THEORETICAL MODELS

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CHAPTER 2

THEORETICAL MODELS

Sandulescu et al [1] in 1980 first predicted exotic decay, the intermediate process between alpha decay and spontaneous fission on the basis of quantum mechanical fragmentation theory [2]. The first experimental evidence of this phenomena was observed by Rose and Jones [3] in 1984. There are mainly two approaches in explaining the exotic decay process: 1) the cluster model and 2) the fission model. In cluster model, the cluster is preformed in the parent nucleus before it penetrates the nuclear interacting barrier. In fission model the nucleus deforms continuously as it penetrates the nuclear barrier and reaches the scission configuration after running down the Coulomb barrier.

2.1 The Quantum Mechanical Fragmentation Theory

The Quantum Mechanical Fragmentation Theory (QMFT) is able to describe cold fusion, cold fission and exotic decay from a unified point of view [4,5,6,7]. The unifying result of this theory is the closed shell effects of one or both the reaction partners for fusion or that of the decay products for fission and exotic decay. This is achieved by treating the mass asymmetry and the length of the nucleus (or distance between the centres of fragments) as dynamical coordinates. The potential energy and dynamical
masses as functions of these variables were calculated with the asymmetric two center shell model (ATCSM), after minimising the total energy in the deformation of the fragments and the neck between them. The nuclear shape is defined \[8\] in terms of coordinates of relative separation \(R\) (or equivalently the length parameter \(\lambda = L/R_0\) where \(L\) is the length of the nucleus and the \(R_0\) the radius of an equivalent spherical nucleus), quadrupole deformations \(\beta_1\) and \(\beta_2\) of fragments, the necking-in parameter \(\varepsilon\) and the two dynamical collective coordinates of mass and charge asymmetries \(\eta\) and \(\eta_z\)

\[
\eta = \frac{A_1 - A_2}{A} \quad \text{and} \quad \eta_z = \frac{Z_1 - Z_2}{Z}
\]

where mass and charge of the parent are \(A = A_1 + A_2\) and \(Z = Z_1 + Z_2\) respectively. The collective Hamiltonian is

\[
H = T(R, \beta_1, \beta_2, \eta, \eta_z, \dot{R}, \dot{\beta}_1, \dot{\beta}_2, \dot{\eta}, \dot{\eta}_z) + V(R, \beta_1, \beta_2, \eta, \eta_z)
\]

The potential \(V\) is obtained in the standard Strutinsky method \([9,10]\) by using appropriate liquid drop model (LDM) and the single particle states of ATCSM giving the shell correction \(\delta U\). The collective potential \(V\) minimized in \(\beta_1, \beta_2\) and \(\varepsilon\) coordinates is

\[
V(R, \eta, \eta_z) = V_{LDM} + \delta U
\]

The mass parameter \(B_{ij}\) defining the kinetic part \(T\) in \(H\), are the consistently calculated adiabatic cranking masses \([11,12]\). Assuming that the relative motion \(R\) is slow compared to the \(\eta\) and \(\eta_z\) motions (this means that the potential is nearly independent of
and hence $R$ can be taken as a time independent parameter) and that the coupling between $\eta$ and $\eta_z$ is weak, the Hamiltonian is quantised according to the prescription of Pauli [13] and Padolsky[14] to yield the stationary Schrodinger equation.

$$\left\{-\frac{\hbar^2}{2\sqrt{B_{\eta}}} \frac{\partial}{\partial \eta} \frac{1}{\sqrt{B_{\eta}}} \frac{\partial}{\partial \eta} + V(R, \eta)\right\} \Psi^{(v)}_R(\eta) = E_R^{(v)} \Psi^{(v)}_R(\eta) \quad (2.4)$$

Here $\Psi^{(v)}_R(\eta)$ are the vibrational states in the potential $V(\eta)$ and are counted by the quantum numbers $\nu = 0, 1, 2, \ldots$. The ground state ($\nu = 0$) solution of equation (2.4) gives the probability $|\Psi^{(0)}_R(\eta)|^2$, which is proportional to the percentage mass distribution yield,

$$Y(A_z) = |\Psi^{(0)}_R(\eta)|^2 \frac{200}{A} \quad (2.5)$$

The consequences of higher excited states ($\nu \neq 0$) can be included through Boltzmann-like occupation function

$$|\Psi^{(v)}_R|^2 = \sum_{\nu=0}^{\infty} |\Psi^{(v)}_R|^2 \exp(-E_R^{(v)}/\theta) \quad (2.6)$$

with $\theta$ the nucleus temperature in MeV, is related to excitation energy as \cite{15}

$$E^* = \frac{1}{9} A \theta^2 - \theta \quad (2.7)$$

The excitation energy will damp the shell effect as \cite{5}

$$V = V_{\text{LDM}} + \delta U \exp(-\frac{\theta^2}{\theta_0^2}) \quad (2.8)$$
The parameter $\theta_0 = 1.5$ MeV is so chosen that the shell effect vanishes if $E^* \geq 60$ MeV [16]. Similarly mass parameter also varies with temperature but no usable prescription is available to date. A constant (averaged) mass is taken to mean a complete washing of shell effect in it [17].

2.2 Existing models

2.2.1 The fission models

2.2.1.1 Analytical Super Asymmetric Fission Model (ASAFM)

A numerical super asymmetric fission model (NSAFM) was developed by Poenaru et al [18,19,20] since 1979 by extending three variants of liquid drop model [21], finite range of nuclear forces model [22] and Yukawa plus exponential model [23] to system with charge asymmetry different from mass asymmetry and by using phenomological shell correction. Half lives computed within W K B approximation are time consuming. So an analytical relationship for half life [analytical super asymmetric fission model (ASAFM)] was developed [24] and extended to account for angular momentum and small excitation effects. In this model the potential barrier $E(R)$ for the overlapping region is approximated by a second order polynomial in $R$. 
If time is expressed in seconds, energies in MeV and length in fermis, then the logarithm of the half life time for a system is given by

$$\log T = 0.43429(K_{ov} + K_v) - \log E_v - 20.8436$$  \hspace{1cm} (2.9)

here $E_v$ is the zero point vibration energy given by

$$E_v = Q [0.056 + 0.039\exp[(4 - A_2)/2.5]] \quad \text{for} \ A_2 \geq 4 \hspace{1cm} (2.10)$$

The action integral $K_{ov}$ corresponding to overlap region and $K_v$ that for separated configuration are given by the following analytical expressions:

$$K_{ov} = 0.2196\left(E_b^0 A_1 A_2 / A\right)^{1/2} \left(b^2 - a^2\right)^{1/2} - a^2\ln\left(b + \frac{b^2 - a^2}{a}\right)$$  \hspace{1cm} (2.11)

$$K_v = 0.4392\left[Q^0 A_1 A_2 / A\right]^{1/2} R_b \tau_{mc}$$  \hspace{1cm} (2.12)

where $Q^0 = Q + E_v + E^*$, $E^*$ being the fraction of the excitation energy concentrated in the collective mode leading to separation. The quantities $a$ and $b$ are defined below. The interaction energy $E_i$ at the top of the barrier in the presence of the non-negligible angular momentum $\ell h$ is given by

$$E_i = E_c + E_\ell = \frac{1.444Z_1 Z_2}{R_i} + \frac{20.735\ell(\ell + 1)A}{R_i^2 A_1 A_2}$$  \hspace{1cm} (2.13)

The radii of parent, emitted cluster and daughter nuclei are given by $R_j = r_0 A_j^{1/3}$, where $j = 0,1,2$ and $r_0 = 1.2249$ fm. $R_i = R_0 - R_2$ is the initial separation distance, $R_t = R_i + R_2$ is the touching separation distance and $E_b^0 = E_i - Q$ is the barrier height before correction. The outer turning point of W.K.B penetrability is given by
\[ R_b = \frac{R_c E_c}{Q} \left[ \frac{1}{2} + \frac{1}{4} + \frac{Q E_t}{E_c^2} \right]^{1/2} \]  

(2.14)

\[ a = b \left[ \frac{Q - Q}{E_b^0} \right]^{1/2} \quad ; \quad b = R_i - R_i \]  

(2.15)

\[ \tau_{mc} = (c + m - 1)^{1/2} [r(c - r) + m]^{1/2} + \frac{c}{2} \left[ \arcsin \frac{c - 2r}{(c^2 + 4m)^{1/2}} - \arcsin \frac{c - 2}{(c^2 + 4m)^{1/2}} \right] \]

\[ + \sqrt{m} \ln \left[ \frac{2\sqrt{m} [r(c - r) + m]^{1/2} + cr + 2m}{r^{2\sqrt{m}(c + m - 1)^{1/2} + c + 2m}} \right] \]  

(2.16)

where \( r = \frac{R_i}{R_b} \); \( m = \frac{r^2 E_t}{Q} \); \( c = \frac{r E_c}{Q} \)  

(2.17)

\subsection{2.2.1.2 Proximity potential model of Shi and Swiatecki}

In this model [25], the deformation energy barrier for touching and for separated configuration consists of Coulomb repulsion between fragments and nuclear proximity potential of Blocki et al [26,27]. The deformation energy between the contact configuration and the configuration of the parent is approximated by a smooth power law interpolation. The explicit formula for deformation energy is as follows

\[ V(L) = -Q + Z_1 Z_2 e^2/r + V_p(z) \quad \text{for} \quad L > L_C \]  

(2.18)

\[ V(L) = a (L-L_0)^v \quad \text{for} \quad L_0 < L < L_C \]  

(2.19)

Here \( L \) is the major axis (overall length) of the configuration, \( L_0 \) is the diameter of the parent nucleus and \( L_C \) is the sum of fragment diameters. \( Q \) is the energy released. The parameters \( a \) and \( v \) are determined by applying the smooth continuity condition on
potential at touching configuration. Also \( r = L - C_1 - C_2 \), the separation between fragment centers, \( z \) is the distance between the near surfaces of the fragments. \( V_p \), the proximity potential is given by

\[
V_p(z) = K \phi(z/b)
\]

where \( K = 4\pi \bar{R} \gamma b \) \hspace{1cm} (2.20)

Here is \( \phi \) is the universal nuclear proximity function \([27]\) given as

\[
\phi(\xi) \approx -4.41 e^{-\xi/0.7176} \quad \text{for} \quad \xi \geq 1.9475 \hspace{1cm} (2.22)
\]

\[
\phi(\xi) \approx -1.7817 + 0.9270\xi + 0.01696\xi^2 - 0.05148\xi^3 \quad \text{for} \quad 0 \leq \xi \leq 1.9475 \hspace{1cm} (2.23)
\]

In the above \( \xi = z / b \), \( b \) is the width (diffuseness) of nuclear surface \((b \approx 1 \text{fm})\), \( \gamma \) is the specific nuclear surface tension given as \([26]\)

\[
\gamma = 0.9517 \left[ 1 - 1.7826 \frac{(N - Z)^2}{A^2} \right] \quad \text{MeV/fm}^2 \hspace{1cm} (2.24)
\]

The reduced radius \( \bar{R} \) is given by

\[
\bar{R} = \frac{C_1C_2}{C_1 + C_2} \hspace{1cm} (2.25)
\]

where \( C_i \) is the central radii of the fragments given by

\[
C_i \approx R_i - b^2 / R_i \hspace{1cm} (2.26)
\]

The effective sharp radii \( R \) in terms of mass number \( A \) is given by \([26]\)

\[
R = 1.28A^{1/3} - 0.76 + 0.8A^{-1/3} \hspace{1cm} (2.27)
\]

The Gamow penetrability factor \( G \) is given by

\[
G = \exp\left\{ \frac{2}{\hbar} \int_{z_o}^{\infty} \sqrt{2M_rV} \, dz \right\} \hspace{1cm} (2.28)
\]
here \( z_0 \) and \( z_{\text{exit}} \) are approximate zeros of the integrand and \( M_1 \) is the effective mass. The absolute value of life time is given by

\[
\tau = \tau_0 G
\]  
(2.29)

\( \tau_0 \) is the pre exponential factor (frequency factor) taken to be \( 10^{-22} \) s for even \( A \) and \( 10^{-20} \) s for odd \( A \) parent nuclei [28]

**2.2.1.3 Cubic Plus Yukawa plus exponential potential model (CYEM)**

In this model [29] the potential for post scission region taken as a function of center of mass distance \( r \) of the fragments is given as

\[
V(r) = Z_1 Z_2 e^{2/r} + V_n(r) - Q
\]  
(2.30)

Here \( Q \) is the energy released and \( V_n(r) \) is the nuclear interaction energy [23] given as

\[
V_n = -D \left[ F + \frac{r - r_i}{a} \right] r_i \exp[(r - r_i)/a]
\]  
(2.31)

where \( r_i = R_1 + R_2 \) is the sum of sharp radii of fragments. The depth constant \( D \) is given by

\[
D = \frac{4a^3 g_1 g_2 e^{r_i/a} [C_0(0)C_0(2)]^{1/2}}{r_0^2 r_i}
\]  
(2.32)

The constant \( F \) is given by

\[
F = 4 + \frac{r_i}{a} - \frac{f_1}{g_1} - \frac{f_2}{g_2}
\]  
(2.33)
where
\[ g_j = (R_j / a) \cosh(R_j / a) - \sinh(R_j / a) \]  \hspace{1cm} (2.34)
\[ f_j = (R_j / a)^2 \sinh(R_j / a) \]  \hspace{1cm} (2.35)
\[ C_x(j) = a_x (1 - K_x I_j^2) \]  \hspace{1cm} (2.36)
\[ I_j = (N_j - Z_j) / A_j \hspace{1.5cm} (j = 1, 2) \]  \hspace{1cm} (2.37)

Here \( r_0 = 1.16 \text{ fm}, \ a = 0.68 \text{ fm}, \ a_x = 21.13 \text{ MeV} \) and \( K_x = 2.3 \)

For the overlap region the potential is approximated by a 3rd order polynomial as [29]
\[ V(r) = -E_v + \lfloor V(r_i) + E_v \rfloor \left[ S_1 \left[ \frac{r - r_i}{r_i - r_i} \right]^2 - S_2 \left[ \frac{r - r_i}{r_i - r_i} \right]^3 \right] \]  \hspace{1cm} (2.38)

The zero point vibration energy \( E_v \) is given by [30]
\[ E_v = \frac{\pi \hbar (2Q/\mu)^{1/2}}{2 (C_1 + C_2)} \]  \hspace{1cm} (2.39)

Here \( \mu \) is the reduced mass. The central radii of fragments \( C_1 \) and \( C_2 \) are given by
\[ C_i = 1.18 A_i^{1/3} - 0.48 \hspace{1cm} (i = 1, 2) \]  \hspace{1cm} (2.40)

The half life time of the system is given by the formula [31]
\[ T = \frac{1.4333 \times 10^{-21}}{E_v} [1 + \exp(K)] \]  \hspace{1cm} (2.41)

Where \( K = \frac{2}{\hbar} \int [2B_r(r)V(r)]^{1/2} dr + \frac{2}{\hbar} \int [2B_r(r)V(r)]^{1/2} dr \)  \hspace{1cm} (2.42)

The limits \( r_a \) and \( r_b \) are two approximate zeros of the integrand and the effective mass \( B_r(r) \) is taken to be deformation dependent [32].
2.2.2 The cluster models

2.2.2.1 Microscopic model of Blendowske et al

In this model [33,34] the potential $U(R)$ used is the sum of semi empirical heavy ion potential, Coulomb potential and centrifugal potential given as

$$U(R) = V(R) + V_{\text{Coul}}(R) + V_{L}(R)$$  \hspace{1cm} (2.43)

The Coulomb potential is $Z_1 Z_2 e^2 / r$ and centrifugal potential $V_L = \hbar^2 L(L+1)/2M$ where $L$ is the angular momentum and $M$ is the reduced mass. The semi empirical heavy ion potential [35] is given as

$$V(R) = -(50 \text{MeV} / \text{fm}) \frac{R_1 R_2}{R_1 + R_2} \exp\left\{-(R - R_1 - R_2)/a\right\}$$  \hspace{1cm} (2.44)

with $R_j = 1.233 A_j^{1/3} - 0.978 A_j^{-1/3} \text{fm}$ \hspace{1cm} (j = 1, 2); $a = 0.63 \text{fm}$ (2.45)

The decay constant $\lambda_0$ is calculated in WKB approximation as

$$\lambda_0 = (\nu / 2 R_i) P \quad ; \quad P = \exp\left\{-2 \int_{R_i}^{R_o} dR [(2M / \hbar^2) (U(R) - Q)]^{1/2}\right\}$$  \hspace{1cm} (2.46)

here $Q$ is the $Q$ value of the reaction. $R_i$ and $R_o$ are inner and outer turning points where $U(R) = Q$. For pre factor $(\nu / 2 R_i)$ Blendowske et al [33] assumed a kinetic energy $\frac{1}{2} M \nu^2$ inside the barrier of $10^2$ MeV for alpha particle and a scaled value
$10^2 \left( \frac{14}{4} \right) \text{MeV}$ for $^{14}\text{C}$ cluster. In the microscopic approach the decay constant $\lambda$ is given as the product

$$\lambda = \lambda_0 S \quad (2.47)$$

The spectroscopic factor $S$ is the probability of the cluster to be preformed in the parent nuclei and is given by the overlap of nucleon states in the cluster with those of the parent nucleus [33].

### 2.2.2.2 Preformed cluster model (PCM)

This model [36] consists of a two steps mechanism 1) cluster formation 2) tunneling of confining nuclear interaction barrier. If $P_0$ is the cluster formation probability (in ground state), $P$ the barrier penetration probability and $v$ the assault frequency, then decay constant $\lambda$ and half life time $T_{1/2}$ are given as

$$\lambda = P_0 v P \quad (2.48)$$

$$T_{1/2} = \ln 2 / \lambda \quad (2.49)$$

Introducing dynamical collective coordinates of mass and charge asymmetries $\eta$ and $\eta_z$

$$\eta = \frac{A_1 - A_2}{A} \quad ; \quad \eta_z = \frac{Z_1 - Z_2}{Z} \quad (2.50)$$
the quantum mechanical probability of finding fragments $A_1$ and $A_2$ (with fixed charges $Z_1$ and $Z_2$) at a relative separation $R$ is given by the solution of the following stationary Schrödinger equation in $\eta$

$$\left[-\frac{\hbar^2}{2\sqrt{B_{\eta\eta}} \eta} \frac{\partial}{\eta} \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} + V(\eta, \eta_z, R)\right] \Psi(\eta) = E_{gs} \Psi(\eta) \quad (2.51)$$

here $V(\eta, \eta_z, R)$ is the collective interaction potential energy and $B_{\eta\eta}$ is the mass parameter [37]. On proper scaling and normalization of above equation the cluster formation probability in ground state is

$$P_0(A_z) = |\Psi(\eta)|^2 \sqrt{\frac{2}{A}} \quad (2.52)$$

The collective interaction energy is taken as the sum of experimental binding energies, the Coulomb and the nuclear proximity potential.

$$V(\eta, \eta_z, R) = -\sum_{i=1}^{2} B_i (A_i, Z_i) + \frac{Z_1 Z_2 e^2}{R} + V_p \quad (2.53)$$

The proximity potential $V_p$ is given as [26]

$$V_p = 4\pi \gamma b \frac{C_1 C_2}{C_1 + C_2} \phi \left(\frac{z}{b}\right) \quad (2.54)$$

With the nuclear surface tension coefficient

$$\gamma = 0.9517[1 - 1.7826 (N - Z)^2 / A^2] \quad \text{MeV/fm}^2 \quad (2.55)$$

$\phi$, the universal proximity potential of pocket formula of Blocki et al [26]

$$\phi(\zeta) = -(1/2)(\zeta - 2.54)^2 - 0.0852(\zeta - 2.54)^3 \quad \text{for } \zeta \leq 1.2511 \quad (2.56)$$

$$\phi(\zeta) = -3.437\exp(-\zeta/0.75) \quad \text{for } \zeta \geq 1.2511 \quad (2.57)$$
with ζ = (C-C₁-C₂)b where width (diffuseness) of nuclear surface b ≈ l and Sissmann central radius Cᵢ of fragments related to sharp radii Rᵢ is Cᵢ ≈ Rᵢ - \( \frac{b^2}{Rᵢ} \). The semi empirical formula Rᵢ in terms of mass number Aᵢ is given as

\[ Rᵢ = 1.28Aᵢ^{1/3} - 0.76 + 0.8Aᵢ^{-1/3} \]  (2.58)

The assault frequency ν is given as \( ν = (2E₂/\mu)^{1/2} / R₀ \), where R₀ is the radius of parent nucleus, E₂ the kinetic energy of the emitted cluster and μ the reduced mass. The tunneling probability P consists of three contributions: 1) Pᵢ, the penetrability from touching configuration Rᵢ to Rᵢ, here Rᵢ is given by \( V(Rᵢ) = V(Rᵢ) \) 2) Wᵢ, the deexcitation probability at Rᵢ and 3) the penetrability Pᵦ from Rᵢ to Rᵦ. Here Rᵦ is given by \( V(Rᵦ) = Q \).

Then tunneling probability P is given by \( P = PᵢWᵢPᵦ \)  (2.59)

According to Greiner and Scheid [38] the deexcitation probability vary exponentially with excitation energy, \( Wᵢ = \exp(-bEᵢ) \). For clusters heavier than alpha particle b = 0 i.e \( Wᵢ = 1 \).

The penetrability Pᵢ and Pᵦ are defined as

\[ Pᵢ = \exp\left\{-\frac{2}{\hbar}\int_{Rᵢ}^{Rᵦ} [2\mu(V - V(Rᵢ))]^{1/2} dR\right\} \]  (2.60)
These integrals can be solved analytically [36].

### 2.2.2.3 Cluster model of Buck et al

The parent nucleus is viewed as consisting of a daughter nucleus core (mass $A_1$) and a preformed cluster (mass $A_2$). This model [39,40] describes the interaction between them in terms of a simple local potential $V_N(r)$ that may be obtained from a double folding integral involving their respective densities $\rho_1(r_1)$ and $\rho_2(r_2)$ and an effective nucleon-nucleon potential $U(|r_1-r_2|)$. Hence

$$V_N(r) = \int \int \rho_1(r_1) \rho_2(r_2) U(|r + r_2 - r_1|) d^3r_1d^3r_2$$  \hspace{2cm} (2.62)

This may be approximated by a simple potential of the form

$$V_N(r) = \frac{-V_0[1 + \cosh(R/a)]}{[\cosh(r/a) + \cosh(R/a)]}$$ \hspace{2cm} (2.63)

of depth $V_0$ and nonzero diffuseness $a$. The radius $R$ is given as

$$R = 1.04(A_1^{2/3} + A_2^{2/3})^{1/2}$$ \hspace{2cm} (2.64)

The potential acting between cluster and core is given by

$$V(r) = V_N(r) + V_C(r) + \frac{\hbar^2}{2\mu r^2} \left( L + \frac{1}{2} \right)^2$$ \hspace{2cm} (2.65)
Here $\mu$ the reduced mass, $V_C(r)$ is the Coulomb potential and 3rd term represent the Langer modified centrifugal barrier. The classical turning points ($r_1$, $r_2$ and $r_3$ in the order of increasing distance from origin) are found by numerical solution of the equation $V(r) = Q$, where $Q$ is the $Q$ value of the reaction. If $L$ is very small $r_1$ is close to zero and if nuclear term $V_N(r)$ is neglected in the asymptotic region, $r_3$ can be found by solving the resulting quadratic equation.

The decay width $\Gamma$ can be calculated using the procedure of Gurvitz et al [41]

$$\Gamma = PF \frac{\hbar^2}{4\mu} \exp \left( -2 \int_{r_1}^{r_2} K(r) dr \right)$$  \hspace{1cm} (2.66)

where $P$ is the preformation probability and semi classical wave number $K(r)$ is given by

$$K(r) = \left[ \frac{2\mu}{\hbar^2} |Q - V(r)| \right]^{1/2}$$  \hspace{1cm} (2.67)

$F$ is the semi classical bound state normalization factor given by

$$F \int_{r_1}^{r_3} \frac{1}{K(r)} \cos \left[ \int_{r_1}^{r} K(r') dr' - \frac{\pi}{4} \right] dr = 1$$  \hspace{1cm} (2.68)

The half life time $T_{1/2}$ in terms of width $\Gamma$ can be calculated using the relation

$$T_{1/2} = \frac{\hbar}{2\Gamma} \ln 2$$  \hspace{1cm} (2.69)
2.2.2.4 Double folded Michigan-three-Yukawa potential model

Sandulescu et al [42] studied heavy ion emission using double folded Michigan-three-Yukawa potential (M3Y) potential [43] by including deformation effect of the fragments. The nuclear plus Coulomb interaction between the fragments is given by M3Y potential as

\[ V_{\text{M3Y}}(R) = \int dr_1 dr_2 \rho_1(r_1) \rho_2(r_2) v(r_{12}) \]  (2.70)

which contains the corresponding nucleon-nucleon interaction [44] as

\[ v(r_{12}) = v_{00}(r_{12}) + J_{00} \delta(r_{12}) + v_{01}(r_{12}) \tau_1 \tau_2 \]  (2.71)

the central component of M3Y force in eqn (2.71) is

\[ v_{00}(r) = \left[ 7999 \frac{e^{-4r}}{4r} - 2134 \frac{e^{-2.5r}}{2.5r} \right] \text{ MeV} \]  (2.72)

and isospin part has the form

\[ v_{01}(r) = \left[ -4885.5 \frac{e^{-4r}}{4r} + 1175.5 \frac{e^{-2.5r}}{2.5r} \right] \text{ MeV} \]  (2.73)

The second term in eqn (2.71) approximates the single nucleon exchange effects through a zero pseudopotential, \( J_{00} = -262 \text{ MeV fm}^3 \). The spin-spin \( v_{10} \) and spin-isospin \( v_{11} \) components are disregarded since their final contributions tend to be small [43].

The two final nuclei are viewed as coaxial spheroids ("nose to nose" configuration) with nuclear density
\[ \rho(r) = \rho_0 \left( 1 + \exp \frac{1}{a} \left( r - \frac{R}{c} \left[ 1 + \beta_2 Y_2^0 (\cos \theta) \right] \right) \right)^{-1} \]

with constant \( \rho_0 \) fixed by normalizing the proton and neutron densities to proton number \( Z \) and neutron number \( N \) respectively. Here \( \beta_2 \) is the quadrapole deformation and \( c \) is the usual constant which ensure volume conservation condition

\[ \int d^3r = \frac{4\pi}{3} R^3 \]  

(2.75)

from which it follows

\[ c(\beta_2) = \left[ 1 + \frac{3}{4\pi} \beta_2 + \frac{1}{14\pi} \sqrt{\frac{5}{4\pi}} \beta_2^3 \right]^{1/3} \]

(2.76)

The two parameters of fermi density are chosen as \( R = 0.95 A^{1/3} \) fm and \( a = 0.67 \) fm for light clusters and \( R = 1.19 A^{1/3} \) fm and \( a = 0.63 \) fm for heavy daughter nuclei.

The penetrability \( P \) through the double folded potential barrier is given as

\[ P = \exp \left\{ -\frac{2}{\hbar} \int_{R_i}^{R_o} \sqrt{2\mu[V(R) - Q]} dR \right\} \]

(2.77)

where \( R \) is the distance between the fragment mass centers, \( \mu \) is the reduced mass, \( R_i \) and \( R_o \) are the inner and outer turning points defined by

\[ V(R_i) = V(R_o) = Q \]

(2.78)

The half life time is given by

\[ T_{1/2} = \frac{\ln 2}{\nu P_0 P} \]

(2.79)

Here \( \nu \) is the collision frequency and \( P_0 \) is the preformation factor.
2.3 The Present model

For the touching and for the separated configuration of the cluster and daughter we took the Coulomb and proximity potential as the interacting barrier. From the touching configuration and down to the parent central radius (overlap region), we use simple power law interpolation as done by Shi and Swiatecki [25]. Figures 2.1 and 2.2 represent potential energy barrier for the emission of $^4$He and $^{12}$C from $^{120}$Nd isotope. Inclusion of the proximity potential reduces the height of the barrier, which closely agrees with experiments. Table 2.1 gives the comparison of the barrier height calculated by us taking proximity potential (present) with 1) those obtained by liquid drop model (LDM) [21] 2) by Cubic and Yukawa plus exponential potential model (CYEM) [29] and 3) experimental values obtained by the relation [18]

$$V(r) = 10.107 + 0.1021Z_1Z_2 - Q$$  \hspace{1cm} (2.80)

It is clear that LDM overestimates the barrier by about 10 MeV but as in the case of CYEM, the present model is able to reproduce experimental values, which are uncertain by about 2 MeV [45].

The proximity potential was first used by Shi and Swiatecki in an empirical manner and extensively used over a decade now by Gupta and collaborators [36] in the preformed cluster model (PCM), which is based on the "pocket formula" of Blocki et al [26]. In the present model we use another formulation of the proximity potential [27]. We
have taken the cluster formation probability as unity for all the clusters irrespective of their masses. So the present model differs from the PCM by a factor $P_0$, the cluster formation probability. But we have included the contribution of the overlap region in the barrier penetrability calculation. In the present model assault frequency is calculated for each parent-cluster combination which is associated with zero point vibration energy, but in Shi and Swiatecki’s model it is taken as $10^{22}$ for even $A$ and $10^{20}$ for odd $A$ parent. For the zero point vibration energy we use semi empirical formula of Poenaru et al [24].

### 2.3.1 Details of the model

The interacting barrier for a parent exhibiting exotic decay is given by

$$V = Z_1 Z_2 e^3 / r + V_p(z) \quad \text{for } z > 0$$

(2.81)

Here $Z_1$ and $Z_2$ are atomic numbers of daughter and emitted cluster, $r$ is the distance between the fragment centers, $z$ is the distance between the near surfaces of the fragments and $V_p$ is the proximity potential given by [26]

$$V_p(z) = 4\pi\gamma b \left( \frac{C_1 C_2}{C_1 + C_2} \right) \phi \left( \frac{z}{b} \right)$$

(2.82)

with the nuclear surface tension coefficient,

$$\gamma = 0.9517[1 - 1.7826(N - Z)^2 / A^2] \quad \text{MeV/fm}^2$$

(2.83)

Here $N$, $Z$ and $A$ represent the neutron, proton and mass numbers of the parent respectively.

$\phi$, the universal proximity potential is given as [27]
\[ \phi(\varepsilon) = -4.41e^{-\varepsilon^{0.7176}} \text{ for } \varepsilon \geq 1.9475 \] (2.84)

\[ \phi(\varepsilon) = -1.7817 + 0.9270\varepsilon + 0.01696\varepsilon^2 - 0.05148\varepsilon^3 \text{ for } 0 \leq \varepsilon \leq 1.9475 \] (2.85)

with \( \varepsilon = z/b \), where the width (diffuseness) of nuclear surface \( b = 1 \) and Siissmann central radii \( C_i \) related to sharp radii \( R_i \) is \( C_i = R_i - \frac{b^2}{R_i} \). For \( R_i \) we use the semi empirical formula in terms of the mass number \( A_i \) as [26]

\[ R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3} \] (2.86)

The barrier penetrability \( P \) is given as

\[ P = \exp\left\{ -\frac{2}{h} \int \sqrt{2\mu(V - Q)} dz \right\} \] (2.87)

The mass parameter is replaced by the reduced mass \( \mu = mA_1A_2/A \) where \( m \) is the nucleon mass and \( A_1 \) and \( A_2 \) represent the mass numbers of the daughter and the emitted cluster respectively. The inner and the outer turning points \( \varepsilon_i \) and \( \varepsilon_f \) are defined as \( V(\varepsilon_i) = V(\varepsilon_f) = Q \), where \( Q \) is the energy released.

The half life time is given by \( T_{1/2} = \ln 2/\lambda = \ln 2/\nu P \) (2.88)

Here \( \lambda \) is the decay constant and the assault frequency, \( \nu = 2E_\nu/h \). The empirical zero point vibration energy \( E_\nu \) is given as [24]

\[ E_\nu = Q \left[ 0.056 + 0.039\exp\left(\frac{4-A_2}{2.5}\right) \right] \text{ for } A_2 \geq 4 \] (2.89)
Fig. 2.1 Potential energy barrier for the emission of $^4$He from $^{120}$Nd isotope.

Fig. 2.2 Potential energy barrier for the emission of $^{12}$C from $^{120}$Nd isotope.
Table 2.1. Comparison of interacting barrier height calculated by the present model with that by other models and with experimental values.

<table>
<thead>
<tr>
<th>Parent</th>
<th>Emitted nuclei</th>
<th>Daughter nuclei</th>
<th>Q value (MeV)</th>
<th>Barrier height (MeV)</th>
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