CHAPTER 2

ODD MEAN LABELINGS OF GRAPHS*

2.1 INTRODUCTION

A graph in this paper shall mean a simple finite graph without isolated vertices. The terminology and notions used here are in the sense of Harary [18]. A labeling of a graph G is an assignment f of labels to either the vertices or the edges of G that induces for each edge uv in the former a label depending on the vertex labels $f(u)$ and $f(v)$ and in the latter for each vertex u a label depending on the labels of the edges incident with it. The oldest and more popular vertex labeling is the one introduced by Rosa [30] in 1967 and R.B. Gnanajothi [14] introduced odd graceful graphs. S. Somasundaram and R. Ponraj [36] introduced the concept of mean graphs. Motivated by these works, we define odd mean labelings of graphs and investigate the odd mean behaviour of certain standard graphs.

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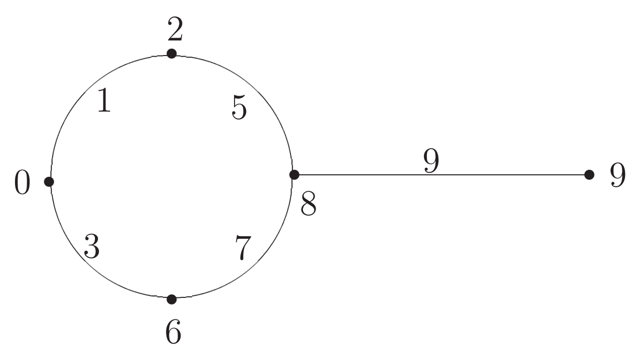
2.2 ODD MEAN LABELINGS

Definition 2.2.1. A graph G with p vertices and q edges is said to be odd mean if there exists a function f from the vertex set of G to $\{0, 1, 2, 3, \ldots, 2q-1\}$ satisfying f is 1-1 and the induced map f^* from the edge set of G to $\{1, 3, 5, \ldots, 2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd,} \end{cases}$$

is a bijection.

The following is a simple example of odd mean graph.



2.3 SOME PRELIMINARY THEOREMS

In this section we prove some basic theorems on odd mean graph.

Result 2.3.1. *If G is a harmonious graph with harmonious labeling f, then* $\sum_{v \in V(G)} d(v)f(v) = \binom{q}{2} (mod\ q)$.

Motivated by this, we prove the following theorems.

Theorem 2.3.2. *Let G be an odd mean graph with odd mean labeling f. Let t be the number of edges whose one vertex label is even and the other is odd. Then $\sum_{v\in V(G)} d(v)f(v) + t = 2q^2$ where $d(v)$ denotes the degree of a vertex.*

Proof. We have

$$f^*(xy) = \begin{cases} \frac{f(x)+f(y)}{2} & \text{if } f(x)+f(y) \text{ is even} \\ \frac{f(x)+f(y)+1}{2} & \text{if } f(x)+f(y) \text{ is odd.} \end{cases}$$

$$\begin{aligned} \sum_{v\in V(G)} d(v)f(v) &= 2\left(\sum_{xy\in E(G)} f^*(xy) - \frac{t}{2}\right) \\ &= 2(1+3+5+\cdots+(2q-1)) - t \\ &= 2q^2 - t. \end{aligned}$$

Hence $\sum_{v\in V(G)} d(v)f(v) + t = 2q^2$.

As an illustration, we consider $P_6 = v_1v_2v_3v_4v_5v_6$.

Define $f : V(P_6) \to \{0, 1, 2, 3, \ldots, q\}$ by

$f(v_1) = 0, f(v_2) = 2, f(v_3) = 4, f(v_4) = 6, f(v_5) = 8$ and $f(v_6) = 9$.

Clearly f is an odd mean labeling. Here $t = 1$.

Now,

$$\begin{aligned}\sum d(v)f(v)+t &= 1\times 0+2\times 2+2\times 4+2\times 6+2\times 8+1\times 9+1\\ &= 4+8+12+16+9+1\\ &= 50\\ &= 2\times 5^2\\ &= 2q^2.\end{aligned}$$

Hence, $\sum\limits_{v\in V(G)} d(v)f(v)+t = 2q^2$.

Hence the theorem. □

Corollary 2.3.3. *If G is an odd mean graph with odd mean labeling f, then* $\sum\limits_{v\in V(G)} d(v)f(v) \geq 2q^2-q$.

Proof. Follows from the above theorem and using $t \leq q$. □

Corollary 2.3.4. *Let G be a 2-regular odd mean graph. Let f be any odd mean labeling of G and $x \in \{0,1,2,3,\ldots,(2q-1)\} - f(V(G))$. Then $x \leq \frac{2q^2-q}{2}$.*

Proof. Since G is 2-regular, $deg(v) = 2$ for all $v \in V(G)$. By Corollary 2.3.3, we have

$$\begin{aligned}2q^2-q &\leq \sum_{v\in V(G)} d(v)f(v)\\ &= 2\sum_{v\in V(G)} f(v)\end{aligned}$$

$$
\begin{aligned}
&= 2\left(\sum_{v\in V(G)} f(v) + x\right) - 2x \\
&= 2(0+1+2+3+\cdots+(2q-1)) - 2x \\
&= \frac{2(2q-1)2q}{2} - 2x \\
&= 4q^2 - 2q - 2x \\
2x &\le 2q^2 - q \\
\Rightarrow x &\le \frac{2q^2-q}{2}.
\end{aligned}
$$

Hence the proof. □

Theorem 2.3.5. *Any path is an odd mean graph.*

Proof. Let P_n be the path $P_n : u_1, u_2, u_3, \ldots, u_n$.

Define $f : V(P_n) \to \{0, 1, 2, \ldots, 2n-3\}$ by

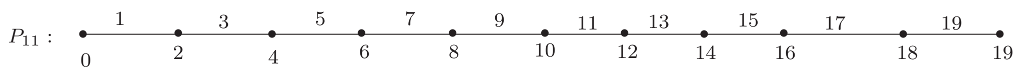
$f(u_i) = 2i - 2 (1 \le i \le n-1)$ and $f(u_n) = 2n - 3$. The label of the edge $u_{i-1}u_i$ is $2i - 3 (2 \le i \le n)$.

Obviously $f(u_1) = 0$ and $f(u_n) = 2n - 3$. Thus f is a function from $V(P_n)$ to the set $\{0, 1, 2, 3, \ldots, 2n-3\}$. Clearly f is one-one.

For $1 \le i \le n-1$, the vertex labels of $f(u_i)$ are in the set, $A = \{0, 2, 4, 6, \ldots, 2n-4\} \cup \{2n-3\}$. The set A has n labels. For $2 \le i \le n$, the edge labels of $f(u_{i-1}u_i)$ is the set $B = \{1, 3, 5, 7, \ldots, 2n-3\}$. The set B has n labels.

Hence P_n is an odd mean graph. □

Example 2.3.6. Odd mean labeling of P_{11}.



Theorem 2.3.7. *C_n is an odd mean graph if $n \equiv 0(mod\ 4)$.*

Proof. Let $C_n : v_1v_2v_3 \ldots v_nv_1$ be the given cycle where $n \equiv 0(mod\ 4)$.

Define $f : V(G) \rightarrow \{0, 1, 2, 3, \ldots, (2n-1)\}$ by

$$f(u_i) = \begin{cases} 4i-4 & \text{if } 1 \leq i \leq n/2 \text{ and } i \text{ is odd} \\ 4i-6 & \text{if } 1 < i \leq n/2 \text{ and } i \text{ is even} \\ 4n+3-4i & \text{if } n/2 < i < n \text{ and } i \text{ is odd} \\ 4n+6-4i & \text{if } n/2 < i \leq n \text{ and } i \text{ is even.} \end{cases}$$

The label of the edge u_iu_{i+1} is $\begin{cases} 4i-3 & \text{if } 1 \leq i \leq n/2 \\ 4n+3-4i & \text{if } n/2 < i < n \end{cases}$ and the label of the edge u_nu_1 is 3.

$$\begin{aligned} \min_{u \in V(G)} f(u) &= \min_{u \in V(G)} \{f(u_i) : 1 \leq i \leq n\} \\ &= 4i-4 \text{ if } 1 \leq i \leq \frac{n}{2} \text{ and} \\ &\quad i \text{ is odd and } i = 1 \\ &= 0 \\ \max_{u \in V(G)} f(u) &= \max\{f(u_i) : 1 \leq i \leq n\} \\ &= 4n+3-4i \text{ if } \frac{n}{2} < i < n, i \text{ is odd and} \\ &= \quad i = \frac{n}{2} + 1 \end{aligned}$$

$$= 4n + 3 - 4(\frac{n}{2} + 1)$$
$$= 2n - 1.$$

Thus f is a function from $V(G)$ to the set $\{0, 1, 2, 3, \ldots, (2n-1)\}$. Clearly f is one-one.

Next we find the vertex labels of $f(u_i)$.

For $1 \leq i \leq \frac{n}{2}$ and i is odd, the labels of $f(u_i)$ are in the set, $A_1 = \{0, 8, 16, \ldots, 2n-8\}$. The set A_1 has $\frac{n}{4}$ labels.

For $1 \leq i \leq \frac{n}{2}$ and i is even, the labels of $f(u_i)$ are in the set, $A_2 = \{2, 10, 18, \ldots, (2n-6)\}$. The set A_2 has $\frac{n}{4}$ labels.

For $\frac{n}{2} < i < n$ and i is odd, the labels of $f(u_i)$ are in the set, $A_3 = \{2n-1, 2n-5, \ldots, 7\}$. The set A_3 has $\frac{n}{4}$ labels.

For $\frac{n}{2} < i \leq n$ and i is even, the labels of $f(u_i)$ are in the set, $A_4 = \{2n+2, 2n-2, 2n-6, \ldots, 10\}$. The set A_4 has $\frac{n}{4}$ labels. Thus, the vertex labels of $f(u_i)$ are in the set, $A = A_1 \cup A_2 \cup A_3 \cup A_4$.

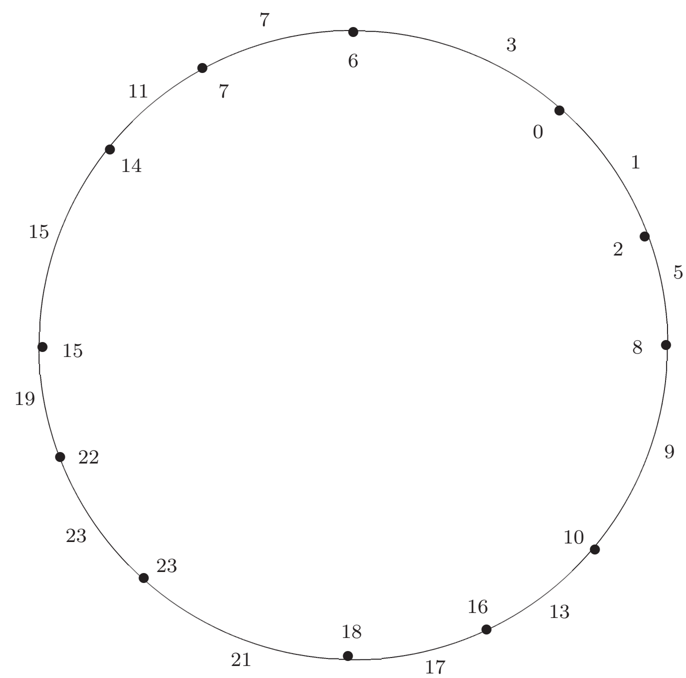
Therefore, the set A has n labels. Next we find the edge labels.

For $1 \leq i \leq \frac{n}{2}$, the labels of $f(u_i u_{i+1})$ are in the set, $B_1 = \{1, 5, 9, \ldots, 2n-3\}$. The set B_1 has $\frac{n}{2}$ labels. For $\frac{n}{2} < i < n$, the labels of $f(u_i u_{i+1})$ are in the set, $B_2 = \{2n-1, 2n-5, 2n-9, \ldots, 7\}$. The set B_2 has $\frac{n-2}{2}$ labels.

The label of $f(u_n u_1)$ is the set $B_3 = \{3\}$. Thus the edge labels of $f(u_iu_{i+1})$ are in the set $B = B_1 \cup B_2 \cup B_3$. Therefore the set B has n labels.

Hence G is an odd mean graph. $\square$

Example 2.3.8. Odd mean lableing of C_{12}.



Definition 2.3.9. $C_4 \oplus P_n$ is the graph obtained by joining an end point of the path P_n to a vertex of the cycle C_4.

Theorem 2.3.10. *$C_4 \oplus P_n$, is an odd mean graph for all positive integer n.*

Proof. Let the vertices of P_n be $v_1, v_2, v_3, \ldots, v_n$ and the vertices of C_4 be $v_{n+1}, v_{n+2}, v_{n+3}, v_{n+4}$.

Define $f : V(G) \to \{0, 1, 2, 3, \ldots, (2n+7)\}$ by

$f(v_i) = 2i - 2(1 \leq i \leq n), f(v_{n+1}) = 2n, f(v_{n+2}) = 2n + 2, f(v_{n+3}) = 2n + 7$ and $f(v_{n+4}) = 2n + 6$.

The label of the edge $v_i v_{i+1}$ is $2i - 1(1 \leq i \leq n)$ and $f(v_{n+1}v_{n+2}) = 2n + 1, f(v_{n+2}v_{n+3}) = 2n + 5, f(v_{n+3}v_{n+4}) = 2n + 7, f(v_{n+4}v_{n+1}) = 2n + 3$.

Obviously $f(v_1) = 0$ and $f(v_{n+3}) = 2n + 7$. Thus f is a function from $V(G)$ to the set $\{0, 1, 2, 3 \ldots, 2n + 7\}$. Clearly f is one-one.

Next we find the vertex labels.

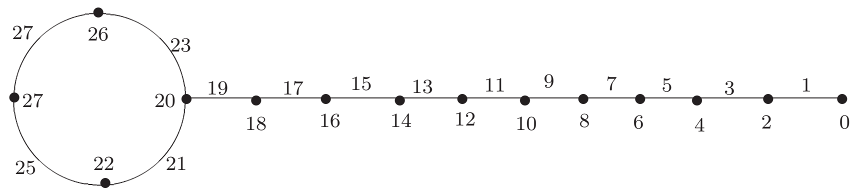
For $1 \leq i \leq n$, the labels of $f(v_i)$ are in the set, $A_1 = \{0, 2, 4, \ldots, 2n - 2\}$. The set A_1 has n labels. The other labels of $f(v_{n+1}), f(v_{n+2}), f(v_{n+3}), f(v_{n+4})$ respectively are in the set, $A_2 = \{2n, 2n+2, 2n+7, 2n+6\}$. The set A_2 has 4 labels. Thus the vertex labels of $f(v_i)$ are in the set $A = A_1 \cup A_2$. Thus the set A has $n+4$ labels.

Next we find the edge labels $f(v_i v_{i+1})$.

For $1 \leq i \leq n$, the labels of $f(v_i v_{i+1})$ are in the set, $B_1 = \{1, 3, 5, \ldots, 2n - 1\}$. The set B_1 has n labels. The other labels of $f(v_{n+1}v_{n+2})$, $f(v_{n+2}v_{n+4})$, $f(v_{n+3}v_{n+4})$, $f(v_{n+4}v_1)$ respectively are in the set, $B_2 = \{2n + 1, 2n + 5, 2n + 7, 2n + 3\}$. The set B_2 has 4 labels. Thus the edge labels are in the set $B = B_1 \cup B_2$. The set B has $n + 4$ labels.

Hence G is an odd mean graph. □

Example 2.3.11. Odd mean labeling of $C_4@P_{10}$.



Theorem 2.3.12. *nC_4 is an odd mean graph.*

Proof. Let $V(nC_4) = \{v_j^i : 1 \leq j \leq 4 \text{ and } 1 \leq i \leq n\}$.

Define $f : V(nC_4) \to \{0, 1, 2, 3, \ldots, 8n-1\}$ by

$f(v_1^{(i)}) = 8i - 8(1 \leq i \leq n), f(v_2^{(i)}) = 8i - 6(1 \leq i \leq n)$, $f(v_3^{(i)}) = 8i - 1, (1 \leq i \leq n) f(v_4^{(i)}) = 8i - 2(1 \leq i \leq n)$.

The label of the edge $v_1^{(i)}v_2^{(i)}$ is $8i-7(1 \leq i \leq n)$. The label of the edge $v_1^{(i)}v_4^{(i)}$ is $8i-5(1 \leq i \leq n)$. The label of the edge $v_2^{(i)}v_3^{(i)}$ is $8i-3(1 \leq i \leq n)$. The label of the edge $v_3^{(i)}v_4^{(i)}$ is $8i-1(1 \leq i \leq n)$.

Obviously $f(v_1^{(1)}) = 0$ and $f(v_3^{(n)}) = 8n-1$. Thus f is a function from $V(G)$ to the set $\{0, 1, 2, 3, \ldots, 8n-1\}$. Clearly f is one-one.

Next we find the vertex labels $f(v_j^{(i)})$.

For $1 \leq i \leq n$, the labels of $f(v_1^{(i)})$ are in the set $A_1 = \{0, 8, 16, 24, \ldots, 8n-8\}$. The set A_1 has n labels.

For $1 \leq i \leq n$, the labels of $f(v_2^{(i)})$ are in the set $A_2 = \{2, 10, 18, \ldots, 8n-6\}$. The set A_2 has n labels.

For $1 \leq i \leq n$, the labels of $f(v_3^{(i)})$ are in the set $A_3 = \{7, 15, 23, \ldots, 8n-1\}$. The set A_3 has n labels.

For $1 \leq i \leq n$, the labels of $f(v_4^{(i)})$ are in the set $A_4 = \{6, 14, 22, \ldots, 8n-2\}$. The set A_4 has n labels.

Thus the vertex labels of $f(v_j^{(i)})$ are in the set $A = A_1 \cup A_2 \cup A_3 \cup A_4$. The set A has $4n$ labels.

Next we find the edge labels.

For $1 \leq i \leq n$, the labels of $f(v_1^{(i)} v_2^{(i)})$ are in the set, $B_1 = \{1, 9, 17, \ldots, 8n-7\}$. The set B_1 has n labels.

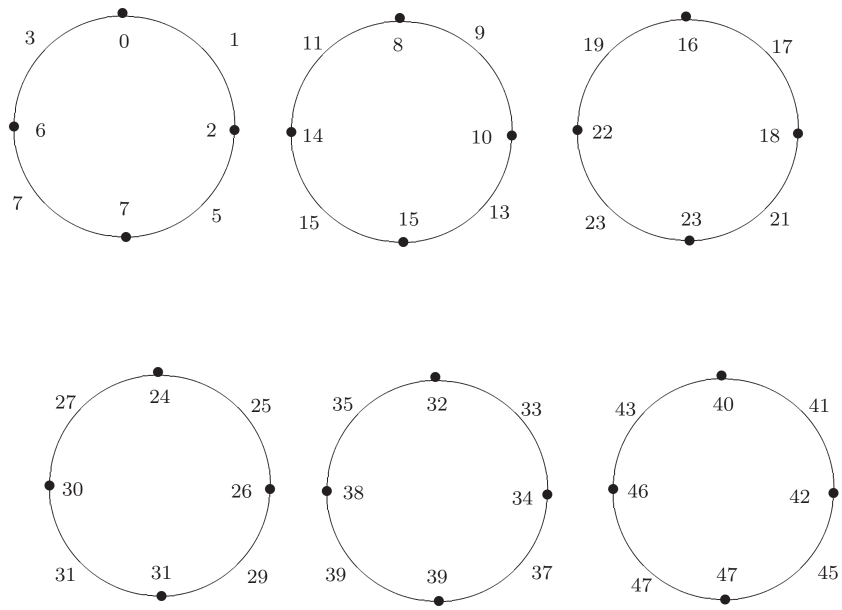
For $1 \leq i \leq n$, the labels of $f(v_1^{(i)} v_4^{(i)})$ are in the set, $B_2 = \{3, 11, 19, \ldots, 8n-5\}$. The set B_2 has n labels.

For $1 \leq i \leq n$, the labels of $f(v_2^{(i)} v_3^{(i)})$ are in the set, $B_3 = \{5, 13, 21, \ldots, 8n-3\}$. The set B_3 has n labels.

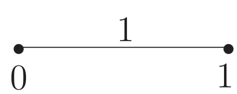
For $1 \leq i \leq n$, the labels of $f(v_3^{(i)} v_4^{(i)})$ are in the set, $B_4 = \{7, 15, 23, \ldots, 8n-1\}$. The set B_4 has n labels. Thus the edge labels are in the set $B = B_1 \cup B_2 \cup B_3 \cup B_4$. The set B has $4n$ labels.

Hence nC_4 is an odd mean graph. $\square$

Example 2.3.13. Odd mean labeling of $6C_4$.



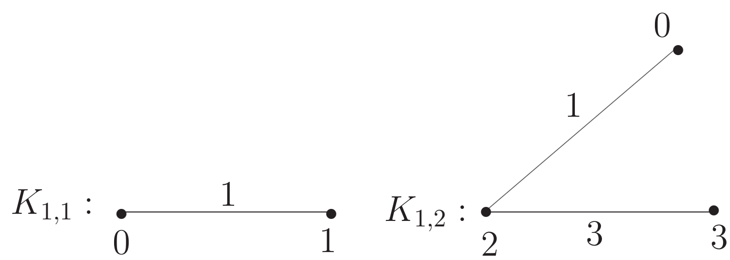
Note. Clearly K_1 and K_2 are odd mean graphs, the labeling being



Theorem 2.3.14. *K_n is not an odd mean graph for $n \geq 3$.*

Proof. Suppose $K_n (n \geq 3)$ is an odd mean graph. To get the edge label $2q-1$, we must have $2q-1$ and $2q-2$ as the labels of adjacent vertices. Let u and v be the vertices whose labels are $2q-1$ and $2q-2$ respectively. To get the edge label 1, we must have 0 and 1 as vertex labels (or) 0 and 2 as vertex labels of adjacent vertices. In either case 0 must be a label of some vertex, say w. Now the edges uw and vw get labels q and $q-1$ which are consecutive integers. This contradiction proves that K_n is not an odd mean graph for $n \geq 3$. □

Note. Clearly $K_{1,1}$ and $K_{1,2}$ are odd mean graphs, the labeling being



Theorem 2.3.15. *If $n \geq 3, K_{1,n}$ is not an odd mean graph.*

Proof. Let $\{V_1, V_2\}$ be the bipartition of $K_{1,n}$ with $V_1 = \{u\}$. To get the edge label $2q-1$, we must have $2q-1$ and $2q-2$ as the labels of adjacent vertices. Thus either $2q-1$ or $2q-2$ must be a label of u. In both cases, since $n \geq 3$, there will be no edge with label 1. This contradiction proves that $K_{1,n}$ is not an odd mean graph. □

Theorem 2.3.16. *$K_{2,n}$ is an odd mean graph for all n.*

Proof. Let $\{V_1, V_2\}$ be the bipartition of $K_{2,n}$ with $V_1 = \{u, v\}$ and $V_2 = \{u_1, u_2, u_3, \ldots, u_n\}$.

Define $f : V(K_{2,n}) \to \{0, 1, 2, \ldots, (4n-1)\}$ by

$f(u) = 0,\ f(v) = 4n-1$, and $f(u_i) = 4i-2, (1 \leq i \leq n)$.

The label of the edge uu_i is $2i-1 (1 \leq i \leq n)$ and the label of the edge vu_i is $2n+2i-1 (1 \leq i \leq n)$.

Obviously $f(u) = 0$ and $f(v) = 4n-1$. Thus f is a function from the set $V(K_{2,n})$ to the set $\{0, 1, 2, 3, \ldots, 4n-1\}$. Clearly f is one-one.

Next we find the vertex labels.

For $1 \leq i \leq n$, the labels of $f(u_i)$ are in the set $A_1 = \{2, 6, 10, 14, \ldots, 4n-2\}$. The set A_1 has n labels.

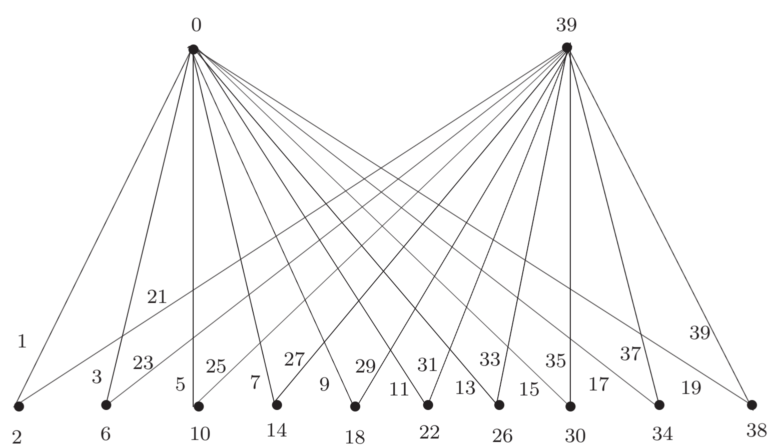
The other labels are $f(u) = 0$ and $f(v) = 4n - 1$. Thus the vertex labels are in the set $A = A_1 \cup \{0, 4n-1\}$. Therefore the set A has $n + 2$ labels.

Next we find the edge labels $f(uu_i), f(vu_i)$.

For $1 \leq i \leq n$, the labels of $f(uu_i)$ are in the set $B_1 = \{1, 3, 5, \ldots, (2n-1)\}$. The set B_1 has n labels. For $1 \leq i \leq n$, the labels of $f(vu_i)$ are in the set, $B_2 = \{2n+1, 2n+3, 2n+5, \ldots, 4n-1\}$. The set B_2 has n labels. Thus the edge labels are in the set $B = B_1 \cup B_2$. The set B has $2n$ labels.

Hence $K_{2,n}$ is an odd mean graph. □

Example 2.3.17. Odd mean labeling of $K_{2,10}$.



Definition 2.3.18. K_2 with n pendent edges attached at each point is called a bistar and is denoted by $B_{n,n}$.

Theorem 2.3.19. *The Bistar $B_{n,n}$ is an odd mean graph for all n.*

Proof. Let $V(K_2) = \{u, v\}$ and u_i, v_i be the vertices adjacent to u and v respectively $(1 \leq i \leq n)$.

Define $f : V(B_{n,n}) \to \{0, 1, 2, \ldots, (4n + 1)\}$ by

$f(u) = 0, f(v) = 4n + 1, f(u_i) = 4i - 2(1 \leq i \leq n)$ and $f(v_i) = 4i(1 \leq i \leq n)$.

The label of the edge uv is $2n+1$. The label of the edge uu_i is $2i-1, (1 \leq i \leq n)$. The label of the edge vv_i is $(2n+1)+2i, (1 \leq i \leq n)$.

Clearly $f(u) = 0$ and $f(v) = 4n + 1$. Thus f is a function $f : V(B_{n,n})$ to the set $\{0, 1, 2, \ldots, (4n + 1)\}$. Clearly f is one-one.

Next we find the vertex labels.

For $1 \leq i \leq n$, the labels of $f(u_i)$ are in the set $A_1 = \{2, 6, 10, \ldots, 4n - 2\}$. The set A_1 has n labels. For $1 \leq i \leq n$, the labels of $f(v_i)$ are in the set $A_2 = \{4, 8, 12, 16, \ldots, 4n\}$. The set A_2 has n labels. The other labels are in the set $A_3 = \{0, 4n + 1\}$. The set A_3 has 2 labels. Thus the vertex labels are in the set $A = A_1 \cup A_2 \cup A_3$. The set A has $2n + 2$ labels.

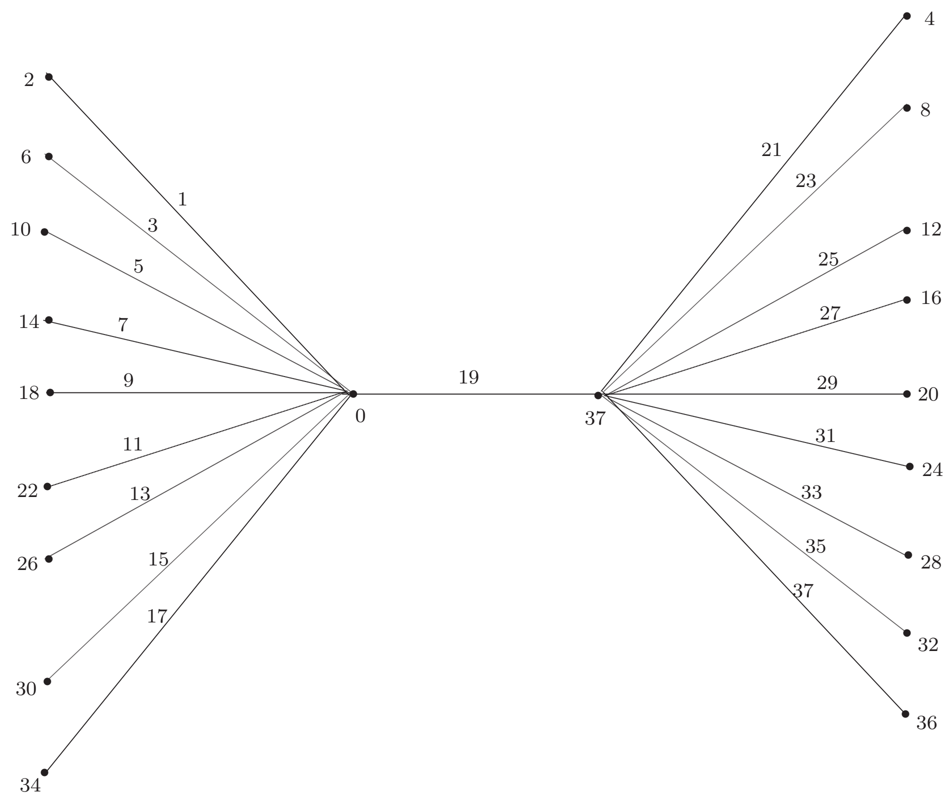
Next we find the edge labels.

For $1 \leq i \leq n$, the labels of $f(uu_i)$ are in the set, $B_1 = \{1, 3, 5, \ldots, 2n - 1\}$. The set B_1 has n labels. For $1 \leq i \leq n$, the labels of $f(vv_i)$ are in the set $B_2 = \{2n+3, 2n+5, 2n+7, \ldots, 4n+1\}$.

The other label is the set, $B_3 = \{2n+1\}$. The set B_3 has 1 label. The edge labels are in the set $B = B_1 \cup B_2 \cup B_3$. The set B has $2n+1$ labels.

Hence $B_{n,n}$ is an odd mean graph. □

Example 2.3.20. Odd mean labeling of $B_{9,9}$.



Definition 2.3.21. The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p points) and p copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

Remark 2.3.22. $P_n \odot K_1$ is called *comb*.

Theorem 2.3.23. *Combs are odd mean graphs.*

Proof. Let G be the comb obtained from a path $P_n : v_1, v_2, \ldots, v_n$ by joining a vertex u_i to $v_i (1 \leq i \leq n)$.

Define $f : V(G) \to \{0, 1, 2, \ldots, (4n-3)\}$ by

$f(v_i) = 4i - 3 (1 \leq i \leq n)$ and $f(u_i) = 4i - 4 (1 \leq i \leq n)$.

The label of the edge $v_i v_{i+1}$ is $4i - 1 (1 \leq i \leq n-1)$. The label of the edge $u_i v_i$ is $4i - 3 (1 \leq i \leq n)$.

$$
\begin{aligned}
\min_{v \in V(G)} f(v) &= \min_{v \in V(G)} \{f(v_i), f(u_i) : 1 \leq i \leq n\} \\
&= \min\{f(u_i) = 4i - 4 : 1 \leq i \leq n\} \\
&= 4i - 4 \text{ if } i = 1 \\
&= 0. \\
\max_{v \in V(G)} f(v) &= \max_{v \in V(G)} \{f(v_i), f(u_i) : 1 \leq i \leq n\} \\
&= \max\{f(v_i) = 4i - 3 : 1 \leq i \leq n\} \\
&= 4i - 3 \text{ if } i = n \\
&= 4n - 3.
\end{aligned}
$$

Thus f is a function from $V(G)$ to the set $\{0, 1, 2, \ldots, 4n-3\}$. Clearly f is one-one.

Next we find the vertex labels $f(v_i)$ and $f(u_i)$.

For $1 \leq i \leq n$, the labels of $f(v_i)$ are in the set, $A_1 = \{1, 5, 9, \ldots, 4n-3\}$. The set A_1 has n labels. For $1 \leq i \leq n$, the labels of $f(u_i)$ are in the set, $A_2 = \{0, 4, 8, 12, \ldots, 4n-4\}$. The set A_2 has n labels. Thus the vertex labels are in the set, $A = A_1 \cup A_2$.

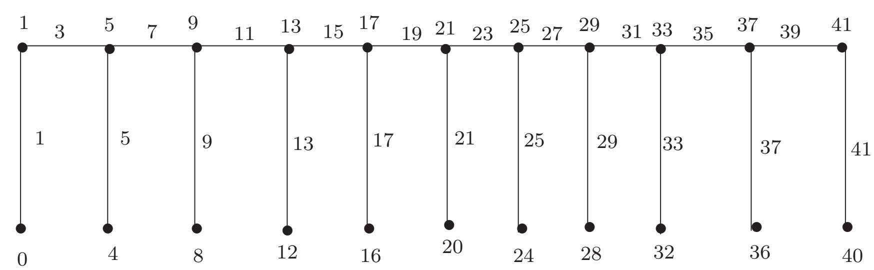
The set A has $2n$ labels.

Next we find the edge labels $f(v_iv_{i+1})$ and $f(u_iv_i)$.

For $1 \le i \le n-1$, the labels of $f(v_iv_{i+1})$ are in the set, $B_1 = \{3, 7, 11, \ldots, 4n-5\}$. The set B_1 has $n-1$ labels. For $1 \le i \le n$, the labels of $f(u_iv_i)$ are in the set, $B_2 = \{1, 5, 9, \ldots, 4n-3\}$. The set B_2 has n labels. Thus the edge labels are in the set $B = B_1 \cup B_2$. The set B has $2n-1$ labels.

Hence G is an odd mean graph. □

Example 2.3.24. Odd mean labeling of Comb.



Theorem 2.3.25. *$P_n \odot K_2$ is an odd mean graph.*

Proof. Let $u_i (1 \le i \le n)$ be the vertices of a path P_n and v_i, w_i be the vertices which are made adjacent with u_i. Then G has $3n-1$ edges.

Define $f : V(G) \to \{0, 1, 2, 3, \ldots, (6n-3)\}$ as follows.

For $1 \le i \le n-1$, let $f(u_i) = \begin{cases} 6i-6 & \text{if } i \text{ is odd} \\ 6i-2 & \text{if } i \text{ is even} \end{cases}$

and let $f(u_n) = 6n-3$.

For $1 \le i \le n-1$, let $f(v_i) = \begin{cases} 6i-4 & \text{if } i \text{ is odd} \\ 6i-8 & \text{if } i \text{ is even} \end{cases}$ and let $f(v_n) = 6n-11$ or $6n-8$ according as n is odd or even.

For $1 \le i \le n-1$, let $f(w_i) = \begin{cases} 6i & \text{if } i \text{ is odd} \\ 6i-4 & \text{if } i \text{ is even} \end{cases}$ and let $f(w_n) = 6n-4$.

The label of the edge u_iu_{i+i} is $6i-1(1 \le i \le n-1)$ and $u_{n-1}u_n$ is $6n-5$ if n is odd. The label of the edge u_iv_i is $6i-5(1 \le i \le n)$ and u_nv_n is $6n-7$ if n is odd. The label of the edge u_iw_i is $6i-3(1 \le i \le n)$.

$$\begin{aligned} \min V(G) &= \min_{V(G)}\{f(u_i), f(v_i), f(w_i) : 1 \le i \le n\} \\ &= \min\{f(u_i) = 6i-6 \text{ if } i = 1\} \\ &= 0. \\ \max V(G) &= \max_{V(G)}\{f(u_i), f(v_i), f(w_i) : 1 \le i \le n\} \\ &= \max\{f(u_n) = 6n-3\} \\ &= 6n-3. \end{aligned}$$

Thus f is a function from $V(G)$ to the set $\{0, 1, 2, 3 \ldots, 6n-3\}$. Clearly f is one-one.

Next we find the vertex labels.

For $1 \le i \le n-1$ and i is odd, the labels of $f(u_i)$ are in the set, $A_1 = \{0, 12, 24, \ldots, 6n-18$ or $6n-12$ according as n is odd or even$\}$. The set A_1 has $\frac{n}{2}$ labels.

For $1 \leq i \leq n-1$ and i is even, the labels of $f(u_i)$ are in the set, $A_2 = \{10, 22, 34, \ldots, 6n-8$ or $6n-14$ according as n is odd or even$\}$. The set A_2 has $\frac{n-2}{2}$ labels. The label of $f(u_n)$ is the set $A_3 = \{6n-3\}$. The set A_3 has 1 label.

For $1 \leq i \leq n-1$ and i is odd, the labels of $f(v_i)$ are in the set $A_4 = \{2, 14, 26, \ldots, 6n-16$ or $6n-10$ according as n is odd or even$\}$. The set A_4 has $\frac{n}{2}$ labels.

For $1 \leq i \leq n-1$ and i is even, the labels of $f(v_i)$ are in the set $A_5 = \{4, 16, 28, \ldots, 6n-14$ or $6n-20\}$. The set A_5 has $\frac{n-2}{2}$ labels. The label of $f(v_n)$ is the set, $A_6 = \{6n-11$ or $6n-8$ according as n is odd or even$\}$. The set A_6 has 1 label.

For $1 \leq i \leq n-1$ and i is odd, the labels of $f(w_i)$ are in the set, $A_7 = \{6, 18, 30, \ldots, 6n-12$ or $6n-6\}$. The set A_7 has $\frac{n}{2}$ labels.

For $1 \leq i \leq n-1$ and i is even, the labels of $f(w_i)$ are in the set, $A_8 = \{8, 20, 32, \ldots, 6n-10$ or $6n-16$ according as n is odd or n is even$\}$. The set A_8 has $\frac{n-2}{2}$ labels. The label of $f(w_n)$ is the set $A_9 = \{6n-4\}$. The set A_9 has 1 label.

The vertex labels of $V(G)$ are in the set, $A = A_1 \cup A_2 \cup A_3 \cup \cdots A_9$. The set A has $3n$ labels.

Next we find the edge labels.

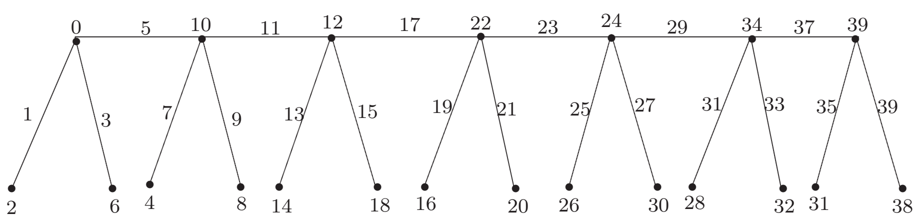
For $1 \leq i \leq n-1$, the labels of the edges $f(u_i u_{i+1})$ are in the set, $B_1 = \{5, 11, 17, \ldots, 6n-5$ or $6n-7$ according as n is odd or even}. The set B_1 has $n-1$ labels.

For $1 \leq i \leq n$, the labels of the edges $f(u_iv_i)$ are in the set, $B_2 = \{1, 7, 13, \ldots, 6n-7$ or $6n-5$ according as n is odd or even}. The set B_2 has n labels.

For $1 \leq i \leq n$, the labels of the edges $f(u_iw_i)$ are in the set $B_3 = \{3, 9, 15, \ldots, 6n-3\}$. The set B_3 has n labels. The edge labels are in the set $B = B_1 \cup B_2 \cup B_3$. The set has $3n-1$ labels.

Hence $P_n \odot K_2$ is an odd mean graph. □

Example 2.3.26. Odd mean labeling of $P_7 \odot K_2$.



Definition 2.3.27. A *quadrilateral snake* is obtained from a path $u_1u_2 \ldots u_n$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and joining v_i and w_i. That is, every edge of the path is replaced by the cycle.

Theorem 2.3.28. *A quadrilateral snake is an odd mean graph.*

Proof. Let Q_n denote the quadrilateral snake obtained from $u_1u_2 \ldots u_n$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and joining v_i and w_i.

Define $f : V(Q_n) \to \{0, 1, 2, 3, \ldots, (8n-9)\}$ as follows:

$$\text{For } 1 \le i \le n, f(u_i) = \begin{cases} 8i-8 & \text{if } i \text{ is odd} \\ 8i-10 & \text{if } i \text{ is even} \end{cases}$$

$f(u_n) = 8n - 9$ if n is odd and $n > 1$.

$$\text{For } 1 \le i \le n-1, f(v_i) = \begin{cases} 8i-6 & \text{if } i \text{ is odd} \\ 8i-4 & \text{if } i \text{ is even} \end{cases}$$

$$\text{For } 1 \le i \le n-1, f(w_i) = \begin{cases} 8i & \text{if } i \text{ is odd} \\ 8i-2 & \text{if } i \text{ is even} \end{cases}$$

$f(w_{n-1}) = 8n - 9$ if n is even and $n \ge 2$.

The label of the edge $u_i u_{i+1}$ is $8i-5$, if $1 \le i \le n-1$. The label of the edge $v_i w_i$ is $8i - 3$, if $1 \le i \le n-1$. The label of the edge $u_i v_i$ is $8i - 7$, if $1 \le i \le n-1$. The label of the edge u_i, w_{i-1} is $8i - 9$ if $2 \le i \le n$.

Obviously $f(u_1) = 0$ and $f(u_n)$ or $f(w_{n-1})$ is $8n - 9$ according as n is odd or n is even. Thus f is a function from $V(Q_n)$ to the set $\{0, 1, 2, 3, \ldots, (8n - 9)\}$.

Next we find the vertex labels.

For $1 \le i \le n$, i is odd and n is odd, the labels of $f(u_i)$ are in the set, $A_1 = \{0, 16, 32, \ldots, 8n - 24, 8n - 9\}$. The set A_1 has $\frac{n+1}{2}$ labels.

For $1 \le i \le n$ and i is even, the labels of $f(u_i)$ are in the set, $A_2 = \{6, 22, 38, \ldots, 8n - 18\}$. The set A_2 has $\frac{n-1}{2}$ labels. The labels of $f(u_i)$ are in the set $A = A_1 \cup A_2$. The set A has n labels.

For $1 \leq i \leq n, i$ is odd and n is even, the labels of $f(u_i)$ are in the set $B_1 = \{0, 16, 32, \ldots, 8n-16\}$. The set B_1 has $\frac{n}{2}$ labels.

For $1 \leq i \leq n, i$ is even and n is even, the labels of $f(u_i)$ are in the set $B_2 = \{6, 22, 38, \ldots, 8n-10\}$. The set B_2 has $\frac{n}{2}$ labels. The vertex labels of $f(u_i)$ are in the set $B = B_1 \cup B_2$. The set B has n labels.

The labels of $f(u_i)$ are in the set, either A or B according as n is odd or even.

For $1 \leq i \leq n-1$ and i is odd, the labels of $f(v_i)$ are in the set $C_1 = \{2, 18, 34, \ldots, 8n-22$ or $8n-14$ according as n is odd or even$\}$. The set C_1 has $\frac{n-1}{2}$ labels.

For $1 \leq i \leq n-1$ and i is even, the labels of $f(v_i)$ are in the set $C_2 = \{12, 28, 44, \ldots, 8n-12$ or $8n-14$ according as n is odd or even$\}$. The set C_2 has $\frac{n-1}{2}$ labels. The labels of $f(v_i)$ are in the set $C = C_1 \cup C_2$. Thus the set C has $(n-1)$ labels.

For $1 \leq i \leq n-1$ and i is odd, the labels of $f(w_i)$ are in the set $D_1 = \{8, 24, 40, \ldots, 8n-16, 8n-10$ or $8n-9$ according as n is odd or even$\}$. The set D_1 has $\frac{n-1}{2}$ labels.

For $1 \leq i \leq n-1$ and i is even, the labels of $f(w_i)$ are in the set $D_2 = \{14, 30, 46, \ldots, 8n-26, 8n-10$ or $8n-9$ according as n is odd or even$\}$. The set D_2 has $\frac{n-1}{2}$ labels. The vertex labels of $f(w_i)$ is the set $D = D_1 \cup D_2$. The set D has $(n-1)$ labels.

Thus the vertex labels of $V(Q_n)$ is the set $E =$ (either A

or $B) \cup C \cup D$. Therefore the set E has $3n-2$ labels.

Next we find the edge labels.

For $1 \le i \le n-1$, the labels of $f(u_iu_{i+1})$ is in the set $F_1 = \{3, 11, 19, \ldots, 8n-13\}$. The set F_1 has $n-1$ labels.

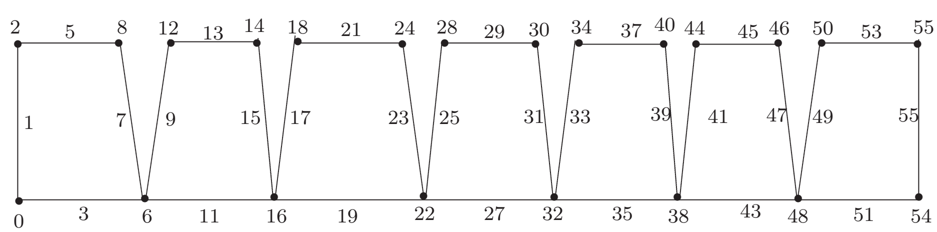
For $1 \le i \le n-1$, the labels of $f(v_iw_i)$ is the set $F_2 = \{5, 13, 21, \ldots, 8n-11\}$. The set F_2 has $n-1$ labels.

For $1 \le i \le n-1$, the labels of $f(u_iv_i)$ is the set $F_3 = \{1, 9, 17, \ldots, 8n-15\}$. The set F_3 has $n-1$ labels.

For $2 \le i \le n$, the labels of $f(u_iw_{i-1})$ is the set $F_4 = \{7, 15, 23, \ldots, 8n-9\}$. The set F_4 has $n-1$ labels. Thus the edge labels $E(Q_n)$ is the set $F = F_1 \cup F_2 \cup F_3 \cup F_4$. The set F has $4n-4$ labels.

Hence Q_n is an odd mean graph. □

Example 2.3.29. Odd mean labeling of Q_8.



Theorem 2.3.30. *The graph G consisting of vertices $u_i, v_i (0 \le i \le n)$ with edges u_iu_{i+1}, v_iv_{i+1} for $i = 0, 1, 2, 3, \ldots, (n-1)$ and u_iv_i for $i = 1, 2, 3, \ldots, (n-1)$ is an odd mean graph.*

Proof. Let G be the given graph with

$V(G) = \{u_i, v_i / i = 0, 1, 2, \ldots, n\}$ and

$E(G) = \{u_i u_{i+1}, v_i v_{i+1} / i = 0, 1, 2, \ldots, (n-1)\}$

$\cup \{u_i v_i / i = 1, 2, 3, \ldots, (n-1\}$.

Define $f : V(G) \to \{0, 1, 2, 3, \ldots, (6n-3)\}$ by

$f(u_i) = 2i, 0 \leq i \leq n, f(v_i) = 4n - 2 + 2i, 0 \leq i \leq n-1$ and $f(v_n) = 6n - 3$.

The label of the edge $u_i u_{i+1}$ is $2i + 1, 0 \leq i \leq n - 1$. The label of the edge $v_i v_{i+1}$ is $4n + 2i - 1, 0 \leq i \leq n - 1$. The label of the edge $u_i v_i$ is $2n + 2i - 1, 1 \leq i \leq n - 1$.

Obviously $f(u_0) = 0$ and $f(v_n) = 6n - 3$. Thus f is a function from $V(G)$ to the set $\{0, 1, 2, 3, \ldots, (6n-3)\}$. Clearly f is one-one.

Next we find the vertex labels.

For $0 \leq i \leq n$, the labels of $f(u_i)$ are in the set $A_1 = \{0, 2, 4, \ldots, 2n\}$. The set A_1 has $n + 1$ labels.

For $0 \leq i \leq n - 1$, the labels of $f(v_i)$ are in the set $A_2 = \{4n - 2, 4n, 4n + 2, \ldots, 6n - 4\}$. The set A_2 has n labels. The other label $f(v_n)$ is $6n - 3$. Thus, the vertex labels are in the set $A = A_1 \cup A_2 \cup \{6n - 3\}$. The set A has $2n + 2$ labels.

Next we find the edge labels of $f(u_i u_{i+1})$, $f(v_i v_{i+1})$ and $f(u_i v_i)$.

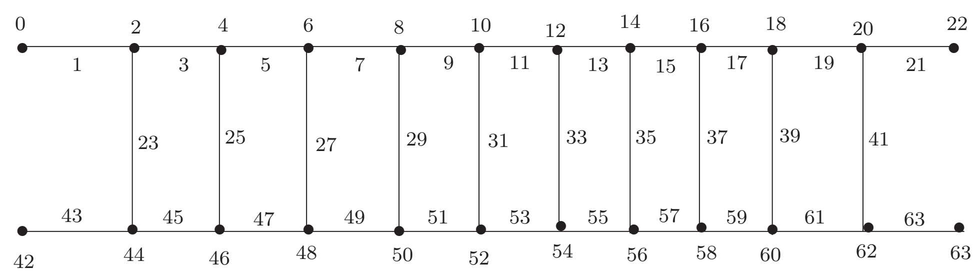
For $0 \leq i \leq n-1$, the labels of $f(u_i u_{i+1})$ are in the set $B_1 = \{1, 3, 5, \ldots, 2n-1\}$. The set B_1 has n labels.

For $0 \leq i \leq n-1$, the labels of $f(v_i v_{i+1})$ are in the set $B_2 = \{4n-1, 4n+1, 4n+3, \ldots, 6n-3\}$. The set B_2 has n labels.

For $1 \leq i \leq n-1$, the labels of $f(u_i v_i)$ are in the set $B_3 = \{2n+1, 2n+3, 2n+5, \ldots, 4n-3\}$. The set B_3 has $(n-1)$ labels. Thus the edge labels are in the set, $B = B_1 \cup B_2 \cup B_3$. Therefore the set B has $3n-1$ labels.

Hence G is an odd mean graph. $\square$

Example 2.3.31. Odd mean labeling of G.



Definition 2.3.32. For two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, $G_1 \times G_2$ is defined as the graph with vertex set as $V_1 \times V_2$ such that two points $u = (u_1, v_1)$ and $v = (u_2, v_2)$ in $V_1 \times V_2$ are adjacent in $G_1 \times G_2$ whenever $u_1 = v_1$ and u_2 is adjacent to v_2 in G_2 or $u_2 = v_2$ and u_1 is adjacent to v_1 in G_1.

Definition 2.3.33. The graph $P_m \times P_n$ is called a *planar grid.*

Theorem 2.3.34. *The planar grid $P_m \times P_n$ is an odd mean graph for $m \geq 2$ and $n \geq 2$.*

Proof. Let $V(P_m \times P_n) = \{a_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(P_m \times P_n) = \{a_{i,j-1}a_{ij} : 1 \leq i \leq m, 2 \leq j \leq n\} \cup \{a_{i-1j}a_{ij} : 2 \leq i \leq m, 1 \leq j \leq n\}$.

Define $f : V(P_m \times P_n) \to \{0, 1, 2, \ldots, (4mn-2m-2n-1)\}$ by

$f(a_{ij}) = 2(j-1), 1 \leq j \leq n$ and $f(a_{ij}) = f(a_{i-1,n}) + 2(n-1) + 2j$ where $2 \leq i \leq m, 1 \leq j \leq n$, $f(a_{mn}) = 4mn - 2m - 2n - 1$. The label of the edge $a_{ij}a_{ij+1}$ is $(i-1)(4n-3) + i + 2j - 2$ where $1 \leq i \leq m, 1 \leq j \leq n-1$. The label of the edge $a_{ij}a_{i+1j}$ is $(2i-2)(2n-1) + 2n - 3 + 2j, 1 \leq i \leq m-1, 1 \leq j \leq n$.

Obviously $f(a_{11}) = 0$ and $f(a_{mn}) = 4mn - 2m - 2n - 1$. Thus f is a function from $V(P_m \times P_n)$ to the set $\{0, 1, 2, 3, \ldots, 4mn-2m-2n-1\}$. Next we find the vertex labels $f(a_{ij})$. For $1 \leq i \leq m, 1 \leq j \leq n$, the vertex labels of $f(a_{ij})$ are arranged as follows. This arrangement has mn labels.

j i	1	2	3	4	......	n
1	0	2	4	6	......	$2n-2$
2	$4n-2$	$4n$	$4n+2$	$4n+4$	......	$6n-4$
3	$8n-4$	$8n-2$	$8n$	$8n+2$	......	$10n-6$
4	$12n-6$	$12n-4$	$12n-2$	$12n$	......	$14n-8$
	$\vdots$					
	$\vdots$					
m	$4mn-2m-4n+2$	$4mn-2m-4n+4,$			......,	$4mn-2m-2n-1$

We have mn labels.

For $1 \leq i \leq m, 1 \leq j \leq n-1$, the labels of the edges $(a_{ij}a_{ij+1})$ are arranged as follows. This arrangement has $m(n-1)$ labels.

1	3	5	7	$\ldots\ldots$	$2n-3$
$4n-1$	$4n+1$	$4n+3$	$4n+5$	$\ldots\ldots$	$6n-5$
$8n-3$	$8n-1$	$8n+1$	$8n+3$	$\ldots\ldots$	$10n-7$
$12n-5$	$12n-3$	$12n-1$	$12n+1$	$\ldots\ldots$	$14n-9$
$\vdots$					
$\vdots$					

$4mn-2m-4n+3$ $4mn-2m-4n+5, \ldots\ldots, 4mn-2m-2n+1$

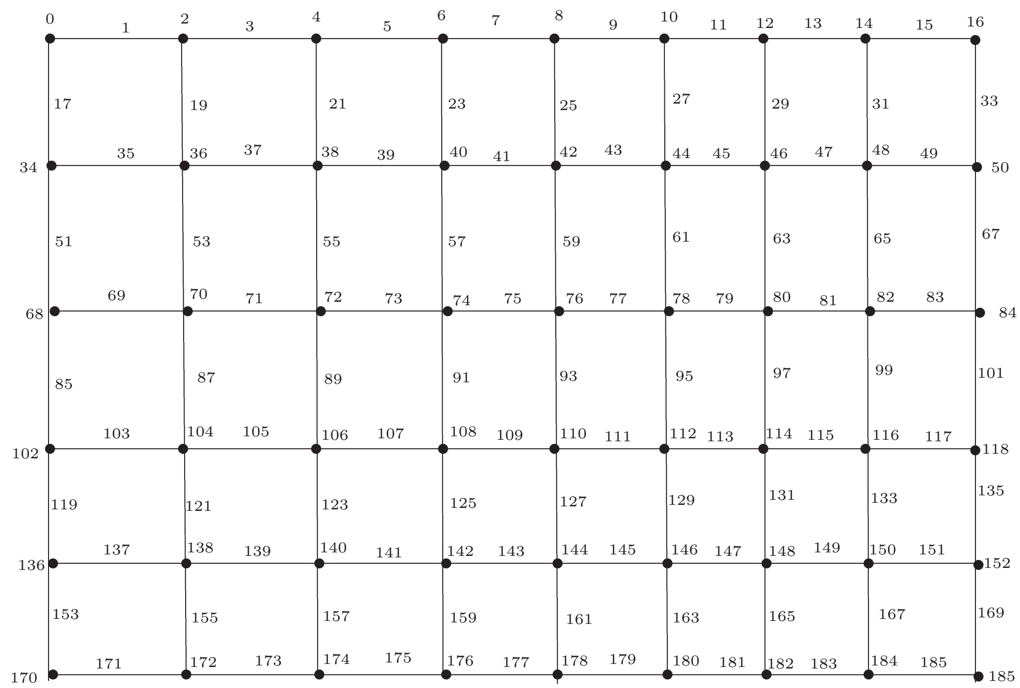
For $1 \leq i \leq m-1, 1 \leq j \leq n$, the labels of edges $f(a_{ij}a_{i+1j})$ are arranged as follows. This arrangement has $(m-1)n$ labels.

$2n-1$	$2n+1$	$2n+3$	$2n+5$	$\ldots\ldots$	$4n-3$
$6n-3$	$6n-1$	$6n+1$	$6n+3$	$\ldots\ldots$	$8n-5$
$10-5$	$10n-3$	$10n-1$	$10n+1$	$\ldots\ldots$	$12n-7$
$14n-7$	$14n-5$	$14n-3$	$14n-1$	$\ldots\ldots$	$16n-9$
$\vdots$					
$\vdots$					

$4mn-2m-6n+3$ $4mn-2m-6n+5, \ldots\ldots, 4mn-2m-4n+1$

Thus we have $m(n-1)+(m-1)n = 2mn-m-n$ edge labels. Hence $P_m \times P_n$ is an odd mean graph for $m \geq 2$ and $n \geq 2$. □

Example 2.3.35. Odd mean labeling of $P_6 \times P_9$.



Remark 2.3.36. $P_n \times P_2$ is called a *ladder* and is denoted by L_n.

Corollary 2.3.37. *L_n is odd mean graph for all n.*

Theorem 2.3.38. *The graph obtained by appending an edge to each vertex of a ladder is an odd mean graph.*

Proof. Let $P_n \times K_2$ be the ladder. Let G be the graph obtained by appending an edge to each vertex of the ladder. Let u_i and v_i be the vertices of the ladder. For $1 \leq i \leq n$, let u_i' and v_i' be the new vertices made adjacent with u_i and v_i respectively.

Define $f : V(G) \to \{0, 1, 2, 3, \ldots, (10n - 5)\}$ by

$f(u_i) = 10i - 8(1 \leq i \leq n), f(v_i) = 10i - 6(1 \leq i \leq n-1), f(u_i') = 10i - 10(1 \leq i \leq n), f(v_i') = 10i - 4, (1 \leq i \leq n)$ and $f(v_n) = 10n - 5$.

The label of the edge u_iu_{i+1} is $10i - 3(1 \leq i \leq n-1)$. The label of the edge v_iv_{i+1} is $10i - 1(1 \leq i \leq n-1)$. The label of the edge u_iu_i' is $10i - 9(1 \leq i \leq n)$. The label of the edge v_i, v_i' is $10i - 5(1 \leq i \leq n)$. The label of the edge u_iv_i is $10i - 7(1 \leq i \leq n)$.

$$\begin{aligned} \min V(G) &= \min\{f(u_i), f(v_i), f(u_i'), f(v_i') : 1 \leq i \leq n\} \\ &= \min\{f(u_i') = 10i - 10 : 1 \leq i \leq n\} \\ &= 10i - 10 \text{ if } i = 1 \\ &= 0. \end{aligned}$$

Obviously $f(v_n) = 10n - 5$. Thus f is a function from $V(G)$ to the set $\{0, 1, 2, 3, \ldots, 10n - 5\}$. Clearly f is one-one.

Next we find the vertex labels.

For $1 \leq i \leq n$, the labels of $f(u_i)$ are in the set $A_1 = \{2, 12, 22, \ldots, 10n - 8\}$. The set A_1 has n labels.

For $1 \leq i \leq n-1$, the labels of $f(v_i)$ are in the set $A_2 = \{4, 14, 24, \ldots, 10n - 16\}$. The set A_2 has $n - 1$ labels.

For $1 \leq i \leq n$, the labels of $f(u_i')$ is the set $A_3 = \{0, 10, 20, \ldots, 10n - 10\}$. The set A_3 has n labels.

For $1 \leq i \leq n$, the labels of $f(v_i')$ is the set $A_4 = \{6, 16, 26, \ldots, 10n - 4\}$. The set A_4 has n labels. The labels of $f(v_n)$ is the set,

$A_5\{10n-5\}$. The set A_5 has 1 label. The vertex label of $f(V(G))$ is the set $A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$. The set A has $4n$ labels.

Next we find the edge labels.

For $1 \leq i \leq n-1$, the labels of the edges $f(u_i u_{i+1})$ is the set $B_1 = \{7, 17, 27, \ldots, 10n-13\}$. The set B_1 has $(n-1)$ labels.

For $1 \leq i \leq n-1$, the labels of the edges $f(v_i v_{i+1})$ is the set $B_2 = \{9, 19, 29, \ldots, 10n-11\}$. The set B_2 has $(n-1)$ labels.

For $1 \leq i \leq n$, the labels of the edge $f(u_i u_i')$ is the set $B_3 = \{1, 11, 21, \ldots, 10n-9\}$. The set B_3 has n labels.

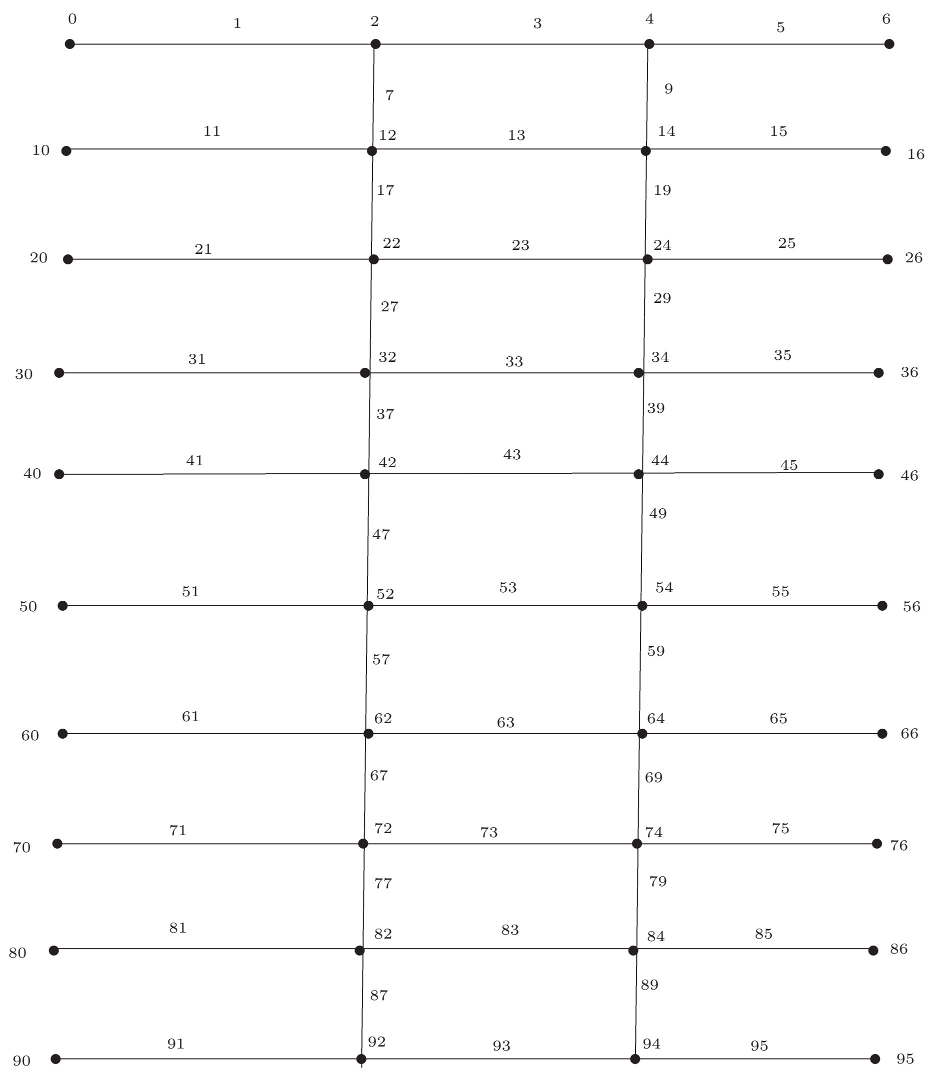
For $1 \leq i \leq n$, the labels of the edge $f(v_i v_i')$ is the set $B_4 = \{5, 15, 25, \ldots, 10n-5\}$. The set B_4 has n labels.

For $1 \leq i \leq n$, the labels of the edge $f(u_i v_i)$ is the set $B_5 = \{3, 13, 23, \ldots, 10n-7\}$. The set B_5 has n labels.

Thus the edge labels are in the set, $B = B_1 \cup B_2 \cup B_3 \cup B_4 \cup B_5$. Therefore the set B has $5n-2$ labels.

Hence G is an odd mean graph. □

Example 2.3.39. Odd mean labeling of G.



Definition 2.3.40. The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge of G by a path of length 2.

Theorem 2.3.41. *$S(L_n)$ is an odd mean graph.*

Proof. Let v_i, u_i be the vertices of a ladder $(1 \leq i \leq n)$. Let v_i' be th newly added vertex between v_i and $v_{i+1}(1 \leq i \leq n-1)$. Let u_i' be the newly added vertex between u_i and $u_{i+1}(1 \leq i \leq n-1)$. Let w_i be the newly added vertex between v_i and u_i. Clearly the graph G has $5n-2$ vertices and $6n-4$ edges.

Define $f : V(G) \to \{0, 1, 2, 3, \ldots, (12n - 9)\}$ by

$f(u_i) = 4i - 4 (1 \leq i \leq n), f(v_1) = 12n - 9, f(v_i) = 12n - 4 - 4i (2 \leq i \leq n), f(u_i') = 4i - 2, 1 \leq i \leq n - 1, f(v_i') = 12n - 6 - 8i, 1 \leq i \leq n - 1$ and $f(w_i) = 12n - 2 - 8i, 1 \leq i \leq n$.

The label of the edge $u_i u_i'$ is $4i - 3 (1 \leq i \leq n - 1)$. The label of the edge $u_i' u_{i+1}$ is $4i - 1 (1 \leq i \leq n - 1)$. The label of the edge $u_i w_i$ is $6n - 2i - 3 (1 \leq i \leq n)$. The label of the edge $v_i w_i$ is $12n - 6i - 3 (1 \leq i \leq n)$. The label of the edge $v_i v_i'$ is $12n - 6i - 5$ $(1 \leq i \leq n - 1)$. The label of the edge $v_i' v_{i+1}$ is $12n - 6i - 7 (1 \leq i \leq n - 1)$.

Obviously $f(u_1) = 0$ and $f(v_1) = 12n - 9$. Thus f is a function from $V(G)$ to the set $\{0, 1, 2, 3, \ldots, (12n - 9)\}$. Clearly f is one-one.

Next we find the vertex labels $f(V(G))$.

For $1 \leq i \leq n$, the labels of $f(u_i)$ is the set $A_1 = \{0, 4, 8, \ldots, 4n - 4\}$. The set A_1 has n labels. The label of $f(v_1)$ is the set $A_2 = \{12n - 9\}$. The set A_2 has 1 label.

For $2 \leq i \leq n$, the labels of $f(v_i)$ is the set $A_3 = \{12n - 12, 12n - 16, 12n - 20, \ldots, 8n - 4\}$. The set A_4 has $(n - 1)$ labels.

For $1 \leq i \leq n - 1$, the labels of $f(u_i')$ is the set $A_4 = \{2, 6, 10, \ldots, 4n - 6\}$. The set A_4 has $(n - 1)$ labels.

For $1 \leq i \leq n-1$, the labels of $f(v_i)$ is the set $A_5 = \{12n-14, 12n-22, \ldots, 4n+2\}$. The set A_5 has $n-1$ labels.

For $1 \leq i \leq n$, the labels of $f(w_i')$ is the set $A_6 = \{12n-10, 12n-18, 12n-26, \ldots, 4n-2\}$. The set A_6 has n labels.

Thus the vertex labels of $f(V(G))$ is the set $A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6$. Therefore the set A has $5n-2$ labels.

Next we find the edge labels.

For $1 \leq i \leq n-1$, the labels of the edge $f(u_iu_i')$ is the set $B_1 = \{1, 5, 9, \ldots, 4n-7\}$. The set B_1 has $(n-1)$ labels.

For $1 \leq i \leq n-1$, the labels of the edge $f(u_i'u_{i+1})$ is the set $B_2 = \{3, 7, 11, \ldots, 4n-5\}$. The set B_2 has $(n-1)$ labels.

For $1 \leq i \leq n$, the labels of the edge $f(u_iw_i)$ is the set $B_3 = \{6n-5, 6n-7, 6n-9, \ldots, 4n-3\}$. The set B_3 has n labels.

For $1 \leq i \leq n$, the labels of the edge $f(v_iw_i)$ is the set $B_4 = \{12n-9, 12n-15, 12n-21, \ldots, 6n-3\}$. The set B_4 has n labels.

For $1 \leq i \leq n-1$, the labels of the edge $f(v_iv_i')$ is the set $B_5 = \{12n-11, 12n-17, \ldots, 6n+1\}$. The set B_5 has $(n-1)$ labels.

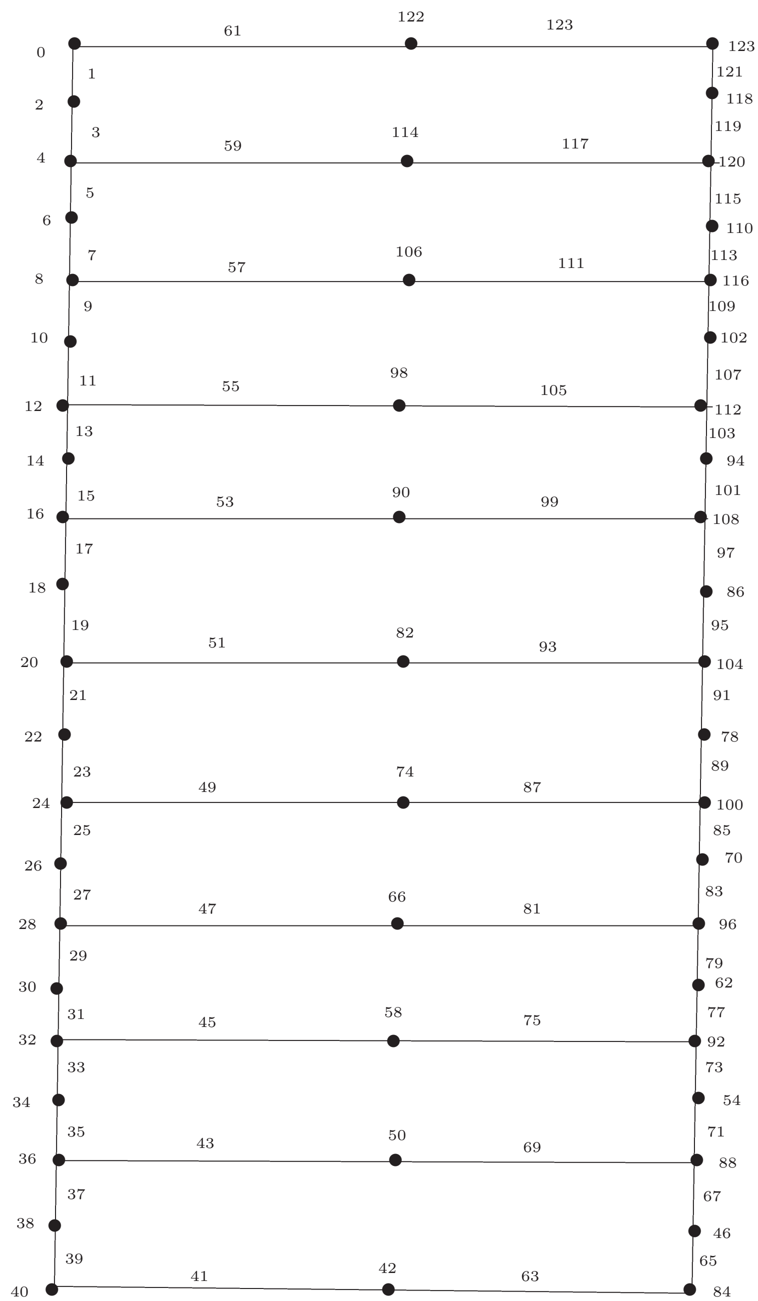
For $1 \leq i \leq n-1$, the labels of the edges $v_i'v_{i+1}$ is the set $B_6 = \{12n-13, 12n-19, \ldots, 6n-1\}$. The set B_6 has $(n-1)$ labels.

Thus the edge labels are in the set $B = B_1 \cup B_2 \cup B_3 \cup$

$B_4 \cup B_5 \cup B_6$. Therefore, the set B has $6n - 4$ labels.

Hence $S(L_n)$ is an odd mean graph. $\square$

Example 2.3.42. Odd mean labeling of subdivision graph $G = S(L_n)$.



2.4 CONCLUSION AND SCOPE

Further we have planned to work k-odd mean labeling and (k, d)-odd mean labeling of any graph.