A CAPACITATED STOCHASTIC LINEAR TRANSSHIPMENT PROBLEM

1. Introduction:

A special class of linear programming problems, known as transportation problems (TPs), arises quite frequently in real-life situations. Deterministic cases of the standard transportation problem and its several variants have been extensively studied and special methods devised for solving them. Some studies [e.g. El-Agizy [1967], Garvin [1960], Williams [1963], Szwarc [1964], Wilson [1972], Gupta and Swarup [1979] etc.] on stochastic transportation problems (STPs) have appeared in the literature. However, it seems that STPs with transshipment have remained unexplored.

The purpose of this chapter is to study the stochastic transshipment problem (STSP) with upper bounds and the objective is to maximize the expected revenue, i.e. total expected revenue minus transportation and transshipment cost.

The contents of this chapter is based on my following paper:

For dealing with uncertainty of demands, we have used the technique applied by Ferguson and Dantzig [1956].

In section 2.6, we develop a computational algorithm for solving a stochastic transshipment problem (STSP) in which additional upper bound restrictions on route capacities are imposed, the upper bound represents the upper limits on the amount that can be shipped over any given route. A numerical example is added in section 2.7 to illustrate the algorithm.
For dealing with uncertainty of demands, we have used the technique applied by Ferguson and Dantzig [1956].

In section 2.6, we develop a computational algorithm for solving a stochastic transshipment problem (STSP) in which additional upper bound restrictions on route capacities are imposed, the upper bound represents the upper limits on the amount that can be shipped over any given route. A numerical example is added in section 2.7 to illustrate the algorithm.

2. Statement of the Problem:

A Orden [1955] proposed a generalized transportation model in which transshipment through intermediate cities is permitted. That is, in this transshipment, instead of shipping direct from source to sink, it may be possible to transship the goods produced at source and destined for some sink reach their ultimate destination via other sources and sinks and are transshipped at these points.

Assume that there are \( m \) sources and \( n \) sinks. The sources are numbered from 1 to \( m \) and the sinks from \( m+1 \) to \( m+n \). Let the amount of product transported from origin \( i \) to destination \( j \) be noted by \( x_{ij} \). The total amount that leaves a source is equal to its production, plus what it transships. Hence, the
the summation indicates that the term \( j = i \) is excluded from
the sum. The total amount that leaves a sink must be equal to
the volume that sink transships, so that
\[
\sum_{j=1}^{m+n} x_{ij} = t_i \quad i = m+1, \ldots, m+n
\]  
(2.2.2)

Similarly, the total amount that arrives at a source, must be
equal to the volume that source transships.
\[
\sum_{i=1}^{m+n} x_{ij} = t_j \quad j = 1, \ldots, m
\]  
(2.2.3)

and the total amount that arrives at a sink must be equal to
the demand at that sink, plus the volume that sink transships.
\[
\sum_{i=1}^{m+n} x_{ij} = b_j + t_j \quad j = m+1, \ldots, m+n
\]  
(2.2.4)

If the cost of shipping per unit from origin \( i \) to
transportation problem we arbitrarily impose an upper bound \( t_i \)
(away), on the amount that can be transshipped at any point, so
that
\[
\begin{align*}
t_i & \leq t_0 & i &= 1, \ldots, m+n \\
t_i &= t_0 - x_{ii} & i &= 1, \ldots, m+n
\end{align*}
\] (2.2.6)

where \( x_{ii} \) is a non-negative slack. After substituting (2.2.6)
in the system (2.2.1) through (2.2.5), we obtain the following

**Problem P\(_{2.1}\):**

\[
\begin{align*}
\min f &= \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij} + \sum_{i=1}^{m+n} l_i t_0 \\
\sum_{j=1}^{m+n} x_{ij} &= \begin{cases} a_i + t_0 & , \quad i = 1, \ldots, m \\
t_0 & , \quad i = m+1, \ldots, m+n \\
\end{cases} \\
\sum_{i=1}^{m+n} x_{ij} &= \begin{cases} t_0 & , \quad j = 1, \ldots, m \\
b_j + t_0 & , \quad j = m+1, \ldots, m+n \\
\end{cases}
\end{align*}
\] (2.2.7)

where \( c_{ii} = -l_i \). The (*) on the summations has disappeared. As
\( t_i \geq 0 \), we must have \( x_{ii} \leq t_0 \), this is guaranteed by equations
(2.2.7), because any \( x_{ii} \) will always appear in one equation
that has \( t_0 \) on the right hand side.
Assume initially a value for $t_0$, which is sufficiently large to ensure that all $x_{1i}$ will be in the optimal basis. Such a value can be easily found as the volume of goods transshipped at any point cannot exceed the total volume of goods produced (or received). Hence, we set,

$$ t_0 = \sum_{i=1}^{m} a_i $$

(2.2.8)
where \( f_j(s_j, y_j) \) is an unknown function that describes the expected revenue from destination \( j \) if a total of \( y_j \) unit is shipped to this destination and \( a_i \) is the amount supply of a homogeneous product available at origin \( i \).

The third term of the right hand side viz. \( \sum_{i=1}^{m+n} l_i t_o \) can be adjusted in the second term as \( t = t - x \) and therefore,

the problem can be written as:
4. Equivalent Deterministic Problem:

Let the demand $b_j$'s at various destinations be independent random variables and the probability distribution of $b_j$ ($j = 1,...,n$) be in increasing order as follows:

<table>
<thead>
<tr>
<th>Demand $b_j$</th>
<th>$b_{1j}$</th>
<th>$b_{2j}$</th>
<th>$b_{Hjj}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob. ($b_j = b_{hj}$) = $p_{bj}$</td>
<td>$p_{1j}$</td>
<td>$p_{2j}$</td>
<td>$p_{Hjj}$</td>
</tr>
<tr>
<td>prob. ($b_j \geq b_{hj}$) = $t_{hj}$</td>
<td>$t_{1j} = \sum_{h=1}^{Hj} p_{hj}$, $t_{2j} = \sum_{h=2}^{Hj} p_{hj}$, ..., $t_{Hjj} = p_{Hjj}$ (=1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Substituting the value of \( f_j(s_j, y_j) \) using (2.4.4) in the

equation (2.3.1) of Problem \( P_{2.2} \), the objective function

becomes:

\[
Z = \sum_{j=1}^{m+n} \sum_{h=1}^{H_j} s_j t_{hj} y_{hj} - \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij}
\]

If \( x_{ij} \) and \( y_{hj} \) are treated as decision variables, the
deterministic equivalent of Problem \( P_{2.2} \) is as follows:

**Problem \( P_{2.3} \):**

\[
\begin{align*}
\text{max.} & \quad Z = \sum_{j=1}^{m+n} \sum_{h=1}^{H_j} F_{hj} y_{hj} - \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij} \\
\text{subject to} & \quad \sum_{j=1}^{m+n} x_{ij} = a_i + t_0, \quad i = 1, \ldots, m \\
& \quad \sum_{j=1}^{m+n} x_{ij} = t_0, \quad i = m+1, \ldots, m+n \\
& \quad \sum_{i=1}^{m+n} x_{ij} = t_0, \quad j = 1, \ldots, m \\
& \quad \sum_{i=1}^{m+n} x_{ij} - \sum_{h=1}^{H_j} y_{hj} = t_0, \quad j = m+1, \ldots, m+n \\
& \quad x_{ij}, y_{hj} \geq 0, \quad (\forall i, j, h) \\
& \quad x_{ij} \leq d_{ij}, \quad (\forall i, j) \\
& \quad y_{hj} \leq R_{hj}, \quad (\forall j, h)
\end{align*}
\]

65
subject to the additional restriction that the solution must satisfy constraints (2.4.3). These constraints can be easily handled by using the following theorem.

**Theorem (2.1):** A feasible solution to the Problem $P_{2.3}$ can always be improved if it violates any of the constraints (2.4.3).

**Proof:** Let $(x^*_i, y^*_h)$ be a feasible solution to Problem $P_{2.3}$ obtained on ignoring the constraints (2.4.3). The value of the objective function of Problem $P_{2.3}$ at this solution is
This result shows that if an optimal solution to the deterministic Problem $P_{2.3}$ is obtained on ignoring the constraints (2.4.3), it shall also satisfy the constraints (2.4.3). Thus, to solve Problem $P_{2.3}$ the constraints (2.4.3) do not restrict our choice and therefore, may be simply ignored.
Absence of the total column equations below the shaded region indicates that there are no row equations for $y_{hj}$ variables.

To obtain the column equations (2.3.5) of Problem $P_{2.3'}$ each $y_{hj}$ has to be multiplied by (-1). For simplicity, (-1) is omitted from $y_{hj}$ boxes.
5. Construction of Initial Basic Feasible Solution:

To start with, let us fix the demands \( b_j \) approximately equal to their expected values such that

\[
\begin{align*}
\sum_{i=1}^{m+n} d_{ij} & \leq b_j \quad (\forall j) \\
\sum_{j=m+1}^{m+n} b_j & = \sum_{i=1}^{m} a_i
\end{align*}
\]

and also such that for all \( j \) except \( j = j^* \) (say).

\[
b_j = \sum_{h=1}^{h_j} R_{hj}
\]

for some \( h_j \leq H \) and for all \( j \) except \( j = j^* \) (the \( b_j \) can always be so split).

With these fixed demands, the upper portion of the Table 4.1 (above the dark region) resembles a \((m+n) \times (m+n)\) standard transportation problem for which an initial basic feasible solution with \(2(m+n)-1\) basic variables may be obtained as follows:

Ignore the upper bounds on \( x_{ij} \)'s and write down the basic feasible solution by the North-West corner rule or any other method used for standard transportation. If this solution satisfies the upper bound constraints, we have hit the target.

If it violates these constraints, however, then we divide the basic variables into two groups.

a) the infeasible variables which violate their upper bounds and

b) the feasible variables which do not violate them.
Now, we discard temporarily the upper bounds on the infeasible variables and replace the original objective function by one that minimizes the sum of the infeasible variables. The existing solution now acts as the initial basic feasible solution for the artificial problem we have just created, and we begin the iterations, keeping in mind the upper bounds on the feasible variables.
have entered enough $y_{hj}$'s so that their sum over $h$ is equal to $b_j$ (fixed earlier).

Obviously, we shall never have to enter $y_{hj}$ below its upper bound except in column $j = j^*$ where the last non-zero
\[ C_{ij} = u_i + v_j - c_{ij} \quad (\forall \text{ non-basic } x_{ij}) \]  
\[ F_{hj} = F_{hj} - v_j \quad (\forall \text{ non-basic } y_{hj}) \]  

Now, for a particular basic feasible solution \((x_{ij}, y_{hj})\), the value of the objective function \((2.3.1)\) is  
\[ Z = \sum_{j=1}^{m+n} \sum_{h=1}^{m+n} F'_{hj} Y_{hj} - \sum_{i=1}^{m+n} C'_{ij} X_{ij} - \left\{ \sum_{i=1}^{m+n} u_i (a_i + t_0) + \sum_{j=1}^{m+n} v_j t_0 \right\} \]  
\[ (2.6.3) \]

But, the relative cost coefficients for basic variables and also the values of the non-basic \(x_{ij}'s\) are zero. As regards
We observe that Z can improved in two possible ways by

a) increasing the non-basic \( x_{ij} \) (or \( y_{hj} \) ) whose \( c_{ij}' \) (or \( f_{hj}' \) ) are positive.

b) decreasing the non-basic \( x_{ij} \) (or \( y_{hj} \) ) whose \( c_{ij}' \) (or \( f_{hj}' \) ) are negative.

Thus a basic feasible solution is optimum iff

\[
\begin{align*}
    c_{ij}' &\leq 0 \quad (\forall \text{ non-basic } x_{ij} \text{ at zero level}), \\
    c_{ij}' &\geq 0 \quad (\forall \text{ non-basic } x_{ij} \text{ at upper bounds}) , \\
    f_{hj}' &\leq 0 \quad (\forall \text{ non-basic } y_{hj} \text{ at zero level}), \\
    f_{hj}' &\geq 0 \quad (\forall \text{ non-basic } y_{hj} \text{ at upper bounds})
\end{align*}
\]

(2.6.5)

If any of the conditions (2.6.5) is violated then the current solution can be improved. The non-basic variable which violates (2.6.5) most severely is selected to enter the basis. The values of the new basic variables are determined by applying \( \delta \) - adjustments. It should, however, be kept in mind that the coefficient of each \( y_{hj} \) in the column equations (2.3.5) is (-1).

The variable to leave the basis is the one which becomes either zero or equal to its upper bound. If two or more basic variables reach zero or their upper bounds simultaneously, then only one of them becomes non-basic. Sometimes, it happen that
the entering variable itself attains upper bound or lower bound (zero) without simultaneously making any of the basic variables zero or equal to its upper bound, the set of basic variables remains unaltered; only their values are changed to allow the so-called entering variable to be fixed at its upper or lower bounds.
For convenience of writing, the non-basic variables at zero level are omitted from the tables and the presence of the smaller boxes indicates a non-basic variables at their upper bounds.

The computational algorithm for determining the optimum solution consists of the following steps.

**Step 1.** Determine the initial/improved basic feasible solution and record it in a working table.

**Step 2.** Obtain the simplex multipliers and relative cost coefficients from equations (2.6.1) and (2.6.2) respectively and record them in the current working table.

**Step 3.** Calculate the value of $Z$ from (2.6.4).
Table (2.7.3): Transshipment from warehouse to warehouse.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>21</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>II</td>
<td>3</td>
<td>21</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>III</td>
<td>4</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Table (2.7.4): Initial Basic Feasible Solution of TP.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>(a_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>10</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>II</td>
<td>6</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>III</td>
<td>8</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

\[b_j = 12, 9\]

Table (2.7.5): Assumed distribution of the demand \(b_j\).
\[ C_{21}' = 0 - 2 - 8 = -10 \leq 0 \]
\[ C_{23}' = -2 + 1 - 5 = -6 \leq 0 \]
\[ C_{31}' = -1 + 0 - 3 = -4 \leq 0 \]
\[ C_{32}' = 2 - 1 - 5 = -4 \leq 0 \]
\[ C_{34}' = -1 + 9 - 8 = 0 \leq 0 \]
\[ C_{41}' = -9 + 0 - 9 = -18 \leq 0 \]
\[ C_{42}' = 2 - 9 - 7 = -14 \leq 0 \]
\[ C_{43}' = 1 - 9 - 8 = -16 \leq 0 \]
\[ C_{45}' = -9 + 4 - 5 = -10 \leq 0 \]
\[ C_{51}' = 0 - 4 - 4 = -8 \leq 0 \]
\[ C_{52}' = 2 - 4 - 2 = -4 \leq 0 \]
\[ C_{53}' = 1 - 4 - 3 = -6 \leq 0 \]
\[ C_{54}' = 2 + 9 - 5 = 0 \leq 0 \]

\[ F_{11}' = 10 - 9 = 1 \geq 0 \]
\[ F_{12}' = 5 - 4 = 1 \geq 0 \]
\[ F_{21}' = 7 - 9 = -2 \leq 0 \]
\[ F_{31}' = 2 - 9 = -7 \leq 0 \]

Here, only \( F_{21}' = -2 \) (marked with asterisk) violates the optimality criterion of (2.6.5) and hence its value is noted in Table (2.7.8). Obviously, the current solution is not optimum and may be further improved by decreasing the value of \( y_{21} \).

**Step 5.** Subtracting \( \theta \) from \( y_{21} \) then the \( \theta \)-adjustments are made, the maximum value of \( \theta^* \) is \( \theta = 1 \).
**Iteration - 2:**

<table>
<thead>
<tr>
<th></th>
<th>21</th>
<th>10</th>
<th>8</th>
<th>-6</th>
<th>3</th>
<th>7</th>
<th>9</th>
<th>3</th>
<th>4</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
<td>0</td>
<td>-6</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>-4</td>
<td>21</td>
<td>21</td>
<td>0</td>
<td>6</td>
<td>27</td>
<td>0</td>
<td>8</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>-18</td>
<td>9</td>
<td>-14</td>
<td>7</td>
<td>0</td>
<td>-10</td>
<td>5</td>
<td>21</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>-8</td>
<td>4</td>
<td>-4</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>21</td>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>-2</td>
<td>-1</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>

\[ u_1 = 2 \]

\[ u_1 = 0 \]

\[ u_1 = 1 \]

\[ u_1 = 7 \]

\[ u_1 = 2 \]
Iteration -1:

Step 1. To obtain the initial basic feasible solution, we fix the demands at \( b_1 = 12 \) and \( b_2 = 9 \) and find the basic feasible solution to the transportation problem by North West corner rule, Table (2.7.4).

Then a standard transshipment problem can be formed (ignoring the upper bounds). We get,

\[
\begin{align*}
    x_{11} &= 21, \quad x_{14} = 10, \quad x_{15} = 2, \quad x_{22} = 21, \quad x_{24} = 4, \\
    x_{25} &= 1, \quad x_{33} = 21, \quad x_{35} = 6, \quad x_{44} = 21, \quad x_{55} = 21.
\end{align*}
\]

This solution violates the upper bound constraints as \( x_{14} \neq 8 \).

To obtain a basic feasible solution to the deterministic capacitated transshipment problem, we temporarily treat all \( x_{ij} \)'s, except the infeasible variable \( x_{24} \), as upper bounded and apply the usual transportation routine to minimize the sum of infeasible variable i.e. to minimize \( x_{24} \) till the infeasibility of \( x_{24} \) is removed. The solution so obtained is as:

\[
\begin{align*}
    x_{11} &= 21, \quad x_{14} = 8, \quad x_{15} = 2, \quad x_{22} = 21, \quad x_{24} = 4, \\
    x_{25} &= 1, \quad x_{33} = 21, \quad x_{35} = 6, \quad x_{44} = 21, \quad x_{55} = 21.
\end{align*}
\]

For the capacitated transshipment problem \( x_{24} = 4 \) is a non-basic variable at its upper bound.

Now, in each column of the working table we assign values to \( y_{i|j} \) variables to their upper bounds (as far as possible) so that we get, \( y_{11} = 9, \ y_{12} = 7, \ y_{21} = 3, \ y_{22} = 2 \ (<R_{22}) \).
This gives the required initial basic feasible solution with basic variables as follows:

\[ x_{11} = 21, \quad x_{14} = 8, \quad x_{15} = 2, \quad x_{22} = 21, \quad x_{24} = 4, \quad x_{25} = 1, \]
\[ x_{33} = 21, \quad x_{35} = 6, \quad x_{44} = 21, \quad x_{55} = 21 \text{ and } y_{22} = 2. \]

It is recorded in Table (2.7.8).

**Step 2.** We determine the simplex multipliers and relative costs from the equations (2.6.1) and (2.6.2). These are entered in Table (2.7.8), (iteration-1).

**Step 3.** The value of \( Z \) is found from equation (2.6.4) as under:

\[
Z = \sum_{j=1}^{m+n} \sum_{h=1}^{H_j} F_{hj} R_{hj} - \sum_{i=1}^{m+n} \sum_{j=1}^{d_{ij}} C_{ij} d_{ij} - \left\{ \sum_{i=1}^{m+n} u_i (a_i t_o) + \sum_{j=1}^{m+n} v_j t_o \right\}
\]

\[
Z = 7(9) + 1(7) + (-2)(3) + 0(4) - \{ 31(0) + 26(-2) + 27 \\
(-1) + 21(-9) + 21(-4) + 21(0) + 21(2) + 21(1) + 21 \\
(9) + 21(4) \}
\]

\[
= 10 - \{-16\}
\]

\[
Z = 26.
\]

**Step 4.** For the non-basic variables, we calculate \( C'_{ij} \) and \( F'_{hj} \) using (2.6.2) as under:

\[
C'_{12} = 2 + 0 - 8 = -6 \leq 0
\]
\[
C'_{13} = 2 - 1 - 3 = -2 \leq 0
\]
Iteration -2:

Step 1. Substituting \( \vartheta = 1 \), the improved basic feasible solution is obtained as given in iteration -2 (Table (2.7.8)).

Step 2. Simplex multipliers and the relative cost coefficients are determined as earlier and recorded in the Table (2.7.8).

Step 3. The value of \( Z \) is calculated from equation (2.6.4) as:

\[
Z = 3(9) + 3(7) + 2(3) + 0(4) - \{ 31(2) + 26(0) + 27(1) + 21(-7) + 21(-2) + 21(-2) + 21(0) + 21(-1) + 211(7) + 21(2) \} \\
= 54 - \{ 89 - 63 \} \\
= 28
\]

Step 4. For the non-basic variables, we calculate \( C'_{ij} \) and \( b'_{ij} \) using (2.6.2) as under:

\[
\begin{align*}
C'_{12} &= 2 + 0 - 8 = -6 \leq 0 \\
C'_{13} &= 2 - 1 - 3 = -2 \leq 0 \\
C'_{21} &= 0 - 2 - 8 = -10 \leq 0 \\
C'_{23} &= -2 + 1 - 5 = -6 \leq 0 \\
C'_{31} &= -1 + 0 - 3 = -4 \leq 0 \\
C'_{32} &= 2 - 1 - 5 = -4 \leq 0 \\
C'_{34} &= -1 + 9 - 8 = 0 \leq 0
\end{align*}
\]
Here, we find that the optimality criterion (2.6.5) is satisfied.

Hence, the optimal solution is as:

\[ Z_{\text{opt}} = 28, \]

\[ x_{11} = 21, \quad x_{14} = 7, \]
\[ x_{15} = 3, \quad x_{22} = 21, \]
\[ x_{24} = 4, \quad x_{25} = 1, \]
\[ x_{33} = 21, \quad x_{35} = 6, \]
\[ x_{44} = 21, \quad x_{55} = 21. \]
Table (2.7.8): Working tables for optimal solution:

Iteration - 1:

<table>
<thead>
<tr>
<th>21</th>
<th>0</th>
<th>-6</th>
<th>8</th>
<th>-2</th>
<th>3</th>
<th>8-θ</th>
<th>2+θ</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>8</td>
<td>0</td>
<td>-6</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>26</td>
</tr>
<tr>
<td>-4</td>
<td>3</td>
<td>-4</td>
<td>5</td>
<td>0</td>
<td>18</td>
<td>8</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>-18</td>
<td>9</td>
<td>-14</td>
<td>7</td>
<td>-16</td>
<td>8</td>
<td>0</td>
<td>-10</td>
<td>21</td>
</tr>
<tr>
<td>-8</td>
<td>4</td>
<td>-4</td>
<td>2</td>
<td>-6</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>21</td>
<td>21</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>9</th>
<th>7</th>
<th>10</th>
<th>1</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-θ</td>
<td>2+θ</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-7</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Iteration - 2:

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>0</td>
<td>-6</td>
<td>8</td>
<td>-2</td>
<td>3</td>
<td>7</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>-10</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>-6</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>-4</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>-18</td>
<td>9</td>
<td>9</td>
<td>-14</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>-8</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>-6</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ui</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>-7</td>
</tr>
<tr>
<td>-2</td>
</tr>
</tbody>
</table>

-2 0 -1 7 2