50} & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\
0 & -p_{63} & 0 & 0 & 1 & -p_{67} & 0 \\
0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10}
\end{bmatrix}
\end{array}
\]

$U_6 = \begin{bmatrix}
-p_{01} & -p_{02} & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\
0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\
-p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
-p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\
0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10}
\end{bmatrix}$

$U_7 = (-1) \begin{bmatrix}
-p_{01} & -p_{02} & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\
0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\
-p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
-p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\
0 & -p_{63} & 0 & 0 & 1 & -p_{67} & 0 \\
0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10}
\end{bmatrix}$
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(on adding all the columns into first column)

\[
\begin{pmatrix}
1 & 0 & -p_{13} & -p_{14} & 0 & 0 & 0 \\
0 & 1-p_{28} & -p_{23} & 0 & 0 & 0 & 0 \\
-p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
-p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} \\
0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} \\
0 & -p_{72} & 0 & 0 & 0 & -p_{76} & 1-p_{7,10}
\end{pmatrix} = U_0
\]

(ii) coefficient of \( m_{02} \)

\[
\begin{pmatrix}
-p_{10} & 1 & -p_{13} & -p_{14} & 0 & 0 & 0 \\
-p_{20} & 0 & -p_{23} & 0 & 0 & 0 & 0 \\
0 & -p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
0 & -p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
-p_{50} & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} & 0 \\
0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} & 0 \\
0 & 0 & 0 & 0 & 0 & -p_{76} & 1-p_{7,10}
\end{pmatrix} = U_0
\]

(on adding all the columns into first column)

\[
\begin{pmatrix}
0 & 1 & -p_{13} & -p_{14} & 0 & 0 & 0 \\
-(1-p_{28}) & 0 & -p_{23} & 0 & 0 & 0 & 0 \\
p_{32} & -p_{31} & -p_{32} & 1-p_{39} & 0 & 0 & -p_{36} & 0 \\
0 & -p_{41} & 0 & 0 & 1 & -p_{45} & 0 & 0 \\
0 & 0 & 0 & -p_{54} & 1 & 0 & -p_{57} & 0 \\
0 & 0 & -p_{63} & 0 & 0 & 1 & -p_{67} & 0 \\
-p_{72} & 0 & 0 & 0 & 0 & -p_{76} & 1-p_{7,10}
\end{pmatrix} = (-1)U_0
\]
Similarly,

(iii) \( \text{Coefficient of } m_{10} = \text{Coefficient of } m_{13} = U_1 \)

(iv) \( \text{Coefficient of } m_{20} = \text{Coefficient of } m_{23} = U_2 \)

(v) \( \text{Coefficient of } m_{31} = \text{Coefficient of } m_{32} = U_3 \)

(vi) \( \text{Coefficient of } m_{41} = \text{Coefficient of } m_{45} = U_4 \)

(vii) \( \text{Coefficient of } m_{50} = \text{Coefficient of } m_{54} = U_5 \)

(viii) \( \text{Coefficient of } m_{63} = \text{Coefficient of } m_{67} = U_6 \)

(ix) \( \text{Coefficient of } m_{72} = \text{Coefficient of } m_{76} = U_7 \)

(x) \( \text{Coefficient of } m_{82} = p_{28} U_2 \)

(xi) \( \text{Coefficient of } m_{93} = p_{39} U_3 \)

(xii) \( \text{Coefficient of } m_{10,7} = p_{7,10} U_7 \)

Thus, we have

\[
D_2'(0) = U_0 (m_{01} + m_{02}) + U_1 (m_{10} + m_{13} + m_{14}) + U_2 (m_{20} + m_{23} + m_{28})
\]
Using the relation $\sum_i m_{ij} = \psi_i$, we get

$$D'(0) = U_0 \psi_0 + U_1 \psi_1 + U_2 \psi_2 + U_3 \psi_3 + U_4 \psi_4 + U_5 \psi_5 + U_6 \psi_6 + U_7 \psi_7$$

$$+ p_{28} U_2 \psi_8 + p_{39} U_3 \psi_9 + p_{7,10} U_7 \psi_{10}$$

Substituting (14) and (15) in (13), we get the expression for the steady state availability of the system starting from state $S_0$.

The expected up time of the system during $(0, t)$, due to a unit in $N$-mode and operative, is given by

$$\mu_{up}(t) = \int_0^t A_0(u) \, du$$

So that

$$\mu^*_{up}(s) = \int_0^s \frac{A^*_0(s)}{s}$$

### 2.7. BUSY PERIOD ANALYSIS

(a) **Due to Generator**

Let $B_1^g(t)$ be the probability that the repairman is busy in the repair of generator at time $t$, when the system initially starts from state $S_1 \in E$. Using elementary probabilistic arguments, we have the following relations:

$$B_0^g(t) = q_{01}(t) \odot B_1^g(t) + q_{02}(t) \odot B_2^g(t)$$

$$B_1^g(t) = q_{10}(t) \odot B_0^g(t) + q_{13}(t) \odot B_3^g(t) + q_{14}(t) \odot B_4^g(t)$$

$$B_2^g(t) = q_{20}(t) \odot B_0^g(t) + q_{23}(t) \odot B_3^g(t) + q_{28}(t) \odot B_8^g(t)$$

$$B_3^g(t) = q_{31}(t) \odot B_1^g(t) + q_{32}(t) \odot B_2^g(t) + q_{36}(t) \odot B_6^g(t) + q_{39}(t) \odot B_9^g(t)$$

$$B_4^g(t) = Z_4(t) + q_{41}(t) \odot B_1^g(t) + q_{45}(t) \odot B_5^g(t)$$
\begin{align*}
B_5^g(t) &= Z_5(t) + q_{50}(t) \otimes B_6^g(t) + q_{54}(t) \otimes B_4^g(t) + q_{35}(t) \otimes B_7^g(t) \\
B_6^g(t) &= Z_6(t) + q_{63}(t) \otimes B_3^g(t) + q_{67}(t) \otimes B_7^g(t) \\
B_7^g(t) &= Z_7(t) + q_{72}(t) \otimes B_2^g(t) + q_{76}(t) \otimes B_6^g(t) + q_{7,10}(t) \otimes B_1^g(t) \\
B_8^g(t) &= q_{82}(t) \otimes B_2^g(t) \\
B_9^g(t) &= q_{93}(t) \otimes B_3^g(t) \\
B_{10}^g(t) &= q_{10,7}(t) \otimes B_7^g(t)
\end{align*}

(1–11)

Where,

\begin{align*}
Z_4(t) &= Z_4(t) = e^{-(\eta + \beta_1)t}, \quad Z_5(t) = e^{-\alpha_1 + 2\alpha_2}t \quad \text{and} \quad Z_7(t) = e^{-(\eta + \beta_2)t}.
\end{align*}

Taking Laplace Transform of the relations (1–11) and then we have

\begin{align*}
B_0^{gs}(s) &= q_{01}^*(s) B_1^{gs}(s) + q_{02}^*(s) B_2^{gs}(s) \\
B_1^{gs}(s) &= q_{10}^*(s) B_0^{gs}(s) + q_{13}^*(s) B_3^{gs}(s) + q_{14}^*(s) B_4^{gs}(s) \\
B_2^{gs}(s) &= q_{20}^*(s) B_0^{gs}(s) + q_{23}^*(s) B_3^{gs}(s) + q_{28}^*(s) q_{82}^*(s) B_2^{gs}(s) \\
B_3^{gs}(s) &= q_{31}^*(s) B_1^{gs}(s) + q_{32}^*(s) B_2^{gs}(s) + q_{36}^*(s) B_6^{gs}(s) + q_{39}^*(s) q_{39}^*(s) B_3^{gs}(s) \\
B_4^{gs}(s) &= Z_4^*(s) + q_{41}^*(s) B_1^{gs}(s) + q_{45}^*(s) B_5^{gs}(s) \\
B_5^{gs}(s) &= Z_5^*(s) + q_{50}^*(s) B_0^{gs}(s) + q_{54}^*(s) B_4^{gs}(s) + q_{57}^*(s) B_7^{gs}(s) \\
B_6^{gs}(s) &= Z_6^*(s) + q_{63}^*(s) B_3^{gs}(s) + q_{67}^*(s) B_7^{gs}(s) \\
B_7^{gs}(s) &= Z_7^*(s) + q_{72}^*(s) B_2^{gs}(s) + q_{76}^*(s) B_6^{gs}(s) + q_{7,10}^*(s) q_{10,7}^*(s) B_7^{gs}(s)
\end{align*}

(12–19)

Solving the resulting set of algebraic equations for $B_0^{gs}(s)$, we get

\begin{align*}
B_0^{gs}(s) &= \frac{N_3(s)}{D_2(s)}
\end{align*}

(20)

Where,

\begin{align*}
N_3(s) &= U_4 Z_4^* + U_5 Z_5^* + U_6 Z_6^* + U_7 Z_7^*
\end{align*}

and $D_2(s)$ is same as in case of availability analysis.
(b) Due to Chillers

Let $B_i^c(t)$ be the probability that the repairman is busy in the repair of chillers at time $t$, when the system initially starts from state $S_{i \in E}$. Using elementary probabilistic arguments, we have the following relations

$$
B_0^c(t) = q_{01}(t) \otimes B_1^c(t) + q_{02}(t) \otimes B_2^c(t)
$$

$$
B_1^c(t) = q_{10}(t) \otimes B_0^c(t) + q_{13}(t) \otimes B_3^c(t) + q_{14}(t) \otimes B_4^c(t)
$$

$$
B_2^c(t) = Z_2(t) + q_{20}(t) \otimes B_0^c(t) + q_{23}(t) \otimes B_3^c(t) + q_{28}(t) \otimes B_8^c(t)
$$

$$
B_3^c(t) = Z_3(t) + q_{31}(t) \otimes B_1^c(t) + q_{32}(t) \otimes B_2^c(t) + q_{36}(t) \otimes B_6^c(t) + q_{39}(t) \otimes B_9^c(t)
$$

$$
B_4^c(t) = q_{41}(t) \otimes B_1^c(t) + q_{45}(t) \otimes B_5^c(t)
$$

$$
B_5^c(t) = q_{50}(t) \otimes B_0^c(t) + q_{54}(t) \otimes B_4^c(t) + q_{57}(t) \otimes B_7^c(t)
$$

$$
B_6^c(t) = q_{63}(t) \otimes B_3^c(t) + q_{67}(t) \otimes B_5^c(t)
$$

$$
B_7^c(t) = q_{72}(t) \otimes B_2^c(t) + q_{76}(t) \otimes B_6^c(t) + q_{7,10}(t) \otimes B_{10}^c(t)
$$

$$
B_8^c(t) = Z_8(t) + q_{82}(t) \otimes B_2^c(t)
$$

$$
B_9^c(t) = Z_9(t) + q_{93}(t) \otimes B_3^c(t)
$$

$$
B_{10}^c(t) = Z_{10}(t) + q_{10,7}(t) \otimes B_7^c(t)
$$

(21–31)

Where,

$$
Z_8(t) = Z_9(t) = Z_{10}(t) = e^{-\beta_2 t}.
$$

Taking Laplace Transform of the relations (21–32) and solving the resulting set of algebraic equations for $B_0^c(s)$, we get

$$
B_0^c(s) = \frac{N_4(s)}{D_2(s)}
$$

(32)

Where,

$$
N_4(s) = \left(Z_2^* + q_{28}^* Z_8^*\right)U_2 + \left(Z_3^* + q_{39}^* Z_9^*\right)U_3 + q_{7,10}^* Z_{10}^* U_7
$$

and $D_2(s)$ is same as in case of availability analysis.
In the long run, the probability that the repairman will be busy in repairing the failed generator is given by

$$B_0^g = \lim_{t \to \infty} B_0^g(t) = \lim_{s \to 0} s B_0^{g*}(s)$$

$$= \frac{N_3(0)}{D'_2(0)}$$

(33)

Where,

$$N_3(0) = \psi_4 U_4 + \psi_5 U_5 + \psi_6 U_6 + \psi_7 U_7$$

and $D'_2(0)$ is same as defined in case of availability analysis.

Similarly, the steady state probability that the repairman will be busy in repairing the failed chillers is given by

$$B_0^c = \frac{N_4(0)}{D'_2(0)}$$

(34)

Where,

$$N_4(0) = (\psi_2 + p_{28} \psi_8) U_2 + (\psi_3 + p_{39} \psi_9) U_3 + p_{7,10} \psi_{10} U_7$$

and $D'_2(0)$ is same as defined in case of availability analysis.

The expected busy period of the repairman in repairing the failed generator and chillers during $(0, t)$ are respectively given by

$$\mu_b^g(t) = \int_0^t B_0^g(u) \, du,$$

So that

$$\mu_b^{g*}(s) = \frac{B_0^{g*}(s)}{s}$$

(35)

and

$$\mu_b^c(t) = \int_0^t B_0^c(u) \, du$$

So that

$$\mu_b^{c*}(s) = \frac{B_0^{c*}(s)}{s}$$

(36)
2.8. PROFIT FUNCTION ANALYSIS

Consider the expected up time of the system when system is operative and expected busy periods of the repairman when he is busy in repair of various failed units, then the expected profit incurred by the system during (0, t) is given by

\[ P(t) = \text{Expected total revenue in (0, t)} - \text{Expected total repair cost in (0, t)} \]

\[ = K_0 \mu_{up}(t) - K_1 \mu_b^g(t) - K_2 \mu_b^c(t) \]

(1)

Where, \( K_0 \) is the revenue per unit up time by the system and \( K_1 \) & \( K_2 \) are respectively the per unit time amounts that is paid to the repairman for repairing the failed generator and chillers.

Therefore, the expected profit per unit time in a steady state is given by

\[ P = \lim_{t \to \infty} \left[ \frac{P(t)}{t} \right] = \lim_{s \to 0} s^2 P^*(s) \]

\[ = K_0 \lim_{s \to 0} s^2 \mu_{up}^*(s) - K_1 \lim_{s \to 0} s^2 \mu_b^g(s) - K_2 \lim_{s \to 0} s^2 \mu_b^c(s) \]

\[ = K_0 \lim_{s \to 0} s A_0^*(s) - K_1 \lim_{s \to 0} s B_0^g(s) - K_2 \lim_{s \to 0} s B_0^c(s) \]

\[ = K_0 A_0 - K_1 B_0^g - K_2 B_0^c \]

(2)

Where \( A_0, B_0^g \) and \( B_0^c \) have been already defined.

2.9. GRAPHICAL STUDY OF THE SYSTEM BEHAVIOUR

For a more concrete study of the system behaviour, we plot the curves for MTSF and profit function w.r.t. \( \alpha_1 \), failure rate of generator, for different values of failure rate of chillers (\( \alpha_2 \)). The curves so obtained are shown in Fig. 2.2 and Fig. 2.3 respectively.

Fig. 2.2 shows the variation in MTSF w.r.t. \( \alpha_1 \) for different values of \( \alpha_2 = 0.01, 0.02 \) and 0.03 when other parameters are kept fixed as \( \theta = 0.02, \eta = 0.10, \beta_1 = 0.05, \beta_2 = 0.06. \) From the figure it is observed that MTSF
uniformly decreases as $\alpha_1$ increases. Also with the increase in the values of $\alpha_2$ MTSF of the system decreases.

Fig. 2.3 shows the curves for profit function. From the curve it is clear that the profit decreases almost with linear trend as the value of $\alpha_1$ increases when other parameters are kept fixed as $K_0 = 8000$, $K_1 = 1000$, $K_2 = 1500$, $\theta = 0.02$, $\eta = 0.10$, $\beta_1 = 0.05$, $\beta_2 = 0.06$. Also the profit tends to decrease as we increase the values of failure rate $\alpha_2$. 