Chapter 9

GA approach for controlling GHG emission from industrial waste in two plant production and reproduction inventory model with interval valued fuzzy pollution parameters

9.1 Introduction

In an imperfect production process, 100% screening is required to identify perfect and defective units. Defective units may be reworked in the same cycle along with the normal production after some time from the initial commencement of production by engaging some additional labour forces and machinery. But all of the defective units cannot be considered for reworking. Some of the defective units should be avoided for rework as they may be of very poor quality and will be expensive to repair. Therefore, a certain percentage of the defective units may be considered for reworking and reworked items are assumed as good as new products. Non-reworkable defective units are treated as rejected items. So many researchers considered reworks in imperfect production sector and some of them are Hayek and Salameh [97], Chiu [35], Chiu et al. [39], Liko et al. [136], Woe et al. [219], Taleizadeh et al. [204], Wang [217] and others.

Generally, consumers dispose of the products after its use or at the end of the life-cycle of
CHAPTER 9. GA APPROACH FOR CONTROLLING GHG EMISSION FROM INDUSTRIAL WASTE IN TWO PLANT PRODUCTION AND REPRODUCTION INVENTORY MODEL WITH INTERVAL VALUED FUZZY POLLUTION PARAMETERS

the product but recovery of the used products may be economically more attractive than disposal. In classical logistic system, material and related information flow (forward flow) is observed until the final products are delivered to the customer. But in reverse logistic system, used and reusable parts are returned from the customers to the producer (backward flow). Environmental consciousness forces companies to initiate such product recovery systems with their disposal (metal, glass, paper). In this way, natural resources can be saved for the future generations, so the firms can contribute to the sustainable development efforts. The importance of reverse logistics has increased significantly in the last two decades for a variety of legislative, environmental, and economic reasons (Fleischmann et al. [72]). Today’s customers are more educated and demanding, and tend to be less inclined to purchase products that are not environmentally friendly. The growth of secondary markets emphasized the importance of reverse logistics (Tibben-Lemke [207]). In the last twenty four years, a lot of research has been investigated on reverse logistics. Authors such as Carter and Elkan [14], Jayaraman et al. [106], Rogers and Tibben-Lemke [176, 177], Dowlatshahi [60], Guice [81], Stock and Mulki [198], and Guice and Var, Wasschenove [82] have described a broad range of reverse logistics systems and structures and analyzed a variety of attendant reverse logistics problems. Implementation of reverse logistics especially in product returns would allow not only for savings in inventory carrying cost, transportation cost, and waste disposal cost due to returned products, but also for the improvement of customer loyalty and future sales. In a broader sense, reverse logistics refers to the distribution activities involved in product returns, source reduction, conservation, recycling, substitution, reuse, disposal, refurbishment, repair and re-manufacturing. Recent reviews on reverse logistics are provided by Ferguson and Toktay [71], Mitra [151, 152], Ahmed and Jaber [4], Schulz [191], Xu [229], Hasanov et al. [91], Kim et al. [120, 121], Naeem et al. [159], Chen and Abrishami [34], Akasaki and Kurut [3], Schulz [191], Rogers et al. [179], Ghosh and Dey [74] and others.

Rise in the temperature of earth, deterioration in Ozone layer, melting of glaciers, numerous natural calamities, phase-shift in environmental clock are the various rays converging at the point of alarm to nature or save environment. This alertness about saving the nature and its resources and realization about environment has given considerable attention to produce green products in all fields worldwide. Along with producing green, reuse and recycle of used products also has been promoted to save the environment as much as possible.

Ever since the fuzzy set was proposed by Zadeh [239] fuzzy numbers have been widely studied, developed and applied to various fields. In fuzzy set, the degree of membership functions of the element in the universe is having a single value: either zero or one. Many times specialists are uncertain about the values of the membership of an element in a set. Hence, it is better to represent the values of the membership of an element in a set by intervals of possible real numbers instead of real numbers. An interval-valued fuzzy set on a universe X is a mapping from X to fall closed sub-intervals of the real interval [0, 1]. This type of fuzzy sets has been intensively investigated, not only its theoretical aspects, but also
its numerous applications. As the parameters relating to the environmental pollution due to industrial waste are not fixed in nature so, in this model, we take them as interval valued fuzzy number.

This chapter deals with the combined effect of manufacturing and re-manufacturing for two types of quality items (item-I and item-II) produced in two different plants (plant-I and plant-II) in the same premises under single management system over a known-finite time horizon with consideration of environment pollution control through industrial waste management. Three types of inventories are involved in this network. The manufactured and re-manufactured items are stored in the first and second inventories. The used items returned from the market and rejected defective units are together collected in the third inventory for raw materials required for re-manufacturing process. The objective of this research is to propose a manufacturing/re-manufacturing policy that would minimize the uses of natural resource as raw materials and minimizes the environmental pollution from the used and non-reworkable defective units by industrial waste management considering pollution parameters as interval-valued fuzzy numbers.

9.2 Notations and Assumptions

The following notations and assumptions have been considered to develop the model:

9.2.1 Notations

The following notations are used throughout the chapter.

- $q_{1i}(t)$: Inventory level of better quality item of $i$th cycle at time $t$ in plant-I.
- $q_{2i}(t)$: Inventory level of less better quality item of $i$th cycle at time $t$ in plant-II.
- $q_{3i}(t)$: Inventory level of the returned and non- reworkable item of $i$th cycle at time $t$.
- $q_{4i}(t)$: Inventory level of disposal item of $i$th cycle at time $t$.
- $\beta_1$: Fraction of the better quality item in plant-I.
- $\beta_2$: Fraction of the less better quality items in plant-II.
- $\delta_1$: Fraction of the re-workable item in plant-I.
- $\delta_2$: Fraction of the re-workable item in plant-II.
- $P_1$: Production rate per unit time of 1st cycle in plant-I.
- $D_1$: Demand rate in each cycle of plant-I.
- $t_1$: Production time in each cycle which is considered as a decision variable.
- $T$: Total time length of each cycle.
- $n$: Total number of cycle.
- $k$: Number of consecutive cycles in which the returned items from a cycle is considered.
- $c_p$: Production cost per unit item per unit time in plant-I.
- $c_p^r$: Production cost per unit item per unit time in plant-II.
- $c_{sr}$: Screening cost per unit item per unit time in plant-I.
CHAPTER 9. GA APPROACH FOR CONTROLLING GHG EMISSION FROM
INDUSTRIAL WASTE IN TWO PLANT PRODUCTION AND REPRODUCTION
INVENTORY MODEL WITH INTERVAL VALUED FUZZY POLLUTION
PARAMETERS

c_{s_t} : Screening cost per unit item per unit time in plant-II.
r_{s} : Reworked cost per unit item per unit time in plant-I.
r_{s^*} : Reworked cost per unit item per unit time in plant-II.
A_{s} : Set-up cost per cycle in plant-I.
A_{s^*} : Set-up cost per cycle in plant-II.
h_{s} : Holding cost per unit item per unit time in plant-I.
h_{s^*} : Holding cost per unit item per unit time in plant-II.
h''_{s} : Holding cost per unit item per unit time in the row material processing unit.
s : Selling price per unit item per unit time in plant-I.
s^* : Selling price per unit item per unit time in plant-II.
c_w : Water pollution cost per unit.
c_t : Cost to control GHG emission per unit.
c_d : Transportation cost per unit to transport the disposal unit for landfills.

E_R_{q} : Emission rate of GHG in landfills.
Q_{d} : Total disposal items.

P_{2i} : Production rate of ith cycle in plant-II, where \( P_{2i} = \lambda P_{2(i-1)} = \lambda^{i-1} P_2 \) with \( P_{21} = P_2 \).

D_{2i} : Demand rate of ith cycle in plant-II, where \( D_{2i} = \mu D_{2(i-1)} = \mu^{i-1} D_2 \) with \( D_{21} = D_2 \).

\( \alpha_{ij} D_{1} \) : Rate of return of item-I in plant-I collected from the market for \( j \)-th cycle when total time horizon consists of \( i \)-th cycles \( (j = 1, 2, \ldots, i) \) and where we take \( \alpha_{1j} = \alpha_{1} \) and \( \alpha_{ij} = \alpha_{ij} \) such that \( \sum_{j=1}^{i} \alpha_{ij} = \alpha_{j}^{h}\frac{h}{h-1} \), \( h > 1 \).

\( \alpha_{ij}^* D_{1} \) : Rate of return of item-I in plant-I collected from the market from the time period \( [(i - 1)T, (i - 1)T + t_{1}] \) during \( [(i - 1)T, iT, iT + t_1] \), where we take \( \alpha_{ij}^* = \alpha_{ij}^*, i = 1, 2, \ldots, n. \)

\( \beta_{ij} D_{2i} \) : Rate of return of item-II in plant-II collected from the market for \( j \)-th cycle when total time horizon consists of \( i \)-th cycles \( (j = 1, 2, \ldots, i) \) and where we take \( \beta_{1j} = \beta_{1} \) and \( \beta_{ij} = \beta_{ij} \) such that \( \sum_{j=1}^{i} \beta_{ij} = \beta_{j}^{h}\frac{h}{h-1} \), \( h > 1 \).

\( \beta_{ij}^* D_{2i} \) : Rate of return of item-II in plant-II collected from the market from the time period \( [(i - 2)T + t_{1}, (i - 1)T + t_{1}] \) during \( [(i - 1)T, (i - 1)T + t_{1}] \), where we take \( \beta_{ij}^* = \beta_{ij}^*, i = 2, 3, \ldots, n. \).

\( c_{p1} \) : Percentage of water pollution per cycle per unit production per unit time produce from plant-I when production rate is \( P_{1} \), where we take \( c_{p1} = c_{p1}^{h} P_{1}^{\eta_{11} - 1}, \), \( c_{p1} > 0, \eta_{11} > 1. \)

\( c_{p2} \) : Percentage of GHG emission per cycle per unit production per unit time produce from plant-I when production rate is \( P_{1} \), where we take \( c_{p2} = c_{p2}^{h} P_{1}^{\eta_{12} - 1}, \), \( c_{p2} > 0, \eta_{12} > 1. \)

\( c_{g1} \) : Percentage of water pollution per cycle per unit production per unit time produce from plant-II when production rate is \( P_{2} \), where we take \( c_{g1} = c_{g1} P_{2}^{\eta_{21} - 1}, \), \( c_{g1} > 0, \eta_{21} > 1. \)

\( c_{g2} \) : Percentage of GHG emission per cycle per unit production per unit time produce from plant-II when production rate is \( P_{2} \), where we take \( c_{g2} = c_{g2} P_{2}^{\eta_{22} - 1}, \), \( c_{g2} > 0, \eta_{22} > 1. \)
9.3. MATHEMATICAL FORMULATION OF THE PROPOSED MODEL

9.2.2 Assumptions

The following assumptions have been made to develop the model.

(i) It is an imperfect production and reproduction inventory model in finite time horizon for two types of items (Item-I and Item-II).

(ii) Item-I is produced with raw materials from natural sources in plant-I and Item-II is produced in plant-II (re-manufacturing process) where the used and non-reworkable defective items are the raw materials. Also we assume that item-II is of less quality than the item-I.

(iii) The manufacturer produced mixture of perfect and defective (imperfect) quality items. Hence, the manufacturer decides to sell the perfect quality item after sorting the items in the inventory. So in this chapter, it is assumed that during the production period, the screening process has been occurred simultaneously which greater than demand rate.

(iv) Water pollution and GHG emission to the environment during production in plant-I and plant-II have been considered. Also we consider GHG emission from the disposal units (Industrial Solid waste) produced from both the plants and used items.

(v) \( n, k, \lambda \) and \( t_1 \) are decision variable.

(vi) The production rate and demand rate of 1st cycle of plant-I and plant-II are constant.

9.3 Mathematical Formulation of the Proposed Model

![Diagram](image)

Figure 9.1: Schematic representation of the proposed model
We have considered an imperfect manufacturing system that produces perfect quality as well as imperfect quality items in plant-I and re-manufacturing of non-reworkable and returned items in plant-II with screening of defective units in both plants. Some imperfect quality products are reworked immediately at a cost to restore it to the original quality. For the development of this model, we assume that there are \( n \) cycles during the finite time horizon \( T \). Here, non-reworkable items and returned items which are used items collected from the market are remanufactured. Before re-manufacturing, some collected non-reworkable items and returned items not to be in a position for re-manufacturing are disposed off at the rate of \((1-\gamma)^{nT} \). This configuration is presented in Figure 9.1.

9.3.1 Formulation for Plant-I

For \( i^{th} \) cycle: \((i = 1, 2, \ldots, n)\)

In this case, the initial stock of the each cycle is zero and starts production with rate \( P_1 \). As production and reworking continues, inventory begins to pile up continuously after meeting demand with rate \( D_1 \). Production of \( i^{th} \) cycle stops at time \((i-1)T + t_1\) and restarts at time \(iT\) for next cycle. Each cycle ends with zero inventory. It then repeats itself. Our problem may be precisely defined as follows. The differential equation of the item-I in the \( i^{th} \) cycle during

![Diagram showing the inventory situation of the integrated model](image)

Figure 9.2: Pictorial representation of inventory situation of the integrated model
9.3. MATHEMATICAL FORMULATION OF THE PROPOSED MODEL

\[ (i-1)T, iT \ (i = 1, 2, \ldots, n) \] is given by

\[
\frac{dq_i}{dt} = \begin{cases} 
\beta_i P_i + \delta_i (1 - \beta_i) P_i - D_i, & (i-1)T \leq t \leq (i-1)T + t_1 \\
-\lambda_i, & (i-1)T + t_1 \leq t \leq iT 
\end{cases}
\]  \hspace{1cm} (9.1)

with \( q_{i,i}(i-1)T = 0 \) and \( q_{i,i}(iT) = 0 \).

The solution of the above differential equation is

\[
q_{i,i}(t) = \begin{cases} 
\beta_i P_i + \delta_i (1 - \beta_i) P_i - D_i \left[ t - (i-1)T \right], & (i-1)T \leq t \leq (i-1)T + t_1 \\
D_i \left( it - t_1 \right), & (i-1)T + t_1 \leq t \leq iT 
\end{cases}
\]

From continuity condition at \( t = (i-1)T + t_1 \), we have

\[
\beta_i + \delta_i (1 - \beta_i) P_i t_1 = D_i T 
\]  \hspace{1cm} (9.2)

The total holding cost of plant-I is given by

\[
THC_1 = \sum_{i=1}^{n} \left[ \int_{(i-1)T+t_1}^{iT} q_i(t) dt + \int_{(i-1)T+t_1}^{iT} q_i(t) dt \right] - \frac{\mu R}{2} \left[ \beta_i + \delta_i (1 - \beta_i) \right] P_1^2 T_1 + D_i (T^2 - 2iT_1)
\]

Amount of water pollution in each production cycle in plant-I = \( c_{p,w} P_1(t_1) \).

Cost incurred for water pollution in plant-I = \( c_{p,w} P_1(t_1) = nc_{p,w} P_1^2 T_1 t_1 \).

Amount of GHG emission in each production cycle in plant-I = \( c_{p,g} P_1(t_1) \).

Cost of GHG emission control in plant-I = \( c_{p,g} P_1^2 T_1 t_1 = nc_{p,g} P_1^2 T_1 t_1 \).

The total cost of the plant-I over the time horizon is

\[
TC_{p,1} = n \left[ c_p + c_{p,g} (1 - \beta_i) + c_{p,w} \right] P_1 t_1 + THC_1 + n A_s + nc_{p,g} P_1^2 T_1 t_1 + nc_{p,g} P_1^2 T_1 t_1
\]

The total sales revenue from plant-I during \((0, nT)\) is given by

\[
TR_{p,1} = \sum_{i=1}^{n} \int_{(i-1)T}^{iT} D_i dl = n s D_i T
\]

9.3.2 Formulation for plant-II

For \( ith \) cycle:\( (i = 1, 2, \ldots, n) \)

In this case, the initial stock of the \( i \)th cycle is zero and starts production with rate \( P_2 \) at time \( (i-1)T + t_1 \). As production with screening and reworking continues, inventory begins to pile up continuously after meeting demand with rate \( P_2 \) and deterioration. Production of \( i \)th cycle stops at time \( iT \). The accumulated inventory of \( i \)th cycle is just sufficient enough to account for demand and deterioration over the interval \([iT, iT + t_1]\). Production restarts at time \( iT + t_1 \) for next cycle. The cycle ends with zero inventory. It then repeats itself. Our problem may be
precisely defined as follows.
The differential equation of the item-II for \(i^{th}\) cycle during \([(i-1)T + t_1, iT + t_1]\) is given by

\[
\frac{dq_2}{dt} = \begin{cases} 
\beta_2 P_{2i} + \delta_2 (1 - \beta_2) P_{2i} - D_{2i}, & (i-1)T + t_1 \leq t \leq iT \\
-D_{2i}, & iT \leq t \leq iT + t_1
\end{cases}
\]

(9.3)

with \(q_{2i}[(i-1)T + t_1] = 0\) and \(q_{2i}[iT + t_1] = 0\).

The solution of the above differential equation is

\[
q_{2i}(t) = \begin{cases} 
\beta_2 P_{2i} + \delta_2 (1 - \beta_2) P_{2i} - D_{2i} |t - [(i-1)T + t_1]|, & (i-1)T + t_1 \leq t \leq iT \\
D_{2i} (iT + t_1 - t), & iT \leq t \leq iT + t_1
\end{cases}
\]

From continuity condition at \(t = iT\), we have

\[
|\beta_2 + \delta_2 (1 - \beta_2)| (T - t_1) P_{2i} = D_{2i} T
\]

(9.4)

The total production cost of plant-II during \((t_1, iT + t_1)\) is given by

\[
TPC_2 = \sum_{i=1}^{n} c'_p \int_{(i-1)T+t_1}^{iT} P_{2i} \, dt = c'_p(T - t_1) \sum_{i=1}^{n} P_{2i} - c'_p(T - t_1) P_2 \frac{\lambda^n - 1}{\lambda - 1}
\]

The total screening cost of plant-II during \((t_1, iT + t_1)\) is given by

\[
SC_2 = \sum_{i=1}^{n} c'_s \int_{(i-1)T+t_1}^{iT} P_{2i} \, dt = c'_s(T - t_1) \sum_{i=1}^{n} P_{2i} - c'_s(T - t_1) P_2 \frac{\lambda^n - 1}{\lambda - 1}
\]

The total reworked cost of plant-II during \((t_1, iT + t_1)\) is given by

\[
RWC_2 = \sum_{i=1}^{n} r'_c \int_{(i-1)T+t_1}^{iT} \delta_2 (1 - \beta_2) P_{2i} \, dt = r'_c \delta_2 (1 - \beta_2) (T - t_1) P_2 \frac{\lambda^n - 1}{\lambda - 1}
\]

The total holding cost of plant-II during \((t_1, iT + t_1)\) is given by

\[
THC_2 = \sum_{i=1}^{n} [h'_t] \int_{(i-1)T+t_1}^{iT} q_{2i}(t) \, dt + \int_{iT}^{iT+t_1} q_{2i}(t) \, dt
\]

\[
\frac{h'_t}{2} \left[ (\beta_2 + \delta_2 (1 - \beta_2)) (T - t_1)^2 \sum_{i=1}^{n} P_{2i} + (2t_1 T - T^2) \sum_{i=1}^{n} D_{2i} \right]
\]

Amount of water pollution in \(i^{th}\) production cycle in plant-II \(= c'_{w} P_{2i}(T - t_1)\).

Cost incurred for water pollution in plant-II = \(c_{w} \cdot c_{w}(T - t_1) \sum_{i=1}^{n} P_{2i}^{w_{2i}}\).

Amount of GHG emission in \(i^{th}\) production cycle in from plant-II = \(c'_{GHG} P_{2i}(T - t_1)\).
9.3. MATHEMATICAL FORMULATION OF THE PROPOSED MODEL

Cost incurred for GHG emission control in plant-II = $c_{f_{2}}^{C}(T - t_{1}) \sum_{i=1}^{n} I_{2i}^{P_{2}}$.

The total cost of the plant-II during $(t_{1}, nT + t_{1})$ is

$$\text{TC}_{P_{2}} = TPC_{2} + RW C_{2} + TSC_{2} + TH C_{2} + nA_{2} + \left( c_{f_{2}}^{C} \sum_{i=1}^{n} I_{2i}^{P_{2}} + c_{f_{2}}^{C} \sum_{i=1}^{n} I_{2i}^{P_{2}} \right)$$

The total sales revenue of plant-II during $(t_{1}, nT + t_{1})$ is given by

$$\text{TSR}_{P_{2}} = s \int_{t_{1}}^{nT+t_{1}} D_{2i} dt = s^{T} \sum_{i=1}^{n} D_{2i} = s^{T} D_{2} \frac{f_{0}^{T} - 1}{\mu - 1}$$

9.3.3 Formulation for Raw Material Processing Unit of Plant-II

For 1st cycle:

The differential equation in the raw material processing unit during $[0, T]$ is given by

$$\frac{dq_{1}}{dt} = \left\{ \begin{array}{l}
\gamma_{1}[\alpha_{11} D_{1} + (1 - \delta_{1})(1 - \beta_{1})P_{1}], & 0 \leq t \leq t_{1} \\
\gamma_{1}[\alpha_{12} D_{1} + \alpha_{11} D_{2} + \beta_{11} D_{21} + (1 - \delta_{2})(1 - \beta_{2})P_{2}], & t_{1} \leq t \leq T 
\end{array} \right.$$  \hspace{1cm} \text{(9.5)}

with $q_{1}(0) = 0$ and $q_{1}(T) = 0$.

The solution of the above differential equation is

$$q_{1}(t) = \left\{ \begin{array}{l}
\gamma_{1}[\alpha_{12} D_{1} + (1 - \delta_{1})(1 - \beta_{1})P_{1}], & 0 \leq t \leq t_{1} \\
\gamma_{1}[\alpha_{12} D_{1} + \alpha_{11} D_{2} + \beta_{11} D_{21} + (1 - \delta_{2})(1 - \beta_{2})P_{2}] - \frac{1}{\gamma_{1}} q_{1}(t - T), & t_{1} \leq t \leq T 
\end{array} \right.$$ \hspace{1cm} \text{for } 0 \leq t \leq T

From continuity condition at $t = t_{1}$, we have

$$\gamma_{1}[\alpha_{12} D_{1} + (1 - \delta_{1})(1 - \beta_{1})P_{1}] = \left\{ \gamma_{1}[\alpha_{12} D_{1} + \alpha_{11} D_{2} + \beta_{11} D_{21} + (1 - \delta_{2})(1 - \beta_{2})P_{2}] - \frac{1}{\gamma_{1}} q_{1}(t - T) \right\}(t_{1} - T)$$

The holding cost in raw material processing unit for first cycle is given by

$$\text{HC}_{31} = \frac{k_{0}^{e}}{2} \int_{0}^{t_{1}} q_{1}(t) dt + \frac{k_{0}^{e}}{2} \int_{t_{1}}^{T} q_{1}(t) dt$$

$$= \frac{k_{0}^{e}}{2} \left\{ \gamma_{1}[\alpha_{12} D_{1}(2Tt_{1} - T^{2}) + (1 - \delta_{1})(1 - \beta_{1})P_{1}t_{1}^{2} - (\alpha_{12} D_{1} + \alpha_{11} D_{2} + \beta_{11} D_{21} + (1 - \delta_{2})(1 - \beta_{2})P_{2})(t_{1} - T)^{2}] \\
+ \left\{ (1 - \gamma_{1}[\alpha_{12} D_{1} + (1 - \delta_{1})(1 - \beta_{1})P_{1}]) P_{2}(t_{1} - T) \right\} \right\}$$

For 2nd cycle:

The differential equation in the raw material processing unit during $[T, 2T]$ is given by

$$\frac{dq_{2}}{dt} = \left\{ \begin{array}{l}
\gamma_{1}[\alpha_{21} D_{1} + (1 - \delta_{1})(1 - \beta_{1})P_{1} + (\beta_{21} + \beta_{12})D_{21}], & T \leq t \leq T + t_{1} \\
\gamma_{1}[\alpha_{21} D_{1} + \alpha_{22} D_{2} + \alpha_{12} D_{21} + \beta_{21} D_{21} + \beta_{22} D_{21} + (1 - \delta_{2})(1 - \beta_{2})P_{2}], & T + t_{1} \leq t \leq 2T 
\end{array} \right.$$  \hspace{1cm} \text{(9.6)}

with boundary conditions $q_{2}[T] = 0$ and $q_{2}[2T] = 0$.

The solution of the above differential equation is

$$q_{2}(t) = \left\{ \begin{array}{l}
\gamma_{1}[\alpha_{21} D_{1} + (1 - \delta_{1})(1 - \beta_{1})P_{1} + (\beta_{21} + \beta_{12})D_{21}], & T \leq t \leq T + t_{1} \\
\gamma_{1}[\alpha_{21} D_{1} + \alpha_{22} D_{2} + \alpha_{12} D_{21} + \beta_{21} D_{21} + \beta_{22} D_{21} + (1 - \delta_{2})(1 - \beta_{2})P_{2}], & T + t_{1} \leq t \leq 2T 
\end{array} \right.$$ \hspace{1cm} \text{for } T \leq t \leq 2T
From continuity condition at \( t = T + t_1 \), we have

\[
\gamma_1[\alpha_1 D_1 + (1 - \delta_1)(1 - \beta_1)P_1 + \alpha_2 D_1 + \alpha_2 D_2 + \alpha_2 D_2]t_1 = \left[ \gamma_1 \left\{ \alpha_1 D_1 + \alpha_2 D_2 + (1 - \delta_2)(1 - \beta_2)P_2 + \alpha_2 D_2 \right\} \right](t_1 - T)
\]

The holding cost in raw material processing unit is given by

\[
HC_{32} = \int_{T+t_1}^{T+2T} q_{32}(t) \, dt
\]

\[
= \frac{h''}{2} \left[ \gamma_1 \left\{ \alpha_1 D_1 + \alpha_2 D_1 + \alpha_2 D_2 \right\} (2T t_1 - T^2) + \left( \alpha_2 D_2 + (1 - \delta_1)(1 - \beta_1)P_1 \right) t_1 \right] + \frac{h''}{2} \left[ \gamma_1 \left\{ -\gamma_1(1 - \delta_2)(1 - \beta_2) \right\} P_2 - \gamma_1(\alpha_1 D_1 + \beta_2 D_2) \right](T - T^2)
\]

For \( i \text{th} \) cycle \((i = 3,4,\ldots,k)\):

The differential equation in the raw material processing unit during \([ (i - 1)T, iT ] \) is given by

\[
\frac{dq_{3i}}{dt} = \begin{cases} 
\gamma_1 \left[ \sum_{j=1}^{i-1} \alpha_{ij} D_1 + (1 - \delta_j)(1 - \beta_j)P_1 \right] + \sum_{j=1}^{i-1} \beta_{ij} D_{2j}, & (i - 1)T \leq t \leq (i - 1)T + t_1 \\
\gamma_1 \left[ \sum_{j=1}^{i-1} \alpha_{ij} D_1 + \alpha_{ij} D_1 + \sum_{j=1}^{i-2} \beta_{ij} D_{2j} + \beta_{ij} D_{2j} \right] + (1 - \delta_i)(1 - \beta_i)P_{2i}, & (i - 1)T + t_1 \leq t \leq iT 
\end{cases}
\]

with \( q_{3i}(i - 1)T = 0 \) and \( q_{3i}(iT) = 0 \)

The solution of the above differential equation is

\[
q_{3i}(t) = \begin{cases} 
\gamma_1 \left[ \sum_{j=1}^{i-1} \alpha_{ij} D_1 + (1 - \delta_j)(1 - \beta_j)P_1 + \sum_{j=1}^{i-2} \beta_{ij} D_{2j} \right] + \beta_{i-1} D_{2(i-1)} + \beta_{i-1} D_{2(i-1)} \right](t - (i - 1)T), & (i - 1)T \leq t \leq (i - 1)T + t_1 \\
\gamma_1 \left[ \sum_{j=1}^{i-1} \alpha_{ij} D_1 + \alpha_{ij} D_1 + \sum_{j=1}^{i-2} \beta_{ij} D_{2j} + \beta_{ij} D_{2j} \right] + (1 - \delta_i)(1 - \beta_i)P_{2i} - P_{2i}, & (i - 1)T + t_1 \leq t \leq iT 
\end{cases}
\]

From continuity condition at \( t = (i - 1)T + t_1 \), we have

\[
\gamma_1 \left[ \alpha_1 D_1 + \alpha_2 D_{2i} + \alpha_2 D_{2(i-1)} + \sum_{j=1}^{i-1} \alpha_{ij} D_1 + \sum_{j=1}^{i-1} \beta_{ij} D_{2j} + (1 - \delta_i)(1 - \beta_i)P_i \right] t_1 = \left[ \gamma_1 \left\{ \alpha_1 D_1 + \alpha_2 D_{2(i-1)} + \sum_{j=1}^{i-1} \beta_{ij} D_{2j} + (1 - \delta_2)(1 - \beta_2)P_{2i} \right\} \right](t_1 - T)
\]

Holding cost of raw material processing unit for \( i \text{th} \) cycle \((i = 3,4,\ldots,k)\) is given by

\[
HC_{3i} = \int_{(i - 1)T + t_1}^{iT} q_{3i}(t) \, dt + h'' \int_{(i - 1)T + t_1}^{iT} q_{3i}(t) \, dt
\]

\[
= \frac{h''}{2} \gamma_1 \left\{ \alpha_1 D_1 + \alpha_2 D_{2i} + \sum_{j=1}^{i-1} \alpha_{ij} D_1 + \sum_{j=1}^{i-2} \beta_{ij} D_{2j} \right\} (2T t_1 - T^2) + \left( \alpha_2 D_{2(i-1)} \right) t_1 + \gamma_1 \left\{ \alpha_1 D_1 + \beta_{i-1} D_{2(i-1)} \right\} (t_1 - T)^2 + \frac{h''}{2} \left[ (1 - \delta_1)(1 - \beta_1)P_{1i} \right] (t_1 - T)^2
\]

224
9.3. MATHEMATICAL FORMULATION OF THE PROPOSED MODEL

Holding cost of raw material processing unit for \((k-2)\) cycles is given by

\[
HC_N = \sum_{i=3}^{k} HC_{Di}
\]

\[
= \frac{h''}{2} \gamma_1 \left\{ \left( \frac{(k-2)}{2} \alpha_1 D_1 + \alpha_2 D_2 \right)^{k-2} - 1 \right\} + \sum_{i=3}^{k-1} \sum_{j=3}^{k-1} \alpha_{ij} D_i + \sum_{i=3}^{k-2} \left( (k-2)(1-\delta_i)(1-\beta_i) T_1 \right) T_1
\]

\[
+ \sum_{i=3}^{k-1} \alpha_{2i} D_{2(i-1)} + (k-2)(1-\delta_i)(1-\beta_i) T_1 \right\} T_1 + \frac{h''}{2} \left[ -\gamma_1 \left\{ (k-2) \alpha_1 D_1 \right\} + \sum_{i=3}^{k-1} \beta_{i, i-1} D_{2(i-1)} \right]
\]

For \(i\)th cycle \((i = k + 1, k + 2, \ldots, n)\):

The differential equation in the raw material processing unit during \([ (i-1)T, iT \) is given by

\[
\frac{dq_3}{dt} = \begin{cases} 
\gamma_1 \left\{ \left( \sum_{j=i-1}^{i-k+1} \alpha_{ij} D_j + (1-\delta_i)(1-\beta_i) P_1 \right) \right\} + \sum_{j=i-1}^{i-k+1} \beta_{ij} D_{2j} + \sum_{j=i-1}^{i-k+1} \beta_{ij} \sum_{j=i-1}^{i-k+1} \beta_{ij} D_{2j} + \sum_{j=i-1}^{i-k+1} \beta_{ij} D_{2j} + \beta_{i1} D_{2i} + (1-\delta_i)(1-\beta_i) P_2 - P_2, & (i-1)T < t \leq (i-1)T + t_1 \\
\end{cases}
\]

with \(q_3[(i-1)T] = 0\) and \(q_3[T] = 0\)

The solution of the above differential equation is

\[
q_3(t) = \begin{cases} 
\sum_{j=i-1}^{i-k+1} \alpha_{ij} D_j + (1-\delta_i)(1-\beta_i) P_1 + \sum_{j=i-1}^{i-k+1} \beta_{ij} D_{2j} + \sum_{j=i-1}^{i-k+1} \beta_{ij} \sum_{j=i-1}^{i-k+1} \beta_{ij} D_{2j} + \beta_{i1} D_{2i} + (1-\delta_i)(1-\beta_i) P_2 - P_2, & (i-1)T < t \leq (i-1)T + t_1 \\
\end{cases}
\]

From continuity condition at \( t = (i-1)T + t_1 \),

\[
\gamma_1 \left\{ \left( \sum_{j=i-1}^{i-k+1} \alpha_{ij} D_j + (1-\delta_i)(1-\beta_i) P_1 \right) \right\} + \sum_{j=i-1}^{i-k+1} \beta_{ij} D_{2j} + \sum_{j=i-1}^{i-k+1} \beta_{ij} \sum_{j=i-1}^{i-k+1} \beta_{ij} D_{2j} + \beta_{i1} D_{2i} + (1-\delta_i)(1-\beta_i) P_2 - P_2, & (i-1)T < t \leq (i-1)T + t_1
\]

Holding cost of raw material processing unit for \(i\)th cycle \((i = k + 1, k + 2, \ldots, n)\) is given by

\[
HC_{Di} = \frac{h''}{2} \int_{(i-1)T}^{iT} q_3(t) dt + H' \int_{(i-1)T}^{iT} q_3(t) dt
\]

\[
= \frac{h''}{2} \gamma_1 \left\{ \left( \sum_{j=i-1}^{i-k+1} \alpha_{ij} D_j + (1-\delta_i)(1-\beta_i) P_1 \right) \right\} + \sum_{j=i-1}^{i-k+1} \beta_{ij} D_{2j} + \sum_{j=i-1}^{i-k+1} \beta_{ij} \sum_{j=i-1}^{i-k+1} \beta_{ij} D_{2j} + \beta_{i1} D_{2i} + (1-\delta_i)(1-\beta_i) P_2 - P_2, & (i-1)T < t \leq (i-1)T + t_1
\]

\[
+ \frac{H'}{2} \left\{ 1-\gamma_1 \left( 1-\delta_2 \right) \right\} \left( P_2(t_1 - T)^2
\]

225
CHAPTER 9. GA APPROACH FOR CONTROLLING GHG EMISSION FROM INDUSTRIAL WASTE IN TWO PLANT PRODUCTION AND REPRODUCTION INVENTORY MODEL WITH INTERVAL VALUED FUZZY POLLUTION PARAMETERS

Holding cost in raw material processing unit for \((n-k)\) cycles is given by

\[
HCN' = \sum_{i=k+1}^{n} HC_{Si}
\]

\[
= \frac{h^p}{2} \gamma_1 \left[ \left( (n-k) \alpha_1 D_1 + \alpha_2 D_2 \frac{\beta_1}{\mu} - 1 \right) + \sum_{j=k+1}^{n} \sum_{i=k+1}^{i-1} \beta_{ij} D_{ij} \right] (2T_1 - T^2) + \left( (n-k)(1-\delta)(1-\beta_1)P_1 \right) + \left( (n-k)(1-\delta)(1-\beta_2)P_2 \right) \frac{\lambda^{n-k} - 1}{\lambda - 1} (T_1 - T)^2
\]

Therefore, total holding cost in raw material processing unit is given by

\[
THC_3 = THC_3 + THC_3 + HCN + HCN'
\]  \hspace{1cm} (9.9)

9.3.4 Formulation of Disposal Units

For 1st cycle:
The differential equation in the disposal units during \([0, T]\) is given by

\[
\frac{dQ_{i1}}{dt} = \begin{cases} 
(1-\gamma_1) \left[ \alpha_1 D_1 + (1-\delta)(1-\beta_1)P_1 \right], & 0 \leq t \leq t_1 \\
(1-\gamma_1) \left[ \alpha_1' D_1 + \alpha_1 D_1 + \beta_1 D_{21} + (1-\delta)(1-\beta_2)P_{21} \right], & t_1 \leq t \leq T
\end{cases}
\]

\[
Q_{i1} = (1-\gamma_1) \left[ \alpha_1 D_1 + (1-\delta)(1-\beta_1)P_1 \right] t_1 + (1-\gamma_1) \left[ \alpha_1' D_1 + \alpha_1 D_1 + \alpha_2 D_2 + (1-\delta)(1-\beta_2)P_{21} \right] (T-t_1)
\]

For 2nd cycle:
The differential equation in the disposal units during \([T, 2T]\) is given by

\[
\frac{dQ_{i2}}{dt} = \begin{cases} 
(1-\gamma_1) \left[ \alpha_{21} D_1 + \alpha_{22} D_1 + (1-\delta)(1-\beta_i)P_i \right] + \beta_{21} D_{21}, & T \leq t \leq T + t_1 \\
(1-\gamma_1) \left[ \alpha_{21} D_1 + \alpha_{22} D_1 + \alpha_{22}' D_1 + \beta_{21} D_{21} \right] + \beta_{22} D_{22} + (1-\delta)(1-\beta_2)P_{22}, & T + t_1 \leq t \leq 2T
\end{cases}
\]
9.3. MATHEMATICAL FORMULATION OF THE PROPOSED MODEL

\[ Q_{42} = \left(1 - \gamma_1\right)\left(\alpha_1 D_1 + \alpha_2 D_2 + (1 - \delta_1)(1 - \beta_1)P_1 + \beta_1 D_21 + \beta_2 D_22\right)t_1 \]
\[ + \left(1 - \gamma_1\right)\left(\alpha_1 + \alpha_2 + \alpha_2' D_1 + \beta_1' D_21 + \beta_2' D_22 + (1 - \delta_2)(1 - \beta_2)P_22\right)(T - t_1) \]
\[ = \left(1 - \gamma_1\right)\left\{\alpha_1 D_1 + \alpha_2 D_2 + \alpha_2' D_1\right\}T + \left\{\alpha_2' D_2 + (1 - \delta_1)(1 - \beta_1)P_1\right\}t_1 \]
\[ + \left\{\alpha_2' D_2 + (1 - \delta_2)(1 - \beta_2)P_22\right\}(T - t_1) \]

For \(i\)th cycle \((i = 3, 4, \ldots, k)\):

The differential equation in the the disposal units during \([(i - 1)T, iT]\) is given by

\[ \frac{dQ_{43}}{dt} = \left\{ \begin{array}{ll}
(1 - \gamma_1)\left[\sum_{j=1}^{i-1} \alpha_j D_j + (1 - \delta_1)(1 - \beta_1)P_j\right] \\
+ \sum_{j=1}^{i-2} \beta_{ij} D_{2j} + \beta_{i-1}(1 - \beta_{i-1})D_{2(i-1)} + \beta_{i-1}' D_{2(i-1)} \end{array} \right\}, \ (i - 1)T \leq t \leq (i - 1)T + t_1 \\
(1 - \gamma_1)\left[\sum_{j=1}^{i-1} \alpha_j D_j + \alpha_2' D_1 + \sum_{j=1}^{i-1} \beta_{ij} D_{2j} \right] \\
+ \beta_{i}' D_{2i} + (1 - \delta_2)(1 - \beta_2)P_{2i}, \ (i - 1)T + t_1 \leq t \leq iT \\
\end{array} \right. 

\[ Q_{45} = \left(1 - \gamma_1\right)\left(\alpha_1 D_1 + \alpha_2 D_2 + \alpha_2' D_2(i-1) + \sum_{j=1}^{i-1} \alpha_j D_j \right) \]
\[ + \sum_{j=1}^{i-2} \beta_{ij} D_{2j} + (1 - \delta_1)(1 - \beta_1)P_1 \right\}t_1 + \left(1 - \gamma_1\right)\left[\alpha_1 D_1 + \alpha_2 D_2 + \alpha_2' D_1 \right] \]
\[ + \sum_{j=1}^{i-1} \alpha_j D_j + \sum_{j=1}^{i-1} \beta_{ij} D_{2j} + (1 - \delta_2)(1 - \beta_2)P_{2i} \right\}(T - t_1) \]
\[ = \left(1 - \gamma_1\right)\left\{\alpha_1 D_1 + \alpha_2 D_2 + \sum_{j=1}^{i-1} \alpha_j D_j + \sum_{j=1}^{i-2} \beta_{ij} D_{2j}\right\}T + \left\{\alpha_2' D_2 + (1 - \delta_1)(1 - \beta_1)P_1 \right\}t_1 \]
\[ + \left\{\alpha_2' D_2 + (1 - \delta_2)(1 - \beta_2)P_{2i}\right\}(T - t_1) \]

For \(i\)th cycle \((i = k + 1, k + 2, \ldots, n)\):

The differential equation in the the disposal units during \([(i - 1)T, iT]\) is given by

\[ \frac{dQ_{45}}{dt} = \left\{ \begin{array}{ll}
(1 - \gamma_1)\left[\sum_{j=1}^{i-k+1} \alpha_j D_j + (1 - \delta_1)(1 - \beta_1)P_j\right] \\
+ \sum_{j=1}^{i-k+1} \beta_{ij} D_{2j} + \beta_{i-k+1}(1 - \beta_{i-k+1})D_{2(i-k)} \end{array} \right\}, \ (i - 1)T \leq t \leq (i - 1)T + t_1 \\
(1 - \gamma_1)\left[\sum_{j=1}^{i-k+1} \alpha_j D_j + \alpha_2' D_1 \right] \\
+ \sum_{j=1}^{i-k+1} \beta_{ij} D_{2j} + \beta_{i-k+1} D_{2(i-k)} + (1 - \delta_2)(1 - \beta_2)P_{2i} \right\}, \ (i - 1)T + t_1 \leq t \leq iT \\
\end{array} \right. 

227
\[ Q_d = (1 - \gamma_1) \left[ \alpha_1 D_1 + \alpha_2 D_2 + \alpha_2' D_2(j-1) + \sum_{j=i-k+1}^{i-1} \alpha_{ij} D_1 \right] \\
+ \sum_{j=i-k+1}^{i-2} \beta_{ij} D_2j + (1 - \delta_1)(1 - \beta_1) P_1 t_1 + (1 - \gamma_1) \left[ \alpha_1 D_1 + \alpha_2 D_2 + \alpha_2' D_1 \right] \\
+ \sum_{j=i-k+1}^{i-1} \alpha_{ij} D_1 + \sum_{j=i-k+1}^{i-1} \beta_{ij} D_2j + (1 - \delta_2)(1 - \beta_2) P_2 (T - t_1) \]
\[ = (1 - \gamma_1) \left[ \left\{ \alpha_1 D_1 + \alpha_2 D_2 + \sum_{j=i-k+1}^{i-1} \alpha_{ij} D_1 + \sum_{j=i-k+1}^{i-2} \beta_{ij} D_2j \right\} T + \left\{ \alpha_2 D_2(j-1) \right\} \right] \\
+ (1 - \delta_1)(1 - \beta_1) P_1 t_1 + \left\{ \alpha_1' D_1 + \beta_{i-1} D_2(j-1) + (1 - \delta_2)(1 - \beta_2) P_2 \right\} (T - t_1) \]

\[ Q_4 = \text{Total disposal amount during } [0, nT] \]
\[ = Q_{41} + Q_{42} + \sum_{k=3}^{n} Q_{4k} + \sum_{i=k+1}^{n} Q_{4i} \]
\[ = (1 - \gamma_1) \left[ n \alpha_1 D_1 T + n(1 - \delta_1)(1 - \beta_1) P_1 t_1 + \left\{ \alpha_2 D_2 \left( \frac{b^n-1}{\mu-1} \right) \right\} \right] \]
\[ + \sum_{k=3}^{n} \sum_{j=1}^{i-1} \alpha_{ij} D_1 + \sum_{i=k+1}^{n} \sum_{j=1}^{i-1} \alpha_{ij} D_1 + \sum_{i=k+1}^{n} \sum_{j=1}^{i-1} \beta_{ij} D_2j + \sum_{i=k+1}^{n} \sum_{j=1}^{i-1} \beta_{ij} D_2j \right\} T \]
\[ + t_1 \sum_{i=2}^{n} \alpha_1' D_2(j-1) + \left\{ n \alpha_1' D_1 + \sum_{i=2}^{n} \alpha_1' D_2(j-1) + (1 - \delta_2)(1 - \beta_2) P_2 \left( \frac{X^n-1}{X-1} \right) \right\} (T - t_1) \]

Transportation cost to transport the disposal amount \( Q_4 \) for landfill = \( c_7 Q_4 \).
Total GHG emission from landfill = \( E R_7 Q_4 \).
Total cost to control GHG emission from landfill = \( c_{4} E R_7 Q_4 \).

**9.4 Objectives of the Proposed Model**

The total profit of the system is given by

\[ TP = TS R_{p1} + TS R_{p2} - TC_{p1} - TC_{p2} - THC_3 - c_7 E R_7 Q_4 - c_8 Q_4 \]

Total GHG emission from the system is given by

\[ Z_d = n c_9 D_1^{b^n} t_1 + c_9 (T - t_1) \sum_{i=1}^{n} \frac{D_1^{b^n}}{2^i} + E R_7 Q_4 \]

**Lemma 9.1.** For \( 0 < \alpha_1 < 1, b > 1 \) and positive integer \( k \), the following relation holds:

\[ \alpha_1(b^k - 1) \leq (b - 1), \quad i = 1, 2, \ldots, n - k + 1, \quad \text{and} \quad \alpha_1(b^{n-i+1} - 1) \leq (b - 1), \quad i = n - k + 2, \ldots, n - 1. \]
9.4. OBJECTIVES OF THE PROPOSED MODEL

Proof. \( \alpha_{ri} D = \) Return rate of items from rth cycle to the ith cycle.

Now
\[
\sum_{i=1}^{k+i-1} \alpha_{ri} = \alpha_1 \frac{h_i^k - 1}{h_i - 1}, \quad h_i > 1, \quad i = 1, 2, \ldots, n - k + 1 \text{ (see Appendix 1)}.
\]

and
\[
\sum_{i=1}^{n} \alpha_{ri} = \alpha_1 \frac{h_i^{n+i-1} - 1}{h_i - 1}, \quad h_i > 1, \quad i = n - k + 2, \ldots, n - 1 \text{ (see Appendix 1)}.
\]

Since we consider the return of the items from a cycle to at most \( k \) consecutive cycles, so we must have
\[
\sum_{i=1}^{k+i-1} \alpha_{ri} D < D, \quad i = 1, 2, \ldots, n - k + 1 \text{ and } \sum_{i=1}^{n} \alpha_{ri} D < D, \quad i = n - k + 2, \ldots, n - 1
\]

i.e.,
\[
\sum_{i=1}^{k+i-1} \alpha_{ri} < 1, \quad i = 1, 2, \ldots, n - k + 1 \text{ and } \sum_{i=1}^{n} \alpha_{ri} < 1, \quad i = n - k + 2, \ldots, n - 1
\]

i.e.,
\[
\alpha_1 \frac{h_i^k - 1}{h_i - 1} < 1, \quad i = 1, 2, \ldots, n - k + 1 \text{ and } \alpha_1 \frac{h_i^{n+i-1} - 1}{h_i - 1} < 1, \quad i = n - k + 2, \ldots, n - 1
\]

i.e.,
\[
\alpha_1 (h_i^k - 1) < (h_i - 1), \quad i = 1, 2, \ldots, n - k + 1 \text{ and } \alpha_1 (h_i^{n+i-1} - 1) < (h_i - 1), \quad i = n - k + 2, \ldots, n - 1
\]

Hence the proof. \( \square \)

Lemma 9.2. For \( 0 < \alpha_2 < 1, \ h_i' > 1 \) and positive integer \( k \), the following relation holds:
\[
\alpha_2 (h_i^k - 1) \leq (h_i' - 1), \quad i = 1, 2, \ldots, n - k + 1 \text{ and } \alpha_2 (h_i^{n+i-1} - 1) \leq (h_i - 1), \quad i = n - k + 2, \ldots, n - 1.
\]

Proof. Similar to Lemma 9.1. \( \square \)

Lemma 9.3. For positive integer \( k \), \( 0 < \alpha_1, \alpha_2 < 1 \), and \( h, \ h_i' > 1 \) then optimal value of \( k \)

is given by \( k = \min\{[X_1], [X_2]\} \), where \( X_1 = \frac{\log(h + 1)}{\log(h)} \), \( X_2 = \frac{\log(h_i' + 1)}{\log(h_i')} \) and \( [\ ] \) denote the greatest integer function.

Proof. From Lemma 9.1, we get
\[
\alpha_1 (h_i^k - 1) < (h_i - 1) \text{ i.e., } \alpha_1 < \frac{(h_i - 1)}{(h_i^k - 1)}, \quad i = 1, 2, \ldots, n - k + 1 \quad \text{(9.10)}
\]

and
\[
\alpha_1 (h_i^{n+i-1} - 1) < (h_i - 1) \text{ i.e., } \alpha_1 < \frac{(h_i - 1)}{(h_i^{n+i-1} - 1)}, \quad i = n - k + 2, \ldots, n - 1 \quad \text{(9.11)}
\]

Substituting \( i = (n - k + 2), (n - k + 3), \ldots, (n - 2), (n - 1) \) in (9.11), we get respectively
\[
\alpha_1 < \frac{(h - 1)}{(h - 2 - 1)} , \alpha_1 < \frac{(h - 1)}{(h - 3 - 1)}, \ldots, \alpha_1 < \frac{(h - 1)}{(h - 2 - 1)} , \alpha_1 < \frac{(h - 1)}{(h - 2 - 1)} \quad \text{(9.12)}
\]

229
Since \( \left( \frac{b_i - 1}{b_i} \right) < \left( \frac{b_{i-1} - 1}{b_{i-1}} \right) < \left( \frac{b_{i-2} - 1}{b_{i-2}} \right) < \cdots < \left( \frac{b_1 - 1}{b_1} \right) \), so from (9.10), we get
\[ \alpha_1 < \frac{b_i - 1}{b_i} \] i.e., \( k < X_1 \), where \( X_1 = \frac{1}{\log_b(1 + \frac{b_i - 1}{\alpha_1})} \)

Again from Lemma 9.2, we get
\[ \alpha_2 \left( \frac{b^k - 1}{b^k - 1} \right) < \left( \frac{b - 1}{b^k - 1} \right), \quad i = 1, 2, \ldots, n - k + 1 (9.13) \]

and
\[ \alpha_2 \left( \frac{b^{m-i+1} - 1}{b^{m-i+1} - 1} \right) < \left( \frac{b - 1}{b^{m-i+1} - 1} \right), \quad i = n - k + 2, \ldots, n - 1 (9.14) \]

Substituting \( i = (n - k + 2), (n - k + 3), \ldots, (n - 2), (n - 1) \) in (9.14), we get respectively
\[ \alpha_2 < \frac{b - 1}{b^{k-2} - 1}, \quad \alpha_2 < \frac{b - 1}{b^{k-3} - 1}, \ldots, \alpha_2 < \frac{b - 1}{b^{m-i+1} - 1}, \quad \alpha_2 < \frac{b - 1}{b^{m-k+2} - 1} (9.15) \]

Since \( \left( \frac{b_i - 1}{b_i} \right) \) is integer, \( X_2 = \frac{1}{\log_b(1 + \frac{b_i - 1}{\alpha_2})} \)

Since \( k < X_1, k < X_2 \) and \( k \) is integer positive, so the optimal value of \( k \) is \( \text{min} \{X_1, X_2\} \).

Hence the proof. \( \square \)

**Lemma 9.4.** If \( k_1 = \beta_1 + \delta_1(1 - \beta_1) \) and \( k_2 = \beta_2 + \delta_2(1 - \beta_2) \) then the positive real parameters \( k_1, k_2, \beta_1, \beta_2, \delta_1, \delta_2, D_1, D_2, P_1, P_2 \) satisfying the following relations \( \frac{P_1}{k_1} < k_1, \frac{P_2}{k_2} < k_2 \) and \( \frac{P_1}{k_1} + \frac{P_2}{k_2} = 1 \)

**Proof.** From equation (9.1), we get
\[ \beta_1 P_1 + \delta_1(1 - \beta_1)P_1 - D_1 > 0 \text{ i.e., } \{\beta_1 + \delta_1(1 - \beta_1)\}P_1 > D_1 \text{ i.e., } \frac{P_1}{k_1} < k_1 \]

For \( i = 1 \), we have from equation (9.3),
\[ \beta_2 P_1 + \delta_2(1 - \beta_2)P_1 - D_2 > 0 \text{ i.e., } \{\beta_2 + \delta_2(1 - \beta_2)\}P_2 > D_2 \text{ i.e., } \frac{P_2}{k_2} < k_2 \]

From equation (9.2), we get
\[ \{\beta_1 + \delta_1(1 - \beta_1)\}P_1 t_1 = D_1 T \text{ i.e., } k_1 P_1 t_1 = D_1 T \text{ i.e., } \frac{T}{P_1} = \frac{P_1}{k_1} \]

For \( i = 1 \), we have from equation (9.4), we get
\[ \{\beta_2 + \delta_2(1 - \beta_2)\}P_2(T - t_1) = D_2 T \text{ i.e., } k_2 P_2(T - t_1) = D_2 T \text{ i.e., } 1 - \frac{T}{P_2} = \frac{P_2}{k_2} \text{ i.e., } 1 - \frac{P_1}{k_1} - \frac{P_2}{k_2} = 1 \text{ Hence the proof. } \square \]

**Lemma 9.5.** If the positive real parameters \(\lambda \) and \(\mu \) satisfying the following relations \( P_2, \lambda P_2(1 - 1), D_2 = \mu D_2(1 - 1), i = 1, 2, \ldots, k \) then \(\lambda = \mu \) holds.
9.4. OBJECTIVES OF THE PROPOSED MODEL

Proof. From the continuity condition (9.1), we get
\[
(β_2 + δ_2(1 − β_2))P_2(T − t_i) = D_2T, \quad i = 1, 2, \ldots, n
\]
For \(i = 1, k_2(1 − \frac{1}{k})P_2 = D_2, \) where \(k_2 = (β_2 + δ_2(1 − β_2))\)
Also \(k_2(T − t_i)\lambda^{-1}P_2 = \mu^{k_2−1}D_2T, \quad i = 2, 3, \ldots, k\)
\(\Rightarrow (\frac{\lambda}{\mu})^{k−1}D_2 = D_2 \Rightarrow (\frac{\lambda}{\mu})^{k−1} = 1, \quad i = 1, 2, \ldots, k\)
\(\Rightarrow \lambda = \mu. \) Hence the proof. \(\Box\)

Lemma 9.6. \(P_2\) and \(λ\) satisfying the following relations:
(i) \(\{1 − γ_i(1 − δ_2)(1 − β_2)\}P_2 > \frac{1}{α_1D_1} [α_3D_1 \sum_{j=1}^{k} h_i^{j−1} + α'_iD_1α_2D_2 \sum_{j=1}^{k} h_i^{j−1} \mu^{k−1}] \) when
\(λ ≥ 1 + \frac{α[D_1h^{k−1} + αD_1h^{k−1}]}{(1 + h^{k−1} + \mu^{k−1})α_1D_1}\)
and (ii) \(\{1 − γ_i(1 − δ_2)(1 − β_2)\}P_2 < γ_i[α_1D_1 + α'_iD_1 + α_2D_2] \) when
\(λ ≤ 1 + \frac{α[D_1h^{k−1} + αD_1h^{k−1}]}{(1 + h^{k−1} + \mu^{k−1})α_1D_1}\)

Proof. From equation (9.5), we get
\(γ_i[α_1D_1 + α_1D_1 + β_1D_2 + (1 − δ_2)(1 − β_2)P_2] − P_2 < 0\)
i.e., \(\{1 − γ_i(1 − δ_2)(1 − β_2)\}P_2 > γ_i[α_1D_1 + α_1D_1 + α_2D_2]\)
i.e., \(MP_2 > g_{2e}\) where \(M = \{1 − γ_i(1 − δ_2)(1 − β_2)\}\) and \(g_{2e} = γ_i[α_1D_1 + α_1D_1 + α_2D_2].\)

From equation (9.6), we get
\(γ_i[α_2D_1 + α_2D_1 + α_2D_1 + β_2D_2 + β_2D_2 + β_2D_2 + (1 − δ_2)(1 − β_2)P_2] − P_2 < 0\)
i.e., \(\{1 − γ_i(1 − δ_2)(1 − β_2)\}P_2 > γ_i[α_2D_1 + α_2D_1 + α_2D_2 \sum_{j=1}^{k} h_i^{j−1} \mu^{k−1}]\)
i.e., \(MP_2 > g_{2e}\) where \(g_{2e} = γ_i[α_2D_1 + α_2D_1 + α_2D_2 \sum_{j=1}^{k} h_i^{j−1} \mu^{k−1}]\).

Put \(i = 3 \) in equation (9.7), we get
\(γ_i[α_3D_1 + α_3D_1 + α_3D_1 + α_3D_1 + α_3D_1 + α_3D_2 + (1 − δ_2)(1 − β_2)P_2] − P_2 < 0\)
i.e., \(\{1 − γ_i(1 − δ_2)(1 − β_2)\}P_2 > γ_i[α_3D_1 + α_3D_1 + α_3D_2 \sum_{j=1}^{k} h_i^{j−1} \mu^{k−1}]\)
i.e., \(MP_2 > g_{2e}\) where \(g_{2e} = γ_i[α_3D_1 + α_3D_1 + α_3D_2 \sum_{j=1}^{k} h_i^{j−1} \mu^{k−1}]\).

Proceeding in the way, if we put \(i = k \) in equation (9.7) we get
\(MP_2 > g_k\) where \(g_k = γ_i[α_{k−1}D_1 + α_{k−1}D_1 + α_{k−1}D_1 + α_{k−1}D_1 + α_{k−1}D_1 + α_{k−1}D_2 \sum_{j=1}^{k} h_i^{j−1} \mu^{k−1}]\).

Put \(i = k + 1 \) in equation (9.7), we get
\(γ_i[α_{k+1}D_1 + α_{k+1}D_1 + α_{k+1}D_1 + α_{k+1}D_1 + α_{k+1}D_1 + α_{k+1}D_2 \sum_{j=1}^{k} h_i^{j−1} \mu^{k−1}]\)
\(− P_2(\mu + 1) < 0\)
CHAPTER 9. GA APPROACH FOR CONTROLLING GHG EMISSION FROM INDUSTRIAL WASTE IN TWO PLANT PRODUCTION AND REPRODUCTION INVENTORY MODEL WITH INTERVAL VALUED FUZZY POLLUTION PARAMETERS

\[ M_{P_2} > g_{(k+1)}, \text{ where } g_{(k+1)} = \frac{\gamma_1}{\lambda_2 \lambda_{k+1}} \left\{ \alpha_1 D_1 \sum_{i=1}^{k} h_i^{-1} + \alpha'_1 D_1 \right\} + \frac{\gamma_2 P_{2(\lambda_{k+1})}}{\lambda_{k+1} \lambda_2} \left( \sum_{i=1}^{k} h_i^{-1} \mu^k - \mu^{k-2} (\mu - \mu_1) \right). \]

Put \( i = k + 2 \) in equation (9.7), we get

\[ \gamma_1 \left( \sum_{i=1}^{k+2} \alpha_{(k+2)} D_i + \alpha'_1 (k+2) D_1 + \sum_{j=1}^{k+2} \frac{\beta_{(k+2)} D_{2j}}{\lambda_{(k+2)} \lambda_2} + \lambda_2 P_{2(k+2)} \right) - P_{2(k+2)} < 0 \]

i.e., \( M_{P_2} > g_{(k+2)}, \) where \( g_{(k+2)} \)

\[ \frac{\gamma_1}{\lambda_2 \lambda_{k+1}} \left\{ \alpha_1 D_1 \sum_{i=1}^{k} h_i^{-1} + \alpha'_1 D_1 \right\} + \frac{\gamma_2 P_{2(\lambda_{k+1})}}{\lambda_{k+1} \lambda_2} \left( \sum_{i=1}^{k+2} h_i^{-1} \mu^k - \mu^{k-3} (\mu + \mu_1) + (\mu^2 - \mu_1^2) \right). \]

Proceeding in the way, if we put \( i = n \) in equation (9.7) we get

\[ M_{P_2} > g_n, \text{ where } g_n = \frac{\gamma_1}{\lambda_2 \lambda_{k+1}} \left\{ \alpha_1 D_1 \sum_{i=1}^{k} h_i^{-1} + \alpha'_1 D_1 \right\} + \frac{\gamma_2 P_{2(\lambda_{k+1})}}{\lambda_{k+1} \lambda_2} \left( \sum_{i=1}^{n} h_i^{-1} \mu^k - \mu^{k-3} \sum_{i=1}^{n} h_i^{-1} (\mu^2 - \mu_1^2) \right). \]

Case-I: When \( \lambda \geq 1 + \frac{\alpha_1 D_1 + \alpha_2 D_2}{\alpha_1 \alpha_2 D_1} \) then we have

\[ g_1 \leq g_2 \leq g_3 \leq \cdots \leq g_{n-1} \leq g_n. \]

Therefore \( g_n = \max \{g_1, g_2, \ldots, g_n\} \)

Hence \( g_n \) is maximum when \( \lambda \geq 1 + \frac{\alpha_1 D_1 + \alpha_2 D_2}{\alpha_1 \alpha_2 D_1} \)

Here \( M_{P_2} > g_1, M_{P_2} > g_2, \ldots, M_{P_2} > g_n \) gives \( M_{P_2} > \max \{g_1, g_2, \ldots, g_n\} \)

\[ \Rightarrow M_{P_2} > g_{(k+1)} \text{ when } \lambda \geq 1 + \frac{\alpha_1 D_1 + \alpha_2 D_2}{\alpha_1 \alpha_2 D_1}, \]

\[ \Rightarrow \{1 - \gamma_1(1 - \mu_2)(1 - \beta_2)\} M_{P_2} > \frac{\gamma_1}{\lambda_2 \lambda_{k+1}} \left\{ \alpha_1 D_1 \sum_{i=1}^{k} h_i^{-1} + \alpha'_1 D_1 \right\} \]

when \( \lambda \geq 1 + \frac{\alpha_1 D_1 + \alpha_2 D_2}{\alpha_1 \alpha_2 D_1} \). Hence the proof.

Case-II: when \( \lambda \leq 1 + \frac{\alpha_1 D_1 h_{k-1} + \alpha_2 D_2 h^k}{\alpha_1 + \alpha_2 D_1} \) then we have

\[ g_{n-1} \leq g_{n-2} \leq \cdots \leq g_{k+1} \leq g_k \leq g_{k-1} \leq g_{k-2} \leq \cdots \geq g_1 \]

Therefore \( g_k = \min \{g_1, g_2, \ldots, g_n\} \)

Hence \( g_k \) is minimum when \( \lambda \leq 1 + \frac{\alpha_1 D_1 h_{k-1} + \alpha_2 D_2 h^k}{\alpha_1 + \alpha_2 D_1} \)

Here \( M_{P_2} > g_1, M_{P_2} > g_2, \ldots, M_{P_2} > g_n \) gives \( M_{P_2} > \min \{g_1, g_2, \ldots, g_n\} \)

\[ \Rightarrow M_{P_2} > g_{(k+1)} \text{ when } \lambda \leq 1 + \frac{\alpha_1 D_1 h_{k-1} + \alpha_2 D_2 h^k}{\alpha_1 + \alpha_2 D_1}, \]

\[ \Rightarrow \{1 - \gamma_1(1 - \mu_2)(1 - \beta_2)\} M_{P_2} > \gamma_1 \left[ \alpha_1 D_1 + \alpha'_1 D_1 + \alpha_2 D_2 \right] \text{ when } \lambda \leq 1 + \frac{\alpha_1 D_1 h_{k-1} + \alpha_2 D_2 h^k}{\alpha_1 + \alpha_2 D_1} \]. Hence the proof.
9.4 OBJECTIVES OF THE PROPOSED MODEL

9.4.1 Model in Crisp Environment

Maximize TP
Minimize \( Z_d \)
subject to the conditions,

\[
\begin{align*}
[\beta_1 P_1 + \delta_1 (1-\beta_1) P_2] t_1 &= D_1 T \\
[\beta_2 + \delta_2 (1-\beta_2)] (T-t_1) P_2 &= D_2 T, & i = 1, 2, \ldots, n. \quad (9.16) \\
\alpha_1 (h^k - 1) &< (h-1), & i = 1, 2, \ldots, n-k+1 \quad (9.18) \\
\alpha_2 (h^{n-i+1} - 1) &< (h-1), & i = n-k+2, \ldots, n-1. \quad (9.19) \\
\alpha_3 (h^k - 1) &< (h'-1), & i = 1, 2, \ldots, n-k+1 \quad (9.20) \\
\alpha_4 (h^{n-i+1} - 1) &< (h'-1), & i = n-k+2, \ldots, n-1. \quad (9.21) \\
n, k, \lambda, t_1 &> 0
\end{align*}
\]

9.4.2 Model in Fuzzy Environment

Here we take the parameters \( c_{\alpha}, c_{\beta} \) and \( ER_d \) as interval valued fuzzy number. Then the above problem in fuzzy environment is

Maximize \( \widetilde{TP} = TSP_{\alpha_1} + TSP_{\beta_2} - \widetilde{TC}_{\alpha_1} - \widetilde{TC}_{\beta_2} - THC_3 - c_{\alpha} \widetilde{ER}_d Q_4 - c_{\beta} Q_4 \)
Minimize \( \widetilde{Z}_d = \tilde{m}_d P_{\alpha_1}^{\tilde{l}} t_1 + c_{\alpha} (T-t_1) \sum_{i=1}^{n} P_{\beta_2}^{\tilde{l}} + \tilde{E} \tilde{R}_d Q_4 \)
subject to, the conditions stated above in (9.16) to (9.21), where

\[
\begin{align*}
\widetilde{TC}_{\alpha_1} &= n \left\{ \tilde{c}_{\alpha} + \tilde{\epsilon}_d (1-\beta_1) + \tilde{c}_{\beta} \right\} P_1 t_1 + THC_1 + n A_3 + n c_{\alpha} \tilde{c}_{\alpha} P_{\alpha_1}^{\tilde{l}} t_1 + n c_{\beta} \tilde{c}_{\beta} P_{\beta_2}^{\tilde{l}} t_1 \\
\widetilde{TC}_{\beta_2} &= TPC_2 + RW C_2 + TSC_2 + THC_2 + n A_4 + c_{\alpha} \tilde{\epsilon}_d (T-t_1) \sum_{i=1}^{n} P_{\beta_2}^{\tilde{l}} + c_{\beta} \tilde{\epsilon}_d (T-t_1) \sum_{i=1}^{n} P_{\beta_2}^{\tilde{l}}.
\end{align*}
\]

Taking \( \tilde{c}_d = \left[ \tilde{c}_{\alpha}^{L}, \tilde{c}_{\alpha}^{R} \right], \tilde{\epsilon}_d = \left[ \tilde{\epsilon}_{\alpha}^{L}, \tilde{\epsilon}_{\alpha}^{R} \right], \tilde{E} \tilde{R}_d = \left[ \tilde{E} \tilde{R}_d^{L}, \tilde{E} \tilde{R}_d^{R} \right] \), we have

\[
\begin{align*}
\widetilde{TC}_{\alpha_1} &= \left[ \widetilde{TC}_{\alpha_1}^{L}, \widetilde{TC}_{\alpha_1}^{R} \right], \widetilde{TC}_{\beta_2} = \left[ \widetilde{TC}_{\beta_2}^{L}, \widetilde{TC}_{\beta_2}^{R} \right], \tilde{Z}_d = \left[ \tilde{Z}_d^{L}, \tilde{Z}_d^{R} \right]. \text{(see Appendix F)}
\end{align*}
\]

Then the above multi-objective fuzzy interval valued problem can be stated as

Maximize \( \tilde{TP} = \left[ \tilde{TP}^{L}, \tilde{TP}^{R} \right] \)
Minimize \( \tilde{Z}_d = \left[ \tilde{Z}_d^{L}, \tilde{Z}_d^{R} \right] \)
subject to the conditions stated in (9.16) to (9.21), where

\[
\begin{align*}
\tilde{TP}^{L} &= TSP_{\alpha_1} + TSP_{\beta_2} - \widetilde{TC}_{\alpha_1} - \widetilde{TC}_{\beta_2} - THC_3 - c_{\alpha} \tilde{E} \tilde{R}_d Q_4 - c_{\beta} Q_4, \\
\tilde{TP}^{R} &= TSP_{\alpha_1} + TSP_{\beta_2} - \widetilde{TC}_{\alpha_1} - \widetilde{TC}_{\beta_2} - THC_3 - c_{\alpha} \tilde{E} \tilde{R}_d Q_4 - c_{\beta} Q_4.
\end{align*}
\]

The interval valued multi-objective problem is transformed into following:

Maximize \( \tilde{TP} = \omega_1 \tilde{TP}^{L} + (1-\omega_1) \tilde{TP}^{R}, \quad 0 < \omega_1 < 1 \)
Minimize \( \tilde{Z}_d = \omega_2 \tilde{Z}_d^{L} + (1-\omega_2) \tilde{Z}_d^{R}, \quad 0 < \omega_2 < 1 \)
subject to, the conditions stated in (9.16) to (9.21) (see Appendix F).
CHAPTER 9. GA APPROACH FOR CONTROLLING GHG EMISSION FROM
INDUSTRIAL WASTE IN TWO PLANT PRODUCTION AND REPRODUCTION
INVENTORY MODEL WITH INTERVAL VALUED FUZZY POLLUTION
PARAMETERS

Equivalent Crisp Model
Using Lemma (2.3), the above fuzzy multi-objective problem is transformed into crisp
multi-objective problem:
Maximize $E[T^P] = \omega_1 E[T^\hat{P}] + (1 - \omega_1)E[T^\hat{P}_1]$, $0 < \omega_1 < 1$
Minimize $E[Z_g] = \omega_2 E[Z_g^\hat{P}] + (1 - \omega_2)E[Z_g^\hat{P}_2]$, $0 < \omega_2 < 1$
subject to, the conditions stated in (9.16) to (9.21). (see Appendix F).

9.5 Numerical Illustration

The proposed model of the production and reproduction inventory system has been developed
with the help of following numerical example in this section. The values of the parameters
of the model, considered in these numerical examples are not elected from any real life
case study, but these values have been seems to be realistic. The example have been solved to find
optimal values of $\hat{n}$, $\hat{K}$, $\hat{A}$, $\hat{t}_1$ and $\hat{T}$ along with the optimal expected total profit of the system.
To illustrate the solution, we consider an inventory system with the following input data:

**Crisp input data:** $\hat{\beta}_1 = 0.89$, $\hat{\beta}_2 = 0.85$, $\hat{\delta}_1 = 0.74$, $\hat{\delta}_2 = 0.75$, $\gamma_1 = 0.81$, $\alpha_1 = 0.09$,
$\alpha_2 = 0.11$, $\alpha'_1 = 0.08$, $\alpha'_2 = 0.07$, $\hat{h} = 1.05$, $\hat{h}_1 = 1.07$, $\hat{\eta}_g = 1.02$, $\hat{\eta}_2g = 1.09$, $\hat{\eta}_v = 1.06$,
$\eta_2v = 1.08$, $c_g = 1.05$, $c_v = 1.02$, $c_p = 1.0$, $c'_p = 1.5$, $c_t = 0.62$, $c_c = 1.5$, $\hat{h}_c = 1.4$,
$\hat{h}_c' = 1.3$, $\hat{A}_c = 2.0$, $\hat{A}_c' = 1.3$, $\hat{c}_c = 0.41$, $\hat{c}_c' = 0.5$, $\hat{r}_c = 1.1$, $\hat{r}_c' = 1.2$, $s = 38.0$,
s$\hat{t} = 38.0$, $\hat{P}_1 = 56$, $\hat{P}_2 = 54$, $D_1 = 29$, $D_2 = 26$.

**Fuzzy input data:** $\hat{c}_g = (0.75, 0.76, 0.78)$, $\hat{c}_v = (0.79, 0.81, 0.82)$, $\hat{c}_p = (0.68, 0.69, 0.70)$,
$\hat{c}_c = (0.70, 0.71, 0.73)$, $\hat{E}[\hat{R}_g] = (0.02, 0.03, 0.05)$, $\hat{E}[\hat{R}_v] = (0.06, 0.08, 0.09)$.

Applying the solution procedure, the optimal solution given in the following Table 9.1.

<table>
<thead>
<tr>
<th>Number of cycle ($n$)</th>
<th>Number of consecutive cycles ($\hat{n}$)</th>
<th>$\lambda$</th>
<th>Production time length of each cycle ($\hat{t}_1$)</th>
<th>Expected total profit ($E[T^\hat{P}]$)</th>
<th>Expected GHG emission ($E[Z_g]$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3</td>
<td>1.3135</td>
<td>3.5250</td>
<td>6.6122</td>
<td>29658.1329</td>
</tr>
</tbody>
</table>

Table 9.1: Optimal result of the illustrated model when $\hat{\rho} = 0.4$, $\hat{w}_1 = 0.3$, $\hat{w}_2 = 0.6$

- Table 9.1 shows that the optimal production time of each cycle is 3.5250 unit, time length of each cycle is 6.6122 unit, expected total profit is 29658.1329 unit and expected GHG emission 1213.7692 unit.

9.5.1 Sensitivity Analysis

Using the same data as that in above Example, we next study the sensitivity of the optimal
production time of each cycle, time length of each cycle, expected total profit and expected
GHG emission to change the values of the different parameters associated with the model. The computational results are reported in the following Table 9.2, 9.3 and 9.4.

Table 9.2: Sensitivity analysis of $E[\tilde{T}]$ and $E[\tilde{Z}_d]$ w.r.t. $\beta_1$, $\beta_2$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Production time $\bar{t}$</th>
<th>Time length of each cycle $T$</th>
<th>Expected total profit $E[\tilde{T}]$</th>
<th>Expected GHG emission $E[\tilde{Z}_d]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1 = 0.86$, $\beta_2 = 0.85$</td>
<td>3.5128</td>
<td>6.5931</td>
<td>28514.7215</td>
<td>1426.3527</td>
</tr>
<tr>
<td>$\beta_1 = 0.86$, $\beta_2 = 0.85$</td>
<td>3.5250</td>
<td>6.6122</td>
<td>29658.1329</td>
<td>1213.7692</td>
</tr>
<tr>
<td>$\beta_1 = 0.85$, $\beta_2 = 0.87$</td>
<td>3.5664</td>
<td>6.6414</td>
<td>30142.1276</td>
<td>1134.5192</td>
</tr>
</tbody>
</table>

- Table 9.2 shows that when the fraction of the better quality item $\beta_1$ and $\beta_2$ in plant-I and plant-II are respectively increases, expected total profit ($E[\tilde{T}]$) increase and expected GHG emission ($E[\tilde{Z}_d]$) decrease.

Table 9.3: Sensitivity analysis of $E[\tilde{T}]$ and $E[\tilde{Z}_d]$ w.r.t. $\delta_1$, $\delta_2$, $r_e$, $r_e'$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Reworked cost $(\delta_1, \delta_2)$</th>
<th>Production time $\bar{t}$</th>
<th>Time length of each cycle $T$</th>
<th>Expected total profit $E[\tilde{T}]$</th>
<th>Expected GHG emission $E[\tilde{Z}_d]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1 = 0.73$, $\delta_2 = 0.78$, $r_e = 1.45$, $r_e' = 1.50$</td>
<td>3.5250</td>
<td>6.5813</td>
<td>28564.2143</td>
<td>1328.2314</td>
<td></td>
</tr>
<tr>
<td>$\delta_1 = 0.73$, $\delta_2 = 0.75$, $r_e = 1.45$, $r_e' = 1.50$</td>
<td>3.5250</td>
<td>6.5813</td>
<td>28564.2143</td>
<td>1328.2314</td>
<td></td>
</tr>
<tr>
<td>$\delta_1 = 0.75$, $\delta_2 = 0.75$, $r_e = 1.45$, $r_e' = 1.50$</td>
<td>3.5250</td>
<td>6.5813</td>
<td>28564.2143</td>
<td>1328.2314</td>
<td></td>
</tr>
</tbody>
</table>

- Table 9.3 shows that when the reworked rate and average reworked cost $(\delta_1, r_e)$ and $(\delta_2, r_e')$ in plant-I and plant-II are respectively simultaneously increase, initially expected total profit ($E[\tilde{T}]$) increases, after that expected total profit ($E[\tilde{T}]$) decrease due to some portion of defective item to be reworked with a minimum rework cost and rest portion of defective item to be reworked with a large amount rework cost. Also expected GHG emission ($E[\tilde{Z}_d]$) decrease. So any manufacturer company can find the optimal reworked rate from this study.

Table 9.4: Sensitivity analysis of $E[\tilde{T}]$ and $E[\tilde{Z}_d]$ w.r.t. $\gamma_1$, $c_p$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Production time $\bar{t}$</th>
<th>Time length of each cycle $T$</th>
<th>Expected total profit $E[\tilde{T}]$</th>
<th>Expected GHG emission $E[\tilde{Z}_d]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1 = 0.76$, $c_p = 12$</td>
<td>3.5250</td>
<td>6.5834</td>
<td>28746.5823</td>
<td>1347.8479</td>
</tr>
<tr>
<td>$\gamma_1 = 0.81$, $c_p = 15$</td>
<td>3.5250</td>
<td>6.6122</td>
<td>29658.1329</td>
<td>1213.7692</td>
</tr>
<tr>
<td>$\gamma_1 = 0.84$, $c_p = 19$</td>
<td>3.5250</td>
<td>6.6371</td>
<td>31254.8579</td>
<td>1140.2584</td>
</tr>
<tr>
<td>$\gamma_1 = 0.89$, $c_p = 24$</td>
<td>3.5250</td>
<td>6.6497</td>
<td>32521.2314</td>
<td>1047.1725</td>
</tr>
</tbody>
</table>
CHAPTER 9. GA APPROACH FOR CONTROLLING GHG EMISSION FROM INDUSTRIAL WASTE IN TWO PLANT PRODUCTION AND REPRODUCTION INVENTORY MODEL WITH INTERVAL VALUED FUZZY POLLUTION PARAMETERS

- Table 9.4 shows that when percentage of amount of disposal unit \((1 - \gamma)\) decreases and the corresponding raw material producing cost per unit \((c_2)\) simultaneously increases, initially the expected total profit \((E[T^P])\) increases, after that the expected total profit \((E[T^P])\) decreases. Because at first increasing raw material producing cost per unit more increasing the amount of raw material unit. As a result, the expected total profit \((E[T^P])\) increases. But latter, though raw material producing cost per unit increases, the amount of raw material unit does not increase as much as previous due to more defectiveness. Also expected GHG emission \((E[\dot{Z}_0])\) decrease. So from this study, any manufacturer company can find the optimal percentage of amount of disposal unit and the corresponding raw material producing cost per unit.

9.6 Conclusion

This chapter focused on the combined effect of imperfect production and reproduction over finite time horizon. Both the manufactured and re-manufactured used items are returned from the market and some portion of them are used as raw-materials for the re-manufacturing process to save the natural resources for future and protect environment from pollution by the used items after the end of their life cycle. Some portion of non-reworkable defective units from both the plants is also used as raw-materials of plant-II for the same purpose. Generally, the items produced in re-manufacturing process are less quality than the items produced in normal manufacturing process. So with respect to quality measure, two types of items are produced in manufacturing (plant-I) and re-manufacturing process (plant-II). In the proposed model, we maximized the expected total profit from both the plants and simultaneously minimized the GHG emission from the industrial waste during production and from the used and non-reworkable defective units by industrial waste management. Our main aim of this model is to minimize the disposal cost by minimizing the disposal amount (ISW) from both the plants so that GHG emission is less to the environment. Here, we consider the parameters relating to the environmental pollution due to industrial waste as interval valued fuzzy number since they are not fixed by nature. Finally, a numerical example has been illustrated to study the feasibility of the model. Also sensitivity analysis has been carried out to draw some managerial insights of the model.
Part V

Studies on Imperfect Production

Inventory System in Fuzzy Stochastic Environment