INTRODUCTION

1.1. RELIABILITY- BACKGROUND, MEANING AND SCOPE

Reliability as a human attribute has been praised for a very long time. For technical systems, however, the reliability concept has not applied for more than 50 years. It emerged with a technological meaning just after World War II, it was then used in connection with comprising operational safety of one, two and four engine aeroplanes. Then the reliability was measured as the number of accidents per hour of flight time. The adage “Necessity is the mother of invention” is also true in case of reliability. The importance of reliability and quality control was born out as the demands of modern technology used in World War II. After World War II, attempts were made to study the proportion of flights which deemed successful for different configurations and average numbers of failures were determined, which lead to the concept of reliability.

Proper research work in the field of reliability was started around 1950. During this period some areas of reliability, particularly, life testing problems and electronic/missile reliability problems received great deal of attention from statistician and engineers. In 1950’s Britain and Japan began to take keen interest in applying the reliability principles to their products. A commercial organization ARNIC was set up in 1950 by airline of Ohio which achieved remarkable success in improving reliability of different types of tubes. In 1952, the department of defense of U.S. established a committee on reliability called “Advisory Group on Reliability of Electronic Equipment (AGREE)”, which published its first report on reliability that included certain reliability specifications such as minimum acceptability limits, reliability test requirements and effects of storage on reliability etc.

A number of text books on the subject matter reliability were published in 1960’s. IEEE transactions on reliability was the first journal on reliability came out in 1963. During the last three decades a quite good amount of
research work and remarkable progress has been seen in the field of reliability.

For the last thirty years, the common people become aware of the uncooperative situations arising due to complexity in a system and reliability started attracting the industrial managers, researchers and scholars of all over the world. They contribute a lot of work in this pioneers field. The researchers not only found the reliability of the system but also evaluated other measures in these directions.

Reliability is an important branch of Mathematics and Statistics and is developing fast in accordance with the needs of changing technology. Scientists, engineers, mathematicians and statisticians all over the world have been actively working in the field of reliability, specially for last two decades. Various Universities have adopted reliability as a subject in their curriculum.

1.2. PERFORMANCE MEASURES

In this section, we concentrate and explain some of the performance measures of a system which are of interest to the system’s designing and analysis. We confine ourselves to those measures that are proposed and discussed in the literature. It is convenient to begin with the following definition –

Let a dichotomous variate $Z(t)$ denoting the state of the system at time $t$, be defined as

$$Z(t) = \begin{cases} 
1 & \text{if the system operates at time } t \\
0 & \text{otherwise.}
\end{cases}$$

Then the stochastic process $\{Z(t), t \geq 0\}$ can be used to define various performance measures as follows–

(a) RELIABILITY

Reliability is the probability of an item operating for a given amount of time without failure. More generally, reliability is the capability of parts, components, equipments, products and systems to perform their required
functions for desired periods of time without failure in specified environments and with a desired confidence.

Mathematically, if a random variable $T$ denotes the life time of the system, then the reliability function of the system at time $t$ is given by

$$R(t) = P[T > t] = \int_{t}^{\infty} f(u) \, du$$

where $f(\cdot)$ and $F(\cdot)$ are the p.d.f. and c.d.f. of the life time of the system $T$ respectively.

The reliability is always a function of time. It also depends on environmental conditions, which may or may not vary with time. The following assumptions are made

(i) $R(0) = 1$, since the device is assumed to be perfect at time $t = 0$.

(ii) $R(\infty) = 0$, since no device can work forever without failure.

(iii) $R(t)$ is a non-increasing function between time limits 0 to 1.

(b) AVAILABILITY

Availability is the probability that an item will be able to function (i.e. not failed or undergoing repair) when called upon to do so. More specifically, it is the probability that the system will be able to operate within tolerance limits at a given instant $t$ and is also called operational readiness.

Symbolically, in terms of $Z(t)$ it is defined as

$$A(t) = P[Z(t) = 1]$$

We also note that reliability is a uniformly non-increasing function defined in an interval while availability is a function defined at a time epoch.

(c) INTERVAL AVAILABILITY

The expected fraction of a given interval of time for which the system is able to operate within tolerances is known as the interval availability. For the
interval $(0, t)$, the interval availability of the system can be obtained by using its pointwise availability as under

$$\bar{A}(t) = \frac{1}{t} \int_0^t A(u) \, du = \frac{\mu_{up}(t)}{t}$$

Where, $A(u)$ represents pointwise availability of the system at epoch $u$ and $\mu_{up}(t)$ is the expected up time of the system during $(0,t)$.

**ASYMPTOTIC OR STEADY-STATE AVAILABILITY**

Steady-state availability is the probability that in the long-run, the system operates satisfactorily. Symbolically, the steady-state availability is

$$A(\infty) = \lim_{t \to \infty} A(t) = \lim_{s \to 0} s A'(s)$$

This measure is suitable for those systems which are operating continuously e.g. radar system.

The difference between the measures of reliability and availability are as follows-

(i) The reliability is an interval function while availability is a point function describing the behaviour of the system at a specified epoch.

(ii) The reliability function precludes the failure of the system during the interval under consideration, while availability functions does not impose any such restriction on the behaviour of the system.

**MEAN SOJOURN TIME IN A STATE**

The expected time taken by the system in a particular state before transiting to any other state is known as mean sojourn time or mean survival time in that state.

Let $T_i$ be the sojourn time in any state $S_i$, then mean sojourn time $\psi_i$ in state $S_i$ is given by

$$\psi_i = \int P[T_i > t] \, dt$$
MEAN TIME TO SYSTEM FAILURE (MTSF)

The time taken by a system to reach into the failed state first time is known as time to system failure (TSF) and its expected value is termed as mean time to system failure. Sometimes it is also known as mean time to first failure of the system.

To obtain it we regard the failed state as absorbing state i.e. once the system enters into the failed state(s) it remains there forever.

Let $T$ be the survival or life time of the system and $f(\cdot)$, $F(\cdot)$ be the p.d.f. and c.d.f. respectively of life time $T$, then

$$MTSF = E[T] = \int_0^\infty t f(t) \, dt = \int_0^\infty t dF(t) = \int_0^\infty t d[1 - R(t)] = \int_0^\infty t dR(t)$$

$$= \int_0^\infty R(t) \, dt$$

Where, $R(t) = 1 - F(t)$ is the reliability function of the system at time $t$.

If $R^*(s)$ is the Laplace transform of the reliability function $R(t)$, then MTSF is given by

$$E(T) = \lim_{s \to 0} \int_0^\infty e^{-st} R(t) \, dt$$

$$= \lim_{s \to 0} R^*(s) = \lim_{s \to 0} \left[ \frac{1}{s} - F^*(s) \right]$$

$$= \lim_{s \to 0} \left[ \frac{1 - \tilde{F}(s)}{s} \right]$$

$$= - \frac{d}{ds} \tilde{F}(s) \bigg|_{s=0} \quad \{ \because \tilde{F}(s) \bigg|_{s=0} = 1 \}$$

Where, $F^*(s)$ and $\tilde{F}(s)$ denote respectively the Laplace and Laplace stieltjes transforms of c.d.f. of time to system failure $T$. 
(g) FAILURE

Failure of an item means unreliability. In the field of reliability theory, a unit is said to be operative, if it works or functions within specified tolerance limits. If the functioning of a unit is either seriously disturbed or has completely stopped, we say that the unit is failed. Failure of a unit is either due to partial or total damage or due to the change in operating conditions such that its operation is affected.

A deviation in the properties from the prescribed condition is considered as a fault. A state of fault is known as “failure”.

An item is considered as failed under one of the following three conditions-

- When it becomes completely inoperable.
- When it is still operable, but is no longer able to perform as required e.g. a 12 volt battery providing 3 volt instead of 12.
- When a sudden serious deterioration makes the item unsafe for its further use.

(h) PHASES OF FAILURE

In general, an item may experience any of three phases of failure during its complete life cycle of operation (a part from the failure due to change in operating and environmental conditions).

(i) Initial failure

When we put a large collection of units into operation, it is likely that there are a large number of failures initially. The early failures are called initial failures or infant mortality. These failures are due to manufacturing defects, such as bad assembly, weak parts, defective designs etc. Since the defective parts of the unit are eliminated during the initial period, this period is known as the debugging or burn in period.
(ii) Random failure (Chance failure)

These failures occur during the middle (useful life) of an item due to sudden stress accumulations beyond the design strength of the item. Such failures can never be predicted and so it is almost impossible to eliminate them. Thus the failure during this phase is often called random failures, chance failures or catastrophic failures. This phase is characterised by a constant failure rate. The effect of these types of failures can be minimised by duplicating of the components (also referred to as redundancy).

(iii) Wear-out failure

With the passage of time, the unit (device) begins to deteriorate. A gradual change in the values of the parameters determining the performance of the unit occurs and when these parameters go beyond the limit admissibility, the unit fails. This region is called the wear-out region and such kinds of failures are called wear-out failures. Wear-out failures can be eliminated up to some extent through proper preventive maintenance and replacement policies. The method is applied to the so called ‘one shot’ equipment, such as missiles, which are used only once during their life time.

All the above three phases of failures are shown in Fig. 1.1. The curve shown in the figure is known as “Bath tub curve” because of its shape. From actual point of view, the failure phenomenon in an item is very analogous to the mortality of death phenomenon in human being. The comparison is given in the following table-

<table>
<thead>
<tr>
<th>Phase</th>
<th>Cause for system failure</th>
<th>Cause for human death</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Original defect (defective designing, manufacturing and assembly)</td>
<td>Birth defect</td>
</tr>
<tr>
<td>2.</td>
<td>Random (chance)</td>
<td>Accident</td>
</tr>
<tr>
<td>3.</td>
<td>Wear and tear (wear out)</td>
<td>Age</td>
</tr>
</tbody>
</table>
**Bath Tub Curve**

Fig. 1.1

(i) **INSTANTANEOUS FAILURE RATE OR HAZARD RATE**

It is the conditional probability rate that the system will fail during the time interval \((t, t+h)\) given that it was operating during \((0, t)\). It is defined as the limit of the failure rate when the interval length approaches zero i.e.

\[
r(t) = \lim_{h \to 0} \frac{F(t+h)-F(t)}{h[1-F(t)]} = \frac{f(t)}{1-F(t)} = \frac{f(t)}{R(t)}
\]

Here \(F(t)\), \(f(t)\) and \(R(t)\) are the c.d.f., p.d.f. and reliability function of the system respectively. Sometimes \(r(t)\) is called the force of mortality or hazard rate.

The inter-relationship among the functions \(f(t)\), \(F(t)\), \(R(t)\) and \(r(t)\) are shown in table (1.1).
### Table 1.1

<table>
<thead>
<tr>
<th>Expressed by</th>
<th>F(t)</th>
<th>f(t)</th>
<th>R(t)</th>
<th>r(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(t)</td>
<td></td>
<td>$\int f(u) , du$</td>
<td>1−R(t)</td>
<td>$1−e^{-\int r(u) , du}$</td>
</tr>
<tr>
<td>f(t)</td>
<td>$\frac{d}{dt} F(t)$</td>
<td></td>
<td>$-\frac{d}{dt} R(t)$</td>
<td>$r(t) \cdot e^{-\int r(u) , du}$</td>
</tr>
<tr>
<td>R(t)</td>
<td>1−F(t)</td>
<td>$\int f(u) , du$</td>
<td></td>
<td>$-\int f(u) , du$</td>
</tr>
<tr>
<td>r(t)</td>
<td>$\frac{dF(t)}{1-F(t)} \frac{d}{dt}$</td>
<td>$\int f(u) , du$</td>
<td>$-\frac{d}{dt} \log R(t)$</td>
<td></td>
</tr>
</tbody>
</table>

### (j) EXPECTED PROFIT EARNED BY THE SYSTEM

Let $K_0$ and $K_1$ be the revenue and repair cost per unit time of the system. Then the expected profit $P(t)$, earned by the system during an interval $(0, t)$, is given by

$$P(t) = K_0 \mu_{up}(t) - K_1 \mu_b(t)$$

Where,

$\mu_{up}(t) = \text{Expected up time of the system during } (0, t)$.

$\mu_b(t) = \text{Expected busy period of the repair facility during } (0, t)$.

### 1.3. SOME STATIC SYSTEM CONFIGURATIONS

A system is an arbitrary device consisting of parts, components or units having known reliabilities. It is assumed that components fail independently to each other. The configuration and nature of the system must be known so that the effect of failure of each component or unit on the system can be determined.

Suppose that a system with life time $T$ consists of $n$-different components $C_1, C_2, \ldots, C_n$ with life time $T_i$ of the $i^{th}$ component. At time $t$ the system reliability is

$$R(t) = P[T > t]$$
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and reliability of \( i^{th} \) component is

\[
R_i(t) = P[T_i > t]
\]

The system structures generally considered are given below

(a) **SERIES CONFIGURATION**

The simplest and perhaps the most common configuration in the reliability analysis is the series configuration. A system having \( n \)-components is said to have series configuration, if the failure of any one component causes the entire system is failure. The simplest example of series configuration is the Deepawali or Christmas glow bulbs, where failure of any bulb leads to entire failure.

In series configuration, the reliability of the system \( R(t) \) is the probability that the failure time of each component is greater than \( t \), \( i.e. \)

\[
R(t) = P[T > t] = P[\text{Min.} (T_1, T_2, \ldots, T_n) > t] = P[T_1 > t] P[T_2 > t] \ldots P[T_n > t]
\]

\[
= \prod_{i=1}^{n} P[T_i > t]
\]

\[
\Rightarrow R(t) = \prod_{i=1}^{n} R_i(t)
\]

Where, \( R_i(t) \) is the reliability of \( i^{th} \) component.

If \( r_i(t) \) denotes instantaneous failures rate of \( i^{th} \) unit and \( r(t) \) is that of the system failure rate, then using the relation between reliability and hazard rate, we have

\[
R(t) = \exp \left[- \int_0^t r(u) \, du \right] = \prod_{i=1}^{n} \exp \left[- \int_0^t r_i(u) \, du \right] = \exp \left[- \int_0^t \left( \sum_{i=1}^{n} r_i(u) \right) \, du \right]
\]

or \( r(t) = \sum_{i=1}^{n} r_i(t) \)

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Thus, in series system the components reliabilities are multiplied to obtain the system reliability and components hazard functions are added to obtain the system hazard function.

An n-component series configuration is shown below-

![Series Configuration Diagram]

(b) **PARALLEL CONFIGURATION**

A system having n-components is said to have parallel configuration if failure of the system occurs only when all the components of the system stop functioning. The configuration is based on the assumption that all the components of the system are active and load sharing.

In this configuration, all the n components are arranged in parallel form and the system fails when all the components fail. In parallel system the reliability of the system is

\[
R(t) = 1 - \prod_{i=1}^{n} [1 - R_i(t)]
\]

Where, \( R_i(t) \) is the reliability of \( i^{th} \) component.

A typical n-component parallel system configuration is shown below-

![Parallel Configuration Diagram]
(c) **k-OUT-OF-n CONFIGURATION**

In many problems the system operates if at least k out of n components function, e.g. a bridge supported by n pillars, out of which k pillars are necessary to support the maximum load on the bridge. If all the components are identical and the reliability of each component at time t is p(t) (say), then the system reliability is

\[
R(t) = \sum_{i=k}^{n} \binom{n}{i} [p(t)]^i [1 - p(t)]^{n-i}
\]

It is of interest to note that for k = 1, the structure becomes a parallel system and for k = n, the structure becomes a series system. Therefore series and parallel systems are special cases of a k-out-of-n structure.

1.4. **RELIABILITY IMPROVEMENT**

The manufacturers as well as the user of a system always desire a high reliability. Reliability of a system can be improved upon in several ways. One way of improving reliability of a unit is either duplicate some of the parts/components of the unit or the unit as a whole. The other way is to provide repair and maintenance to the system at the time of need. Some of the important techniques to improve the reliability of a unit/system are given below–

(a) **REDUNDANCY**

In redundant system some additional parts are created for performing the function of a system. Although either one of the component/unit is sufficient for the successful operation of the system, we deliberately use some more components/units so as to increase the probability of success, thus causing the system to be redundant. Thus, redundancy is the creation of new parallel paths in a system structure to improve the system reliability. The redundancy may be classified as-
(i) **ACTIVE REDUNDANCY**

An active redundant system with n units/components is one in which all the units function simultaneously and system operates even when one unit/component operates. In active redundant system all the units fail independently. For example – the human body is two organ active redundant system in respect of many functions e.g. seeing, hearing, breathing, walking, manual work etc.

(ii) **PASSIVE (STANDBY) REDUNDANCY**

A standby redundant system is one in which one unit/component is operative at a time and others are kept in spare are known as standbys. An n independent units/components standby system operates in the following manner: component \( C_1 \) operates until its failure, then component \( C_2 \) is switched on . . . . . . . . . . component \( C_1 \) operates until its failure, . . . . . . . . . . component \( C_n \) operates until its failure and then system is declared as failed. Such a system some times referred to as a sequentially redundant system of order n. This type of system is similar to parallel system of order n except that each component is used one at a time rather than using all of them simultaneously.

(b) **SYSTEM MAINTENANCE**

All recoverable systems which are used for continuous or intermittent service for some period of time are subject to maintenance. Maintenance action can be classified in the following categories

(i) Preventive Maintenance (P.M.)

(ii) Corrective Maintenance (C.M.)

(iii) Repair Maintenance (R.M.)

Preventive maintenance (P.M.) is a sort of repair that is done before the unit actually fails. Usually, a periodic policy for preventive maintenance is adopted. However, it is not always possible to perform this maintenance action exactly at that time when desired and thus one would expect the time at which the P.M. is made to be a random variable with perhaps small
dispersion about the desired time. During such maintenance the system can be either in operative mode or it can be switched off for sometime. It can be assumed that after each maintenance operation, system/unit works as good as new.

Corrective maintenance deals with the system performance when it gives wrong results. Repair maintenance is concerned with increasing system availability by implementation of major changes in the failed components of a unit. The reliability of the system can also be increased by repair maintenance if system consists of at least two units in parallel configuration. On failure, a unit is sent to a repair facility, if available i.e. free, otherwise it queues up for repair. The time taken for preventive maintenance, corrective maintenance and repair maintenance are random variable and it is assumed that the distribution functions of these variables are known and follow certain probability density functions.

There are many systems in which some units are given preference for operation as well as for repair. Such systems are called priority systems and consist of two sub-classes of units. One is the class of priority (p) units and the other is of ordinary (o) units. The p-unit is never kept as standby. When the p-unit fails, it goes into repair immediately if repair facility is free. However, if o-unit is under repair and the p-unit fails, the following policies can be adopted-

(i) **Pre-emptive priority**: The repair of the o-unit is interrupted and its repair is continued as soon as the repair of the p-unit is completed. The resumed repair of o-unit can follow any one of the following rules.

(1) **Pre-emptive resume**: The repair of the o-unit is continued from where it was left.

(2) **Pre-emptive repeat**: The repair of the o-unit is started afresh i.e. the time already spent in the repair of o-unit has no-influence on its repair time now.

(ii) **Non pre-emptive priority**: The repair of the o-unit is continued and the repair of the p-unit is start only when the repair of the o-unit is completed.
(c) **INSPECTION**

The inspection of the system at random epochs is useful to trace out the fault in redundant system, particularly in deteriorating standby systems. When in service, maintenance is impossible or unit failure indication is impractical, units in parallel redundant system must be periodically inspected to make assurance that none of them has failed and that the system still has its existing reliability.

### 1.5. SOME STOCHASTIC PROCESSES

A stochastic process \( \{X(t), t \in T\} \) is a collection of random variables *i.e.* for each \( t \in T \), \( X(t) \) is a random variable. The random variable \( X(t) \) is called the state of the system at time \( t \). For example, \( X(t) \) might be equal to the total number of incoming calls at a telephone exchange up to time ‘\( t \)’, the number of consumers that arrive in a supermarket up to time \( t \). If \( t \) assumes discrete values then the process \( \{X(t), t = 0, 1, 2, \ldots\} \) is called a discrete parametric stochastic process. If \( t \) assumes continuous values then the process \( \{X(t), t \geq 0\} \) is called as a continuous parametric stochastic process. The set of all possible values of \( X(t) \) is called the parametric space \( (S) \) and the set of all possible values of \( t \) is called the parametric space or the index set \( (T) \).

Generally, the stochastic processes encountered in the analysis of a specified complex system can be put into one of the following classes-

(a) **MARKOV PROCESS**

A stochastic process \( \{X(t), t \in T\} \) is said to be Markov process if for every \( n \) and arbitrary time points \( t_1 < t_2 < \ldots < t_n \), we have

\[
P[X(t_n) \leq x_n | X(t_1) = x_1, \ldots, X(t_{n-1}) = x_{n-1}] = P[X(t_n) \leq x_n | X(t_{n-1}) = x_{n-1}]
\]

It is a random process whose future probabilities are determined by its most recent values.
In other words a Markov process is a process having the Markovian property that the conditional probability of the future event depends on present and not on past.

A Markov process whose state space is discrete is called a Markov chain. A Markov chain is called a discrete parameter Markov chain or a continuous parameter Markov chain according as the parametric space is discrete or continuous.

(b) POISSON PROCESS

A Poisson process, named after the French mathematician Siméon-Denis Poisson (1781 - 1840), is a stochastic process which is defined in terms of the occurrences of events.

A stochastic process \( \{N(t), t \geq 0\} \) is said to be a counting process if \( N(t) \) represents the number of events that have occurred by the time \( t \), e.g. number of birth in a locality by the instant \( t \), number of goals that a given foot-ball player has scored by the epoch \( t \) etc. For a counting process, \( N(t) \) must satisfy

(i) \( N(t) \geq 0 \)
(ii) \( N(t) \) is integer valued.
(iii) If \( s \leq t \), then \( N(s) \leq N(t) \) and \( N(t) - N(s) \) represents the number of events that have occurred in the interval \( (s, t) \) and has a Poisson distribution.

The stochastic process \( \{N(t), t \geq 0\} \) is said to be a Poisson Process having with intensity rate \( \lambda > 0 \), if the following postulates are satisfied-

(i) \( N(0) = 0 \)
(ii) The process has stationary and independent increments.
(iii) \( P \left[ N(\Delta t) = 1 \right] = \lambda \Delta t + O(\Delta t) \) and
(iv) \( P \left[ N(\Delta t) \geq 2 \right] = O(\Delta t) \).

There are some examples of Poisson process-

- The number of telephone calls arriving at a switchboard, or at an automatic phone-switching system, may be characterized by a Poisson process.
The number of photons hitting a photodetector, when lit by a laser source, may be characterized by a homogeneous Poisson process. Other sources may show either a bunching or an antibunching of the photons.

The number of web page requests arriving at a server may be characterized by a Poisson process except for unusual circumstances such as coordinated denial of service attacks.

The number of raindrops falling over a wide spatial area may be characterized by a spatial Poisson process.

The arrival of "customers" is commonly modelled as a Poisson process in the study of simple queueing systems.

(c) **REGENERATIVE PROCESS**

Let \( X(t) \) be the state of the system at epoch \( t \). If \( t_1, t_2, \ldots, t_n \) are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process \( [X(t), t = t_1, t_2, \ldots, t_n] \) is called regenerative process. Here, the process beyond \( t_1 \) is a probabilistic replica of the whole process starting at \( t = 0 \). A regenerative state has the property that as soon as the system enters it, its further development is independent of the past history. This process was introduced by Smith.

1.6. **THESIS AT A GLANCE**

The thesis presents the analysis of some stochastic system models of practical importance under a variety but realistic set of assumptions. Important reliability characteristics such as transient and steady state transition probabilities, mean sojourn times in various states, mean time to system failure (MTSF), reliability, pointwise and steady state availabilities, expected busy period of the repair facility, expected profit earned in the interval \((0, t)\) and in steady state etc. are obtained. The subject matter of the thesis is divided into five chapters. The chapter one is purely introductory and other four are analytical.
Introduction

In the first chapter, we present the meaning and scope of reliability in the modern era including important aspects of system configurations, various measures of system effectiveness, important methods of reliability improvement and a brief description of different stochastic processes involved in the analysis of complex systems.

Chapter – 2 contains the study of a blood bank system. The system consists of three subsystems viz. Power supply (P), Chillers (C₁ & C₂) and Generator (G). Both the chillers are identical and work in parallel network whereas either the power supply or generator is required for the functioning of chillers. The generator is considered as cold standby when power is present. The failure and repair rates of different subsystems are taken to be constant with different parameters.

Chapter – 3 is devoted to the study of the profit analysis of a stochastic model which consists of three non-identical units A, B and C. Initially the system starts from state $S_0$ in which unit A is operative and units B & C are kept in cold standby. The preference in operation and repair is given to unit A over units B and C and to unit B over unit C. The failure and repair times of each unit are taken to be bivariate exponential. A single repairman is always with the system to repair a failed unit. Using the regenerative point technique, various measures of system effectiveness are obtained.

Chapter – 4 deals with the cost benefit analysis of a two non-identical unit system model with the concept of correlation between time to preventive maintenance and time taken in preventive maintenance assuming their joint distribution as bivariate exponential. The system consists of two non identical units (unit-1, unit-2). Unit-1 gets preference in operation over unit-2. Initially the system starts from state $S_0$ in which unit-1 is operative & unit-2 is kept as cold stand by. The failure & repair time distributions of both the units are taken as exponential with different parameters. After working some significant time the operative unit goes for preventive maintenance. A repairman is always available with the system to repair a failed unit and for preventive maintenance of an operating unit. The switching device used to
detect the failed unit and to put the standby unit into operation is perfect and instantaneous. The repair and preventive maintenance discipline is first come first served (FCFS).

Chapter – 5 presents the analysis of a complex system with two physical conditions of repairman and inverse Gaussian repair time distribution. The system consists of two subsystems A & B arranged in series configuration. Subsystem A comprises of two identical units in passive redundancy and subsystem B consists of only one unit which is dissimilar of the units of subsystem A. System failure occurs when either both units of subsystem A fail or subsystem B stop functioning. Each unit of the system has two modes – Normal (N) and Total failure (F). A repairman is always available with the system to repair a failed unit. He may be in good/poor physical condition at the time of need with fixed known probabilities p and q respectively. The failure times of the units of both the subsystems are taken as exponential with different parameters. The distribution of time to repair of each unit in both the physical conditions of repairman is taken as Inverse Gaussian with different parameters.

Finally, the thesis also includes the two appendices – A and B. Appendix – A covers the definitions of various mathematical results and transforms used throughout the thesis and Appendix – B contains a list of references.