Chapter 4

An EPQ model with promotional demand in random planning horizon: population varying genetic algorithm approach

4.1 Introduction

One of the key features of an Economic Production Quantity (EPQ) model is that the quality level is not subject to control, i.e., the defective items sometimes may also be produced during the production run time. These defective items may be discarded or sold at a reduced price (cf. Das and Mali [48], Gurnasia et al. [83]) or may be repaired / reworked (cf. Manna et al. [147]) and then that items are reached to the customers. In 2000, Salameh and Jaber [184] developed an EPQ model with imperfect quality items. Then Goyal and Cardenas-Barron [78] extended the EPQ model of Salameh and Jaber [184] considering the rework of imperfect items. Khouja and Melendez [114] and Khouja [115] extended the imperfect production process with flexible production rates to reduce the imperfect rate. Hussein et al. [92] and Sana et al. [185, 186] also discussed the volume of flexibility policy in production. In practice, it is observed that the screening process is essentially required to sort out the imperfect items which are to be reworked. So, in most of the existing literature, the imperfect quantities are either rejected or fully screened in a single cycle for a single item. **But, till now, no one has considered a multi-item multi-cycle EPQ model for imperfect quality items with rework.**

Moreover, the inflation and time value of money play an important role in long time

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business concerns, especially in the developing countries. Therefore, the effect of inflation and time value of money cannot be ignored in real situations. To relax the assumption of inflation, Duzacott [12] and Dey et al. (55), (57) simultaneously developed an EOQ model with a constant inflation rate to all associated costs. From these literature survey, it is known that no one has considered inflation and time value money together in an imperfect production inventory model.

In the present competitive market, to increase the sale of items, the inventory/stock is decoratively displayed to attract the customers. In this context, many researchers considered the stock dependent demand (cf. Chan et al. [20, 22], Das et al. [49]) instead of constant demand (cf. Dey et al. [55], Jana et al. [105]). Wadkva et al. [209] also established the impact of product availability for impulse demand. All these factors are virtually depending on selling price and mode or regularity of the advertisement. Basically, the demand rate may change due to different market conditions. But, there may be some dedicated customers who are accustomed to a particular brand and they wait for that product until it is available. Therefore, the demand of any item must have a minimum value. So, the demand of an existing product can be considered as \( D = A + B \), where \( A \) represents the minimum requirement from the loyal customers who buy this product regularly and \( B \) represents the varying demand from the disloyal customers. The volume of these disloyal customers depends on the attractive contract terms, e.g. long-term relationships, quantity discounts, low selling price for high quality items, terms of payment and delivery. Another relevant and important scenarios are often accompanied by an intensive promotion campaign or advertisement. In all the existing literature, the different demand parametric values are taken by intuitions (Baron and Sana [13]). But, till now, no one has considered the estimation of different demand parameters from market survey in imperfect production inventory model.

Normally, it is seen that the production inventory models are developed in infinite planning horizon (cf. Wang and Chan [218]) because it is assumed that in production inventory process, the related various parameters remain constant over the future infinite time. But, in reality, it is not correct due to several reasons such as variation of inventory costs, changes of product specifications and designs, technological changes due to environmental conditions, availability of product, etc. Moreover, for seasonal products like fruits, vegetables, warm garments, etc. business period may not be finite. Das et al. [49], Ali et al. [5] and others supported this idea. Rather for seasonal products (like, different type of juice, medicines, etc) the planning horizon is not fixed, it varies over the years and may be considered as a random variable with some probability. Moon [155] considered such type of horizon with exponential distribution for an EOQ model. Then, a lot of research works has been done in this field (cf. Roy et al. [182]). Very recently, Jana et al. [105] considered the random planning horizon for an EOQ model with shortages. But, none have considered the random planning horizon for a imperfect production rework model. The detailed comparative statement of the proposed model with the existing literature has been given in Table 4.1.
### 4.2. NOTATIONS AND ASSUMPTIONS

<table>
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<th>Author(s)</th>
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<th>Production rate</th>
<th>Demand rate</th>
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<th>Multi-item</th>
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<td>Das et al. [49]</td>
<td>EPQ</td>
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<tr>
<td>Yang et al. [232]</td>
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<td>advertise and selling</td>
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Table 4.1: Summary of related literature for multi-item EOQ/EPQ models

In this chapter, a multi-item production inventory model with promotional effected demand under space constraints has been formulated. In the underlying EPQ model, the production rate has been considered on the basis of demand of the market and the production cost is taken as Khouja function (Khouja [115]) in the random planning horizon. The present study provides the rework facility of the unsatisfactory products. A constant rate of inflation is also introduced in the uncertain time scale. **To our knowledge, these issues have not been considered together by any researcher earlier.** Moreover, here we perform a numerical study from a market survey with a statistical test which enhances the feasibility of the model.

### 4.2 Notations and Assumptions

The following notations and assumptions have been used to developing the proposed model:

#### 4.2.1 Notations

The following notations are used throughout the entire chapter (for the $j$-th item).

- $q_i^j(t)$: On hand inventory of $i$-th cycle at time $t$ for perfect quality, $(i = 1, 2, ..., N^j)$.
- $q_f^j(t)$: On hand inventory of last cycle at time $t$ for perfect quality.
- $Q^j$: Maximum inventory level for perfect quality.
- $p^j$: The production rate of each cycle.
- $\psi^j$: Advertisement effort.
- $D^j$: Demand rate of each cycle for perfect quality.
- $N^j$: Number of fully accommodated cycles to be made during the prescribed time horizon.
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\( T^i \) : Duration of a complete cycle.
\( t^i_j \) : Duration of production in each cycle.
\( \delta^i_j \) : Percentage of perfect quality.
\( \delta^i \) : Percentage of reworking to be perfect quality items from imperfect quality items.
\( c_p^i \) : Production cost per unit.
\( c_{st}^i \) : Screening cost per unit item.
\( r^i_j \) : Reworking cost per unit item.
\( h^i \) : The inventory holding cost per unit time for perfect quality item.
\( s^i \) : Selling price per unit for perfect quality item.
\( r \) : The discount rate.
\( k \) : The inflation rate which is varied by the social economical situations.
\( R \) : \((=r-k)\) The discount rate minus the inflation rate.
\( H \) : The length of time horizon.
\( M \) : Total number of items.
\( a^j \) : Space required for storing \( j \)-th item per unit.
\( B \) : Total space available in the system.

4.2.2 Assumptions

The following assumptions have been used to develop this model.

(i) In this production system, the multiple items are produced in the form of perfect and imperfect quality. The perfect quality items are directly ready for sale and some of the imperfect quality items are reworked to make as good as perfect. The rest imperfect quality items which may be too much expensive to make as perfect quality items, are disposed.

(ii) In this model the production rate \((P^j)\) has been considered as \(P^j = P_0^j + P_1^j D^j\)
where (a) \(P_0^j\) = Minimum production (which is constant) per unit time and (b) \(P_1^j D^j\) = The portion of the production which varies directly from the market demand per unit time.

(iii) Here, demand of the items are inversely and directly proportional to the selling price and advertisement respectively with constant coefficients which are estimated from the market survey. The mathematical form of the demand rate of each item is:
\( D^j(s^i, s^j) = D_0^j + D_1^j(s^i) + D_2^j(s^j) \), where (a) \(D_0^j\) = minimum demand (which is independent of advertisement effort and selling price) for each cycle.
(b) \(D_1^j(s^i) = A^j \frac{s^i - s_{min}}{s_{max} - s_{min}}\) where \(A^j\) is positive constant and \(s^j \in [s_{min}^j, s_{max}^j]\) for each \(j\).
Here \(\frac{d}{ds^i} D_2^j(s^j) = A^j \frac{d}{ds^i} \frac{s^i - s_{min}}{s_{max} - s_{min}} < 0\). This shows that the demand \(D_2^j(s^j)\) is decreasing function of the selling price \(s^j\). When \(s^j = s_{max}^j\) then \(D_2^j(s^j) = 0\) and when \(s^j = s_{min}^j\) then the market has unlimited demand, i.e., \(D_2^j(s^j) \in [0, \infty]\), for \(s^j \in [s_{min}^j, s_{max}^j]\) for \(j=1,2,\ldots,M\).
4.3 Mathematical Formulation of the Proposed Model

In this model, simultaneously more than one item are produced in both perfect and imperfect forms. The produced items are screened 100% and the repairable items are reworked and
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returned to the inventory of the good items. The perfect quality items are disposed to the customers as per their demand rate (as discussed in assumption (iii)). Such EPQ model is formulated in random time horizon scale which is taken as exponential distribution. The model also assumes the existence of a pair of mutually exclusive events for the last cycle. For the development of this model, we assume the time horizon \( H \) (random variable) to accommodate completely \( N^j \) cycles of equal time period \( T^j \), for \( j \)-th item.

### 4.3.1 Formulation for \( i \)-th (\( 1 \leq i \leq N^j \)) Cycle of \( j \)-th Item

In this case, the initial stock of the \( i \)-th cycle is zero and it starts the production with a rate \( P^j \). As the production and rework continues, the inventory begins to pile up continuously after meeting the demand of rate \( (D^j) \) of the customers. The production of the \( i \)-th cycle continues up to the time \( (i-1)T^j + t^i \) and again it starts at time \( iT^j \) for next cycle. Each cycle ends with zero inventory. During the cycle period \( [(i-1)T^j, iT^j] \), the inventory rate increases with the production rate of perfect item and the rework rate of the imperfect item as well as it decreases along with the demand rate.

Hence, the differential equation to describe the inventory level \( q^j_i(t) \) \( (i-1)T^j \leq t \leq iT^j \) is given by

\[
\frac{dq^j_i(t)}{dt} = \begin{cases} 
\beta^j P^j + \delta^j (1 - \beta^j) P^j - D^j, & (i-1)T^j \leq t \leq (i-1)T^j + t^i \\
-D^j, & (i-1)T^j + t^i \leq t \leq iT^j 
\end{cases}
\]

subject to the boundary conditions: \( q^j_i[(i-1)T^j] = 0 \) and \( q^j_i[iT^j] = 0 \).

The above differential equation is linear with unit integrating factor. Therefore, the inventory level is given by the solution:

\[
q^j_i(t) = \begin{cases} 
\left\{ \beta^j P^j + \delta^j (1 - \beta^j) (P^j + P^j D^j) \right\} (i-1)T^j - t, & (i-1)T^j \leq t \leq (i-1)T^j + t^i \\
\left\{ \beta^j P^j + \delta^j (1 - \beta^j) (P^j + P^j D^j) \right\} t^i, & (i-1)T^j + t^i \leq t \leq iT^j 
\end{cases}
\]  \( \text{ (4.2)} \)

The above \( q^j_i(t) \) is a continuous function on the defined range of interval \( [(i-1)T^j, iT^j], \) therefore

\[
q^j_i[(i-1)T^j + t^i - 0) = q^j_i([(i-1)T^j + t^i + 0) \\
i.e., \{ \beta^j P^j + \delta^j (1 - \beta^j) (P^j + P^j D^j) t^i = D^j T^j \]  \( \text{ (4.3)} \)

Therefore, the present value of production cost of the \( j \)-th item during the \( i \)-th \( (1 \leq i \leq N^j) \) cycle is given by

\[
PC^j = c^j P^j e^{-Rt} \int_{(i-1)T^j}^{iT^j + t^i} P^j e^{-Rt} dt = \frac{c^j P^j}{R} (P^j + P^j D^j) (1 - e^{-Rt^i}) e^{-RG^j} \]
Present value of screening cost of the ith \((1 \leq i \leq N^j)\) cycle is given by
\[
SC_i^j = c_s^j \int_{(i-1)T^j}^{(i)T^j} P^j e^{-R^j t} \, dt = \frac{c_s^j}{R}(P_0^j + P_1^j D^j)(1 - e^{-R^j}) e^{-R(i-1)T^j}
\]

Present value of reworked cost of the ith \((1 \leq i \leq N^j)\) cycle is given by
\[
RC_i^j = \frac{r^j}{R} \int_{(i-1)T^j}^{(i)T^j} \delta^j (1 - \beta^j) P^j e^{-R^j t} \, dt \\
= \frac{r^j}{R} \delta^j (1 - \beta^j) (P_0^j + P_1^j D^j)(1 - e^{-R^j}) e^{-R(i-1)T^j}
\]

Present value of holding cost of the inventory for the ith \((1 \leq i \leq N^j)\) cycle is given by
\[
HC_i^j = h_i^j \int_{(i-1)T^j}^{(i)T^j} q_i^j(t) e^{-R^j t} \, dt + \int_{(i-1)T^j}^{(i)T^j} q_i^j(t) e^{-R^j t} \, dt \\
= \frac{h_i^j}{R^2} \left\{ [\delta^j + \delta^j (1 - \beta^j)] (P_0^j + P_1^j D^j) - D^j \right\} \left\{ 1 - (1 + R^j) e^{-R^j} \right\} e^{-R(i-1)T^j} \\
+ \frac{h_i^j D^j}{R^2} \left\{ e^{-R^j} - (RT^j - \ell_{T^j}^i + 1) e^{-R^j} \right\} e^{-R(i-1)T^j}
\]

Present value of sales revenue for the ith \((1 \leq i \leq N^j)\) cycle is given by
\[
SR_i^j = s^j \int_{(i-1)T^j}^{(i)T^j} D^j e^{-R^j t} \, dt = \frac{D^j s^j}{R} (1 - e^{-R^j}) e^{-R(i-1)T^j}
\]

Therefore, total profit after completing \(N^j\) full cycles is given by
\[
TP(T^j) = \sum_{i=1}^{N^j} \left[ SR_i^j - PC_i^j - SC_i^j - RC_i^j - HC_i^j \right] \\
= \left[ \frac{D^j s^j}{R} (1 - e^{-R^j}) - \frac{1}{R} (c_s^j + c_r^j + r^j \delta^j (1 - \beta^j))(P_0^j + P_1^j D^j)(1 - e^{-R^j}) \right] \\
- \frac{h_i^j}{R^2} \left\{ [\delta^j + \delta^j (1 - \beta^j)] (P_0^j + P_1^j D^j) - D^j \right\} \left\{ 1 - (1 + R^j) e^{-R^j} \right\} \\
- \frac{h_i^j D^j}{R^2} \left\{ e^{-R^j} - (RT^j - \ell_{T^j}^i + 1) e^{-R^j} \right\} \left\{ 1 - e^{-R^j} \right\} \left\{ 1 - e^{-RT^j} \right\} (See \ Appendix \ B).
\]

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Since \( f(h) \) is the p.d.f of the planning horizon \( H \), therefore the expected total profit in \( N^j \) complete cycles is given by

\[
E[TPP(T^j)] = \sum_{\substack{N^j > 0}} \int_{N^j T^j}^{(N^j + 1) T^j} TPP(N^j, T^j) f(h) \, dh
\]

\[
= \left[ \frac{s^j D^j}{R} \left( 1 - e^{-R T^j} \right) - \frac{1}{R} \left( c^j_p + c^j_r + r^j \delta^j (1 - \beta^j) \right) (P^j_0 + P^j D^j) \left( 1 - e^{-R T^j} \right) \right]
\]

\[
- \frac{b^j}{R^2} \left\{ \delta^j (1 - \beta^j) (P^j_0 + P^j D^j) - D^j \right\} \left\{ 1 - \left( 1 + b^j T^j \right) e^{-R T^j} \right\} \frac{e^{-N^j T^j}}{1 - e^{-N^j T^j}}. \quad \text{(See Appendix B)}.
\]

4.3.2 Formulation for Last Cycle

In this case, the initial stock of the last cycle is zero and starts production with rate \( P^j \). As production and rework are continue, inventory begins to pile up continuously after meeting demand with rate \( D^j \). Production and reworking of last cycle stop at time \( N^j T^j + t^j_1 \). During the time \( [N^j T^j, N^j T^j + t^j_1] \), the inventory rate increases with production rate of perfect item, rework rate of the imperfect item and decreases with demand rate. And during the time \( [N^j T^j + t^j_1, (N^j + 1) T^j] \) inventory level decreases with demand rate. Hence, the differential equation describing the inventory level \( q^j_l(t) \) in the interval \( N^j T^j \leq t \leq (N^j + 1) T^j \) is given by

\[
\frac{dq^j_l(t)}{dt} = \begin{cases} 
\beta^j \left( P^j + \delta^j (1 - \beta^j) P^j \right) - D^j, & N^j T^j \leq t \leq N^j T^j + t^j_1 \\
-D^j, & N^j T^j + t^j_1 \leq t \leq (N^j + 1) T^j 
\end{cases}
\]

subject to the boundary conditions: \( q^j_l(N^j T^j) = 0 \) and \( q^j_l((N^j + 1) T^j) = 0 \).

The solution of above differential equation is given by

\[
q^j_l(t) = \begin{cases} 
\left\{ \beta^j \left( P^j_0 + P^j D^j \right) \right\} \left( t - N^j T^j \right), & N^j T^j \leq t \leq N^j T^j + t^j_1 \\
D^j \left\{ (N^j + 1) T^j - t \right\}, & N^j T^j + t^j_1 \leq t \leq (N^j + 1) T^j 
\end{cases}
\] (4.4)

The above equation indicate the amount of stock at any time during the last cycle. Moreover, the parameter \( h \) present in the expression of last cycle is a random variable follows exponential distribution (as discuss in assumption (v)). For simplicity we consider two cases depending upon the cycle length.
4.3. MATHEMATICAL FORMULATION OF THE PROPOSED MODEL

Case-I: when $N^{j}T^{j} \leq h < N^{j}T^{j} + t^{j}_{1}$

![Graphical Representation of Inventory Model for case-I](image)

Figure 4.1: Graphical Representation of Inventory Model for case-I

As like earlier, the present value of production cost of the $j$th item for the last cycle is given by

$$PC_{L_{1}}^{j} = C_{j}^{l} \int_{N^{j}T^{j}}^{h} P^{j} e^{-Rt} \, dt = \frac{c_{j}^{l}}{R} (P_{0}^{j} + P_{1}^{j} D^{j}) (e^{-R N^{j}T^{j}} - e^{-R h})$$

Present value of screening cost of the last cycle is given by

$$SC_{L_{1}}^{j} = C_{j}^{l} \int_{N^{j}T^{j}}^{h} P^{j} e^{-Rt} \, dt = \frac{c_{j}^{l}}{R} (P_{0}^{j} + P_{1}^{j} D^{j}) (e^{-R N^{j}T^{j}} - e^{-R h})$$

Present value of reworked cost of the last cycle is given by

$$RC_{L_{1}}^{j} = r_{j} \int_{N^{j}T^{j}}^{h} \delta^{j} (1 - \beta^{j}) P^{j} e^{-Rt} \, dt = \frac{r_{j}^{j}}{R} \delta^{j} (1 - \beta^{j}) (P_{0}^{j} + P_{1}^{j} D^{j}) (e^{-R N^{j}T^{j}} - e^{-R h})$$

Present value of sales revenue for the last cycle is given by

$$SR_{L_{1}}^{j} = s^{j} \int_{N^{j}T^{j}}^{h} D^{j} e^{-Rt} \, dt = \frac{D_{1}^{j} s^{j}}{R} (e^{-R N^{j}T^{j}} - e^{-R h})$$

Present value of holding cost of the inventory for the last cycle is given by

$$HC_{L_{1}}^{j} = h_{j}^{l} \int_{N^{j}T^{j}}^{h} q_{L_{1}}^{j}(t) e^{-Rt} \, dt$$

$$= \frac{h_{j}^{l}}{R^{2}} \left[ \beta^{j} + \delta^{j} (1 - \beta^{j}) (P_{0}^{j} + P_{1}^{j} D^{j}) - D^{j} \right] \left[ e^{-R N^{j}T^{j}} - (1 + R(h - N^{j}T^{j})) e^{-R h} \right]$$
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**Case-II:** when \( N^jT^j + t^j \leq h < (N^j + 1)T^j \)

![Graphical Representation of Inventory Model for case-II](image)

Similarly, the present value of the production cost for the last cycle is given by

\[
PC_{L2}^j = c_1^j \int_{N^jT^j}^{N^jT^j + t^j} P^j e^{-Rt} dt = c_1^j \frac{P^j}{R} (P_0^j + P_1^jD^j)(1 - e^{-Rt^j})e^{-RN^jT^j}
\]

Present value of screening cost of the last cycle is given by

\[
SC_{L2}^j = c_2^j \int_{N^jT^j}^{N^jT^j + t^j} P^j e^{-Rt} dt = c_2^j \frac{P^j}{R} (P_0^j + P_1^jD^j)(1 - e^{-Rt^j})e^{-RN^jT^j}
\]

Present value of reworked cost of the last cycle is given by

\[
RC_{L2}^j = r^j \int_{N^jT^j}^{N^jT^j + t^j} \delta^j (1 - \beta^j) P^j e^{-Rt} dt = r^j \frac{P^j}{R} \delta^j (1 - \beta^j) (P_0^j + P_1^jD^j)(1 - e^{-Rt^j})e^{-RN^jT^j}
\]

Present value of holding cost of the inventory for the last cycle is given by

\[
HC_{L2}^j = h \left[ a_1^j \int_{N^jT^j}^{N^jT^j + t^j} q_1^j(t)e^{-Rt} dt + \int_{N^jT^j + t^j}^{h} q_1^j(t)e^{-Rt} dt \right]
\]

\[
= h \frac{P^j}{R^2} \left[ (\beta^j + \delta^j (1 - \beta^j)) (P_0^j + P_1^jD^j) - D^j \right] \left[ 1 - e^{-R(N^j + 1)T^j} \right] e^{-RN^jT^j}
\]

\[
+ \frac{h}{R^2} D^j \left[ 1 - R \{(N^j + 1)T^j - h\} \right] e^{-RN^jT^j} + \frac{h}{R^2} \delta^j \left[ \{(N^j + 1)T^j - h\} R - 1 \right] e^{-RN^jT^j} e^{-Rt^j}
\]

Present value of sales revenue for the last cycle is given by

\[
SR_{L2}^j = s^j \int_{N^jT^j}^{h} D^j e^{-Rt} dt = \frac{D^j s^j}{R} (e^{-RN^jT^j} - e^{-Rt^j})
\]

Hence, expected production cost for the last cycle (see Appendix B) is given by

\[
E[PC_{L2}^j] = \sum_{N^j = 0}^{N^j + 1} \int_{N^jT^j}^{(N^j + 1)T^j} PC_{L2}^j f(h) dh
\]

\[
= \frac{P^j}{R} \left[ \frac{P_0^j + P_1^jD^j}{1 - \frac{R}{R + \lambda}} \right] \left[ \frac{R}{R + \lambda} \left( 1 - e^{-(R + \lambda)t^j} \right) - (1 - e^{-Rt^j})e^{-\lambda T^j} \right]
\]

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4.3. Mathematical Formulation of the Proposed Model

Expected screening cost for the last cycle (see Appendix B) is given by

\[ E[SC^j_k] = \sum_{N_T=0}^{\infty} \int_{N_T+j}^{(N_T+j+1)T_j} SC^j_k f(h) \, dh \]

\[ = \frac{c^j_k (P^j_k + P^j_0 D^j)}{R \{ 1 - e^{-(R+\lambda)T_j} \}} \left\{ \frac{R}{R + \lambda} \left[ 1 - e^{-\lambda T^j} \right] - (1 - e^{-R T^j}) e^{-\lambda T^j} \right\} \]

Expected reworked cost for the last cycle (see Appendix B) is given by

\[ E[RC^j_k] = \sum_{N_T=0}^{\infty} \int_{N_T+j}^{(N_T+j+1)T_j} RC^j_k f(h) \, dh \]

\[ = \frac{\delta^j_k (1 - \beta^j)(P^j_0 + P^j_0 D^j)}{R \{ 1 - e^{-(R+\lambda)T_j} \}} \left\{ \frac{R}{R + \lambda} \left[ 1 - e^{-\lambda T^j} \right] - (1 - e^{-R T^j}) e^{-\lambda T^j} \right\} \]

Expected holding cost for the last cycle is given by

\[ E[HC^j_k] = \sum_{N_T=0}^{\infty} \int_{N_T+j}^{(N_T+j+1)T_j} HC^j_k f(h) \, dh + \sum_{N_T=0}^{\infty} \int_{N_T+j+1}^{(N_T+j+1+1)T_j} HC^j_k f(h) \, dh \]

\[ = \frac{h^j_k}{R^2 \{ 1 - e^{-(R+\lambda)T_j} \}} \left\{ \left\{ \beta^j + \delta^j (1 - \beta^j) \{ P^j_0 + P^j_0 D^j \} - D^j \right\} \left\{ 1 - e^{-\lambda T^j} \right\} + \frac{\lambda}{R^2 (R + \lambda)^2} \left\{ \{ R (R + \lambda) T^j + \lambda \} e^{-(R+\lambda)T^j} - \lambda \right\} + \{ 1 + \lambda T^j \} e^{-\lambda T^j} \{ e^{-\lambda T^j} - e^{-\lambda T^j} \} \right\} \]

\[ - \frac{\lambda T^j D^j}{R^2 (R + \lambda)^2 \{ 1 - e^{-(R+\lambda)T_j} \}} \left\{ \lambda \{ e^{-(R+\lambda)T^j} - e^{-\lambda T^j} \} - (R + \lambda) e^{-(R+\lambda)T^j} \right\} \]

\[ + \frac{h^j_k D^j}{R^2 \{ 1 - e^{-(R+\lambda)T_j} \}} \left\{ (T^j - T^j) R + 1 \} \{ e^{-\lambda T^j} - e^{-\lambda T^j} \} e^{-\lambda T^j} \right\} \]

Expected sales revenue from the last cycle (see Appendix B) is given by

\[ E[SR^j_k] = \sum_{N_T=0}^{\infty} \int_{N_T+j}^{(N_T+j+1)T_j} SR^j_k f(h) \, dh \]

\[ = \frac{D^j_k}{R \{ 1 - e^{-(R+\lambda)T_j} \}} \left\{ (1 - e^{-(R+\lambda)T^j}) + \frac{\lambda}{R + \lambda} e^{-(R+\lambda)T^j} - 1 \right\} \]

4.3.3 Objective of the Proposed Model

Therefore, expected total profit in the last cycle for jth item is given by the positive differences of the costs from the sales revenue, i.e.,

\[ E[TP^j_k(T^j)] = E[SB^j_k] - E[RC^j_k] - E[SC^j_k] - E[RC^j_k] - E[HC^j_k] \]
CHAPTER 4. AN EPQ MODEL WITH PROMOTIONAL DEMAND IN RANDOM PLANNING HORIZON: POPULATION VARYING GENETIC ALGORITHM APPROACH

Now, total expected profit during the time horizon is given by adding the total expected profit of \( N^i \) complete cycles and expected total profit for last cycle and summing over the number of items, i.e.,

\[
E[TP] = \sum_{j=1}^{M} E[TP^j_i(T^j)] + \sum_{j=1}^{M} E[TP^j_i(T^j)] \quad (4.5)
\]

In the present chapter the decision manager with its limited space investigate the optimum decision variables \( T^j \) in such a manner that the total expected profit is maximum. Hence the problem becomes:

Maximize \( E[TP] = \sum_{j=1}^{M} E[TP^j_i(T^j)] + \sum_{j=1}^{M} E[TP^j_i(T^j)] \quad (4.6) \)

Subject to the constraint: \( \sum_{j=1}^{M} a^jQ^j \leq B \quad (4.7) \)

4.4 Solution Procedure

The proposed model is numerically solved through following steps:

Step-1: Consider the minimum demand rates of the items \( D^j_i \).

Step-2: Estimate the other demand parameters.

Step-3: Receive the other input parameters.

Step-4: Hypothetically, propose the available space.

Step-5: With the help of above data, decision variables \( T^1, T^2 \) are optimized through GAVP, described in section 2.4.4.

4.5 Numerical Illustration

To find the different input parameters of the demand function, we consider the market survey at “Sarana Steel Furniture, Midnapore, Paschim Medinipur, W.B., India”, for two different items: steel almirah and steel bed. The minimum demand be \( D^j_i \) estimated by considering the minimum demand for the year-2012, \( D^j_{10}=49 \) unit and \( D^j_{11}=47 \) unit.

The second term of demand expression \( D^j_{(s^j)}, s^j_{\text{min}}=Rs.72, s^j_{\text{max}}=Rs.75 \). Other constant parameters \( A^j \) and \( s^j_{\text{max}} \) are estimated using the method curve fitting from the average daily demands of 7 months (during this months the selling prices are different).
4.5. NUMERICAL ILLUSTRATION

Table 4.2: Market survey for estimating \( D^1_D (s^1) \)

<table>
<thead>
<tr>
<th>Markets →</th>
<th>Jan’12</th>
<th>Feb’12</th>
<th>Mar’12</th>
<th>Apr’12</th>
<th>May’12</th>
<th>Jun’12</th>
<th>Jul’12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^1 (Rs.) )</td>
<td>88</td>
<td>89</td>
<td>90</td>
<td>91</td>
<td>92</td>
<td>93</td>
<td>94</td>
</tr>
<tr>
<td>( D^1 (unit) )</td>
<td>55.8</td>
<td>54.6</td>
<td>53.1</td>
<td>52.9</td>
<td>52.3</td>
<td>51.5</td>
<td>51.3</td>
</tr>
</tbody>
</table>

Following Appendix B, the estimated values are: for the first item \( A^1 = 10.20, s^1_{\text{max}} = Rs.98.30 \), and similarly for the 2nd item \( A^2 = 7, s^2_{\text{max}} = Rs.98.00 \). Again, the demand parameter \( k^1 \) present in the third term are also estimated from the daily market demand about 7 months (January, 2007 to July, 2007) (This survey is made in the period, when selling price is fixed at Rs.91) by increasing advertisement efforts month to month.

Table 4.3: Market survey for estimating \( D^2_D (\nu^1) \)

<table>
<thead>
<tr>
<th>Days →</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu^1 )</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>17</td>
<td>20</td>
<td>23</td>
<td>26</td>
</tr>
<tr>
<td>( D^2 (\text{unit}) )</td>
<td>58.3</td>
<td>58.8</td>
<td>59.1</td>
<td>59.5</td>
<td>59.8</td>
<td>60.2</td>
<td>60.8</td>
</tr>
</tbody>
</table>

Following Appendix B, the estimated value: for the 1st item \( k^1 = 6.97 \) and similarly for the 2nd item \( k^2 = 4.0 \). Other input parameters are: \( \lambda = 0.1009, R = 0.39, D^1 = 59.56 \text{unit}, D^2 = 51.41 \text{unit}, \beta^1 = 0.95, \beta^2 = 0.94, \delta^1 = 0.86, \delta^2 = 0.87, \eta_1 = 0.3, \eta_2 = 0.83, L^1 = 6, L^2 = 16, M^1 = 2, M^2 = 2.5, \rho_1 = .03, \rho_2 = .031, c^1_1 = Rs.34, c^2_1 = Rs.28, \nu^1 = 20, \nu^2 = 16, s^1 = Rs.91, s^2 = Rs.95, r^1_c = Rs.2, r^2_c = Rs.2.02, c^1_{1r} = Rs.1.5, c^2_{1r} = Rs.1.25, h^1_L = Rs.2.10, h^2_L = Rs.2.42.

Now, using above input values we get the optimum expected profit which given computational result: Table 4.4.

Table 4.4: Optimum results of the illustrated model

<table>
<thead>
<tr>
<th>ITEM</th>
<th>( T^A (\text{unit}) )</th>
<th>( t^A (\text{unit}) )</th>
<th>( Q^A (\text{unit}) )</th>
<th>( P^A (\text{unit}) )</th>
<th>( E(PC)^A (\text{Rs.}) )</th>
<th>( ET\ P (\text{Rs.}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITEM-1</td>
<td>7.60</td>
<td>3.67</td>
<td>234.08</td>
<td>124.20</td>
<td>37.72</td>
<td>6914.45</td>
</tr>
<tr>
<td>ITEM-2</td>
<td>4.53</td>
<td>1.97</td>
<td>132.85</td>
<td>120.31</td>
<td>31.19</td>
<td></td>
</tr>
</tbody>
</table>

The graphical representation of the fitness value of \( ETP \) for different number of generations are given in the following figure:
CHAPTER 4. AN EPQ MODEL WITH PROMOTIONAL DEMAND IN RANDOM PLANNING HORIZON: POPULATION VARYING GENETIC ALGORITHM APPROACH

Figure 4.3: ETP vs no of generation

Figure 4.4: Selling price vs production rate  Figure 4.5: Selling price vs inventory level

Figure 4.6: Demand rate vs inventory level  Figure 4.7: Production rate vs inventory level
4.5. NUMERICAL ILLUSTRATION

4.5.1 Sensitivity Analysis

The change in the values of parameters may happen due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision-making. The numerical example given in the preceding section, the sensitivity analysis of various parameters such as advertisement efforts ($v_1, v_2$), inflation effect ($R$) and mean ($\lambda$) of exponential distribution has been done. The optimal values of $T^1$ and $T^2$ along with the maximum expected total profit have been calculated for different values of $v_1$, $v_2$, $R$ and $\lambda$. The results of sensitivity analysis are summarized in Table 4.5 and Table 4.6.

<table>
<thead>
<tr>
<th>Advertising effort</th>
<th>Demand rate 1st item ($T^1$)</th>
<th>Demand rate 2nd item ($T^2$)</th>
<th>Expected total profit (ETP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>59.522</td>
<td>51.783</td>
<td>6907.36</td>
</tr>
<tr>
<td>15</td>
<td>59.522</td>
<td>51.800</td>
<td>6908.84</td>
</tr>
<tr>
<td>16</td>
<td>59.522</td>
<td>51.814</td>
<td>6910.15</td>
</tr>
<tr>
<td>17</td>
<td>59.522</td>
<td>51.927</td>
<td>6911.81</td>
</tr>
<tr>
<td>18</td>
<td>59.522</td>
<td>51.839</td>
<td>6912.36</td>
</tr>
<tr>
<td>19</td>
<td>59.540</td>
<td>51.783</td>
<td>6909.62</td>
</tr>
<tr>
<td>15</td>
<td>59.540</td>
<td>51.800</td>
<td>6911.80</td>
</tr>
<tr>
<td>16</td>
<td>59.540</td>
<td>51.814</td>
<td>6912.41</td>
</tr>
<tr>
<td>17</td>
<td>59.540</td>
<td>51.839</td>
<td>6913.15</td>
</tr>
<tr>
<td>18</td>
<td>59.540</td>
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<td>6914.62</td>
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<tr>
<td>20</td>
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</tr>
<tr>
<td>16</td>
<td>59.557</td>
<td>51.814</td>
<td>6914.50</td>
</tr>
<tr>
<td>17</td>
<td>59.557</td>
<td>51.827</td>
<td>6914.81</td>
</tr>
<tr>
<td>18</td>
<td>59.557</td>
<td>51.839</td>
<td>6914.81</td>
</tr>
<tr>
<td>21</td>
<td>59.572</td>
<td>51.783</td>
<td>6915.51</td>
</tr>
<tr>
<td>15</td>
<td>59.572</td>
<td>51.800</td>
<td>6914.09</td>
</tr>
<tr>
<td>16</td>
<td>59.572</td>
<td>51.814</td>
<td>6916.30</td>
</tr>
<tr>
<td>17</td>
<td>59.572</td>
<td>51.827</td>
<td>6917.47</td>
</tr>
<tr>
<td>18</td>
<td>59.572</td>
<td>51.839</td>
<td>6918.51</td>
</tr>
<tr>
<td>22</td>
<td>59.585</td>
<td>51.783</td>
<td>6915.20</td>
</tr>
<tr>
<td>15</td>
<td>59.585</td>
<td>51.800</td>
<td>6916.68</td>
</tr>
<tr>
<td>16</td>
<td>59.585</td>
<td>51.814</td>
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</tr>
<tr>
<td>17</td>
<td>59.585</td>
<td>51.827</td>
<td>6919.16</td>
</tr>
<tr>
<td>18</td>
<td>59.585</td>
<td>51.839</td>
<td>6919.21</td>
</tr>
</tbody>
</table>
4.5.2 Discussion and Managerial Insights

The proposed model and its numerical character help the decision maker to make different managerial decisions. Here, the demands of the items are affected by more than one influence (advertisement and selling price) and the affections are find-out by method of estimation. From Table 4.5, it reveals that with the frequency of advertisement the market demand varies proportionally, which gives more profit for more advertisement. In numerical point of view, the following decisions can also be made, which are also reflected from the figures.

(i) Figure 4.4 shows that higher selling price implies lower production rate.

(ii) Figure 4.5, 4.6 and 4.7 indicate that inventory is a convex function of selling price, demand rate and production rate respectively.

Significant management implications for the practical application of the proposed approach are as follows. The obtained optimal order quantity is a crisp and precise solution, although the inventory problem is stochastic. This is important to decision-makers since one of their major concerns is how to find the precise target values for order quantities when some data are uncertain in terms of stochastic. Moreover, the proposed research work lights to the newly established companies with considerable input parameters.

4.5.3 Comparison between the Demands by ANOVA Test

Comparison test of the demands of the 1st item present in Table 4.2 and Table 4.3 with constant demand $D_0 = 49$ by ANOVA test. For convenience, let $X_1(t) = D_0$, $X_2(t) = D_0(t)$.
in Table 4.2) and $X_{4}(i) = V_{4}(i)$ (presented in Table 4.3), for $i = 1, 2, \ldots, 7$.
Then, the corresponding means are, $\bar{X}_1 = 49$, $\bar{X}_2 = 53.07$, and $\bar{X}_3 = 59.5$.
The total sum of squares be $SS_t = 412.73$, with degree of freedom $df_t = 20$.
Between groups sum of squares be $SS_b = 329.36$, with degree of freedom $df_b = 2$.
And within groups sum squares be $SS_w = 20.37$, with degree of freedom $df_w = 18$.
Therefore, $F = \frac{SS_b}{SS_w}$ with $(k-1, N-k)$ df, where $s_b^2 = \frac{SS_b}{k-1}$ and $s_w^2 = \frac{SS_w}{N-k}$ is given in the following Table 4.7.

Table 4.7: Table for different demand pattern ANOVA test

<table>
<thead>
<tr>
<th>Sources of variation</th>
<th>Sums of squares</th>
<th>df</th>
<th>Variances</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>329.36</td>
<td>2</td>
<td>196.18</td>
<td>173.31</td>
</tr>
<tr>
<td>Within groups</td>
<td>20.37</td>
<td>18</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>349.73</td>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The critical $F$ values $(df=2, 18)$ are quoted below:
$F_{0.05(2, 18)} = 3.55$, $F_{0.01(2, 18)} = 5.61$
As the computed $F$ is found to be higher than the critical $F$ for 0.01 level also, the computed $F$ is significant beyond the 0.01 level ($F < 0.01$). Hence, there is a significant added treatment component between the groups.

4.5.4 Practical Implications
A manufacturing system may be illustrated as follows: Let an automobile industrial company produces two different types of vehicles. The company has a showroom of fixed space to store the vehicles (say, 500 acre land). Each of the items has its different demand rate and other parameters. The decision managers of the company decide that how much of each quantity is produced? and what will be the length or frequency of the production cycle? In such a real life problem, the present model can be implemented.

4.6 Conclusion
For the first time, this model presents a production inventory model for multi-item with constant rate of reworking the defective items in finite time horizon employing the net present value in the objective function. Moreover, the time horizon is randomly distributed. The demands of the items have a promotional effort due to the selling price of the items and frequency of the advertisement. The proposed model provides an optimal cycle length in the random time period, production quantity on the basis of total expected profit. The described model is optimized through the population varying genetic algorithm. Finally, numerical examples are considered through a market survey.