Chapter 3

Event Rates for Atmospheric Neutrinos at Super-Kamiokande

3.1 Introduction

Cosmic rays are particles coming from space and impinge on the Earth’s atmosphere. Almost 90% of all the incoming cosmic ray particles are protons, about 9% are alpha particles and rest of them are heavy nuclei and electrons. These are known as primary cosmic rays. When the primary cosmic-ray particles enter into the atmosphere their interactions produce secondary particles and are called secondary cosmic rays. The spectrum of these secondary peaks lie in the GeV range, but extend to high energy and obey almost a power-law spectrum. As cosmic rays propagate in the atmosphere, they create pions and kaons in interactions with air nuclei. These mesons create atmospheric neutrinos when they decay. They are generally given by the following reactions

\[
\begin{align*}
A_{cr} + A_{air} & \rightarrow \pi^\pm, K^\pm, K^0, \cdots \\
\pi^+ & \rightarrow \mu^+ + \nu_\mu \\
& \quad \quad \quad \quad e^+ + \nu_e + \bar{\nu}_\mu \\
\pi^- & \rightarrow \mu^- + \bar{\nu}_\mu \\
& \quad \quad \quad \quad e^- + \bar{\nu}_e + \nu_\mu \\
\end{align*}
\]

(3.1)
Neutrinos are mainly produced in the following decays of $\mu$'s, $\pi$'s, and $K$'s [138]:

\[
\begin{align*}
\pi^\pm & \rightarrow \mu^\pm \nu_\mu (\bar{\nu}_\mu) \quad (100 \%) \\
\mu^\pm & \rightarrow e^\pm \nu_e (\bar{\nu}_e) \nu_\mu (\bar{\nu}_\mu) \quad (100 \%) \\
K^\pm & \rightarrow \mu^\pm \nu_\mu (\bar{\nu}_\mu) \quad (63.5 \%) \\
& \rightarrow \pi^\pm \pi^0 \quad (21.2 \%) \\
& \rightarrow \pi^0 \mu^\pm \nu_\mu (\bar{\nu}_\mu) \quad (6.6 \%) \quad (K_{3\mu
u}) \\
& \rightarrow \pi^0 e^\pm \nu_e (\bar{\nu}_e) \quad (4.8 \%) \quad (K_{3e
u}) \\
& \rightarrow \pi^\pm \pi^0 \pi^0 \quad (2.5 \%) \quad (K_3\mu
u) \\
K_s^0 & \rightarrow \pi^+ \pi^- \quad (68.6 \%) \\
K_l^0 & \rightarrow \pi^+ \pi^- \pi^0 \quad (12.37 \%) \\
& \rightarrow \pi^0 \mu^\pm \nu_\mu (\bar{\nu}_\mu) \quad (27 \%) \quad (K_{3\mu
u}) \\
& \rightarrow \pi^0 e^\pm \nu_e (\bar{\nu}_e) \quad (36.6 \%) \quad (K_{3e
u}) \\
\end{align*}
\]

(3.2)

The decay of charged $\pi$'s and subsequent $\mu$ decay ($\pi - \mu$ decay):

\[
\begin{align*}
\pi^\pm & \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu) \\
& \rightarrow e^\pm + \nu_e (\bar{\nu}_e) + \bar{\nu}_\mu (\nu_\mu),
\end{align*}
\]

(3.3)

is dominant among these processes.

Looking at the above reactions one can naively say that the flux ratio for $(\nu_e + \bar{\nu}_e)/(\nu_\mu + \bar{\nu}_\mu)$ should be $\frac{1}{2}$ and $\bar{\nu}_\mu/\nu_\mu$ should be 1. However, if the energy of the muon becomes high ($\gtrsim 5$ GeV), muon tends not to decay in the air but to reach the Earth before decaying and it loses energy in the Earth, and decays after almost it gets stopped or it is captured by nuclei in the Earth. This effect reduces the ratio $(\nu_e + \bar{\nu}_e)/(\nu_\mu + \bar{\nu}_\mu)$ above this energy.

There are now several works that exists in literature which simulate the three dimensional atmospheric neutrino flux, e.g. the works of Honda et al.[96, 139], Battistoni et al.[140, 141], Wentz et al.[142], Favier et al.[143], Liu et al.[144] and Gaisser et al.[145]. But all of them are not considered at the same level. The flux given by Honda, Battistoni, and Gaisser are more relevant due to the high statistics in the Monte Carlo simulation. All of them use the AMS-BESS[146, 147] and BESS [148] based primary cosmic ray spectra model. The largest difference among them is in the interaction model. Honda et al. [96, 139] have used DPMJET3 with a little modification, Gaisser et al. [145] uses their own developed interaction model known as TARGET2 while Battistoni et al.[140, 141] uses FLUKA model. In Fig.(3.1), we have given a comparison of the atmospheric neutrino flux predicted by some of the groups. As one can notice that they are very close to each other and the difference is typically around 10 %, and maximum of around 20% below 3 GeV. For our numerical calculations we have taken the atmospheric neutrino flux given by Honda et al. for the Super-Kamiokande site. In the
past, atmospheric neutrinos have played an important role in establishing the phenomenon of neutrino oscillations via observing the dominant oscillation mode in the 2-3 sector of lepton mixing. The atmospheric neutrinos are coming to the earth from all directions, therefore, the path length plays an important role in the observation of oscillations. The incoming direction of a neutrino specifies the distance traveled from production source to the detector. The range of path lengths is from 10 km (depth of the atmosphere) to 13,000 km (diameter of the earth). Since the atmospheric neutrinos contain a wide band of energies, therefore, their interactions are divided into two different categories, one is known as fully contained (FC) event and the other is called partially contained (PC) event. FC events have their interaction vertex inside the fiducial volume of the detector and all produced particles stop within the inner detector and therefore are known as contained events, while in the case of PC event, vertices are also required to be within the fiducial volume, but in this case some of the produced particle tracks escape to the outer detector. The average neutrino energy is around 1 GeV for FC and around 10 GeV for PC events. FC events are further divided into sub-categories based on their visible energy, for example, at the Super-Kamiokande the sub-energy ranges are known as sub-GeV for $E_{vis} < 1.33$ GeV, and multi-GeV for $E_{vis} > 1.33$ GeV. The other broad category of atmospheric neutrino events is upward-going muons, which are created by neutrino interactions in the rock beneath the detector. They either stop inside the detector (upward stopping) or pass through the detector entirely (upward through-going). The average parent
neutrino energies for these events are around 10 GeV and 100 GeV, respectively. In Fig.(3.2) we have shown the distributions of neutrino energies that give rise to four classes of events.

The observation of atmospheric neutrino experiments started with the first reported results from the Kolar Gold Field(KGF) experiment in India, where a set of telescope counters consisting of iron, flash tubes and scintillators were used. Around 100 contained events and 229 neutrino induced horizontal muons were reported by this collaboration[149]. Another experiment that was performed sometimes later was at South Africa, named as South African Neutrino Detector(SAND). However, the real atmospheric neutrino experiments were started in 1980s with large detectors at IMB and Kamiokande. The IMB detector was a 6.9kT water Cerenkov detector located in a salt mine near Cleveland, Ohio while Kamiokande was the first version of water Cerenkov detector at the present Super-Kamiokande site in Japan. The idea that something is wrong in the ratio $R = \frac{\nu_e + \bar{\nu}_e}{\nu_{\mu} + \bar{\nu}_{\mu}}$, first came from the IMB experiment which was later confirmed by the Kamiokande experiment. Later most of the experiments measured $R$, which is known as the ratio of the ratios $R = \frac{\nu_{\mu} + \bar{\nu}_e}{\nu_e + \bar{\nu}_e}$ measured over $(\nu_{\mu} + \bar{\nu}_e)/(\nu_e + \bar{\nu}_e)$ MonteCarlo

which is basically done to minimize the uncertainty arising due to uncertainty in the neutrino flux as well as the cross sectional uncertainties. Naively, one would expect the ratio $R=1$ after looking at the reactions given in Eq.(3.3), while the observed ratio from the different experiments show a value which is much smaller than 1. These numbers from the different experiments have been tabulated in Table-3.1. The most promising results came from the Super-Kamiokande(SK) experiment which established that neutrinos really do oscillate and it changes from one flavor to the other. Super-Kamiokande is a 50 kT water Cerenkov detector, with 22.5 kT of fiducial volume located at a depth of 1 km under the mountain in the Gifu Prefecture, Japan. The detector is cylindrical, 42 m high and 39.3 m in diameter and the neutrino interactions in the water are observed by around 11,000 photomultiplier tubes facing inward and distributed evenly on the entire inner detector. The inner volume is surrounded by an outer detector which is instrumented with around 1900 smaller photomultiplier tubes.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$R(\mu/e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Super-Kamiokande (sub-GeV)</td>
<td>$0.658 \pm 0.016 \pm 0.035$</td>
</tr>
<tr>
<td>Super-Kamiokande (multi-GeV)</td>
<td>$0.702^{+0.032}_{-0.030} \pm 0.101$</td>
</tr>
<tr>
<td>Soudan2 &amp; Kamiokande</td>
<td>$0.69 \pm 0.10 \pm 0.06$</td>
</tr>
<tr>
<td>FREJUS</td>
<td>$1.00 \pm 0.15 \pm 0.08$</td>
</tr>
<tr>
<td>Kamiokande (sub-GeV)</td>
<td>$0.60^{+0.06}_{-0.05} \pm 0.05$</td>
</tr>
<tr>
<td>Kamiokande (multi-GeV) &amp; IMB</td>
<td>$0.57^{+0.08}_{-0.07} \pm 0.07$</td>
</tr>
</tbody>
</table>

Table 3.1: Value of $R(\mu/e)$ from different experiments.
Facing outward to aid in identification and rejection of cosmic ray events and events arising from interactions that took place in the rock surrounding the detector. Energy range that is using the Super-Kamiokande detector varies from a few MeV to up to a few hundreds of GeV. Recently the Super-Kamiokande-I and Super-Kamiokande-II full datasets were re-analyzed after improvements in both the Monte Carlo simulation and the event reconstruction algorithms. Some of the changes to the simulation include are for example, an update of the atmospheric neutrino flux model to the so-called Honda 06 model; various modifications to the neutrino interaction model e.g., change of quasi-elastic and single pion axial mass to \( M_A = 1.2 \) GeV, addition of lepton mass effect for charged-current single pion production, and addition of the pion-less delta decay channel \( \Delta \rightarrow N\gamma \), improvements to the detector simulation model of light reflections and scattering; better tuning of outer detector parameters in the simulation; and improvements in the ring counting algorithm. Furthermore, the analysis of the Super-Kamiokande-III dataset is underway for the atmospheric neutrinos. Also, it has been pointed out that the atmospheric neutrinos (ATM) are sensitive to the neutrino mass hierarchy if \( \theta_{13} \) is sufficiently large due to earth matter effect, mainly in multi-GeV e-like events and a sub-GeV e-like events provide sensitivity to the octant of \( \theta_{23} \) due to oscillations with \( \Delta m^2_{21} \). Now with the development of dedicated long base line (LBL) experiments, emphasis has been laid to have a combined LBL+ATM analysis. Therefore, Super-Kamiokande is entering into Phase-IV of the experiment where the neutrinos from the T2K accelerator (about which we shall talk about
CHAPTER 3. EVENT RATES FOR ATMOSPHERIC NEUTRINOS AT SUPER-KAMIOKANDE

in the next chapter) as well as the neutrinos from the atmosphere will be observed.

In this chapter, we have studied the lepton event rates for the atmospheric neutrinos at Super-Kamiokande with and without the nuclear medium effect and compared our results with the experimental observed events and also with the Monte Carlo predictions for the events used by Super-Kamiokande collaboration [151]. These results have been presented for a 22.5kT water fiducial mass on an exposure of 1489 live days, and we have taken the sub-GeV events in our analysis which have been classified as the region in which lepton’s energy $E_l < 1.33 \text{GeV}$ and minimum observed momenta of electrons and muons are 100MeV and 200MeV respectively [151]. Our Monte-Carlo analysis of the events have been done by considering the nuclear medium effect in the quasielastic, incoherent and coherent pion production processes, as they give the most dominant contribution in the sub-GeV energy region of atmospheric neutrino events. We have used the atmospheric neutrino flux given by Honda et al. [96],[151].

3.2 Total Cross Section

The double differential cross section $\sigma_0(E_l, |\vec{k}'|)$ for the basic reaction

$$\nu_l(k) + n(p) \rightarrow l^-(k') + p(p') ; l = e^-, \mu^-$$

is written as

$$\sigma_0(E_l, |\vec{k}'|) = \frac{|\vec{k}'|^2}{4\pi E_{\nu_l} E_l} \frac{M_n M_p}{E_n E_p} \Sigma |\mathcal{M}|^2 \delta[q_0 + E_n - E_p]$$

where $E_{\nu_l}$, $E_l$, $E_n$ and $E_p$ are the energies of the neutrino, final state lepton, neutron and proton respectively. $M_n$ is the mass of neutron and $M_p$ is the mass of proton. $k'$ is the momenta of lepton and $q_0 = E_{\nu_l} - E_l$ is the energy transfer. $|\mathcal{M}|^2$ is the matrix element square given by Eq.(2.10). When the above process (Eq.3.4) takes place inside the nucleus the nuclear effect becomes important which we have discussed in detail in chapter-2.

We have used the local density approximation to calculate the cross section in the Fermi gas model where the neutrino scatters from a neutron moving in the finite nucleus of neutron density $\rho_n(r)$, with a local occupation number $n_n(p, r)$. In the local density approximation the scattering cross section is written as

$$\sigma(E_l, |\vec{k}'|) = \int 2d\mathbf{r}d\mathbf{p} \frac{1}{(2\pi)^3} n_n(p, r) \sigma_0(E_l, k')$$

where $\sigma_0(E_l, |\vec{k}'|)$ is given by Eq.(3.5).

In the presence of nuclear medium effect, the total cross section $\sigma(E_{\nu_l})$, is written as

$$\sigma(E_{\nu_l}) = -2G_F^2 \cos^2 \theta_C \int_{r_{min}}^{r_{max}} r^2 dr \int_{k_{min}}^{k_{max}} k'dk' \int_{Q_{min}}^{Q_{max}} dQ^2 \int_{Q_{min}}^{Q_{max}} \frac{1}{E_{\nu_l} E_l} L^{(\nu)}_{\mu\nu} J^{RPA}_{\mu\nu}$$

$$ImU_N[E_{\nu_l} - E_l - Q_r - V_c(r), q]$$

(3.7)
3.3. EXPRESSION OF CROSS SECTION FROM DIFFERENT MODELS

where $G_F$ is the Fermi coupling constant, $\theta_C$ is the Cabibbo angle, $Q=-q$ is the momentum transfer, $Q_r$ is the Q-value of the reaction, $U_N$ is the Lindhard function and $V_C$ is the coulomb potential. $L^{(\nu)}_{\mu\nu}$ is the leptonic tensor and $J^{\nu}_{\mu RPA}$ is the modified hadronic tensor when RPA effect is incorporated. Expressions for these tensors are given and discussed in chapter-2.

Using these expressions we have calculated lepton events in our local Fermi gas model. We have also studied the dependence of the total cross section and event rates on the other versions of Fermi gas model used in literature for Monte Carlo simulation of event rates given by Llewellyn Smith [21], Smith & Moniz [22] and Gaisser O'Connell [36]. Here we have given the expression of the cross section prescribed by them [21, 22, 36] which we have used to calculate the total cross section and the event rates.

In the next section we are going to summarize the expression of the cross section used in various models.

3.3 Expression of cross section from different models

3.3.1 Expression used by Llewellyn Smith [21]

In this model the cross section per nucleon is equal to the cross section for a free nucleon multiplied by a factor $\{1 - \frac{D_n}{N}\}$ where

$$D = Z \text{ for } 2x < u - v$$
$$= \frac{1}{2} A \left\{1 - \frac{3x}{4}(u^2 + v^2) + \frac{x^3}{2} - \frac{3}{32x}(u^2 - v^2)^2\right\} \text{ for } u - v < x < u + v$$
$$= 0 \text{ for } x > u + v$$

(3.8)

with $x = \frac{|q|}{2k_F}$, $u = \left(\frac{2N}{A}\right)^{1/3}$, $v = \left(\frac{2Z}{A}\right)^{1/3}$ and N, Z, A are neutron, proton and mass numbers of the initial nucleus respectively, $k_F$ is the Fermi momentum and the three momentum transfer $|q| = \sqrt{(q^2 + m^2)/4M^2 - q^2}$. In this energy range neutrino-nucleus reactions has been calculated in the closure approximation with harmonic-oscillator potentials for closed-shell nuclei.

3.3.2 Expression of the cross section used by Smith and Moniz [22]

The expression of cross section for the process in which a charged lepton of mass $m_l$ is detected at an angle $\theta$ with respect to the incident neutrino, is

$$d^2\sigma = \frac{G^2}{2} \frac{1}{(2\pi)^2} \frac{1}{2|k_1.p|} \frac{dk_2}{2\epsilon_2} \eta_{\mu\nu} W_{\mu\nu}$$

(3.9)

where

$$\eta_{\mu\nu} = Tr\{\gamma_\mu (1 + \gamma_5) F_1 \gamma_\nu (1 + \gamma_5) F_2\}$$

(3.10)
\[ W_{\mu\nu} = (2\pi)^3 \Omega \sum \delta^{(4)}(p - p' - q) \langle p|j^{(-)}_\nu(0)|p'\rangle \langle p'|j^{(+)}_\mu(0)|p\rangle E. \tag{3.11} \]

\[ J^{(\pm)}_\mu(0) = V^{(\pm)}_\mu(0) + A^{(\pm)}_\mu(0) \]

is the nuclear weak current, \( \Omega \) is the quantization volume, \( E \) is the energy of the target, \( m \) is the proton mass and \( G \) is the weak coupling constant.

The form of \( W_{\mu\nu} \) under Lorentz and time reversal invariance is

\[ W_{\mu\nu} = W_1 \delta_{\mu\nu} + \frac{W_2}{m_T^2} p_\mu p_\nu + \frac{W_\alpha}{m_T^2} q_\mu q_\nu + \frac{W_\beta}{m_T^2} (p_\mu q_\nu + p_\nu q_\mu) + \frac{W_8}{m_T^2} \epsilon_{\mu\nu\sigma\tau} p_\sigma q_\tau \tag{3.12} \]

where \( m_T \) is the target mass, \( W_j \) are the form factors which depend only upon the scalars \( q^2 \) and \( q \cdot p \). Using the definition \( \cos \chi = k_2/\varepsilon_2 \cos \theta \), the cross section in the lab frame is given by

\[
\left( \frac{d^2\sigma}{dk_2 d\Omega_2} \right)_\nu = \frac{G^2 k_2^2 \cos^2(\frac{1}{2} \chi)}{2\pi^2 m_T} \left\{ W_2 + \frac{m_T^2}{W_\alpha} W_\alpha \tan^2(\frac{1}{2} \chi) \right. \\
+ \left. (W_\beta + W_8) m_T^2 / (m_T \varepsilon_2 \cos^2(\frac{1}{2} \chi)) - 2W_8/m_T \tan(\frac{1}{2} \chi) \right. \\
\times \sec(\frac{1}{2} \chi) \left[ q^2 \cos^2(\frac{1}{2} \chi) + |q|^2 \sin^2(\frac{1}{2} \chi) + m_T^2 \right] \right\} \tag{3.13} \]

For antineutrino reactions the sign of the form factor \( W_8 \) is reversed. In the lab frame \( W_{\mu\nu} \) can be expressed in terms of the single nucleon matrix elements:

\[
(W_{\mu\nu})_{\text{lab}} = \int dk \mathcal{F}(k,q,\omega) T_{\mu\nu} \tag{3.14} \]

where

\[
\mathcal{F}(k,q,\omega) = \frac{m_T \Omega}{(2\pi)^3} \frac{\delta(\varepsilon_k - \varepsilon_k - q + \omega)n_i(k)(1 - n_f(|k - q|))}{\varepsilon_k \varepsilon_{k-q}} \tag{3.15} \]

and

\[
T_{\mu\nu} = \varepsilon_k \delta_{\mu-k-q} \Omega^2 \sum_{\lambda,\lambda'} \langle k - q|j^{(+)}_\mu(0)|k\lambda\rangle \langle k\lambda|j^{(-)}_\nu(0)|k - q\lambda'\rangle \\
= T_1 \delta_{\mu\nu} + T_2/m^2 k_\mu k_\nu + T_\alpha/m^2 q_\mu q_\nu + T_\beta/m^2 (k_\mu q_\nu + k_\nu q_\mu) \\
+ T_8/m^2 \epsilon_{\mu\nu\sigma\tau} k_\sigma q_\tau \tag{3.16} \]

here single particle form factors \( T_j \) depend only upon \( q^2 \), \( n_i(k) \) is the neutron(proton) momentum distribution for incident neutrinos(antineutrinos), \( n_f(k) \) is the proton(neutron) momentum distribution for outgoing neutrinos(antineutrinos). For quasielastic scattering the Pauli exclusion factor \( (1 - n_f(|k - q|)) \) is included and for a pure Fermi gas model \( n_i(k) = \theta(k_F - |k|) \).

This factor ensures that the recoil nucleon lies outside the Fermi sea. Using the Lorentz transformation properties of \( W_{\mu\nu} \) and \( T_{\mu\nu} \), \( W_j \)'s can be determined in terms of \( T_j \)'s. In lab frame Eq.(3.12) becomes

\[
W_{\mu\nu} = W_1 \delta_{\mu\nu} - W_2 \delta_{\mu4} \delta_{\nu4} + W_\alpha/m_T^2 q_\mu q_\nu + iW_\beta/m_T (\delta_{\mu4} q_\nu + \delta_{\nu4} q_\mu) \\
- iW_8/m_T \epsilon_{\mu\nu\sigma4} q_\sigma \tag{3.17} \]
Now putting each term from Eq. (3.16) in Eq. (3.14) and comparing it with Eq. (3.17). After performing the angular integration, $W_i$’s can be given as

$$
\begin{align*}
W_1 &= a_1 T_1 + \frac{1}{2} (a_2 - a_3) T_2, \\
W_2 &= [a_4 + 2 \omega / |q| a_5 + \omega^2 / |q|^2 a_3 + \frac{1}{2} q^2 / |q|^2 (a_2 - a_3)] T_2, \\
W_\alpha &= m_T^2 / |q|^2 (\frac{1}{2} a_3 - \frac{1}{2} a_2) T_2 + m_T^2 / m T_a + 2 m_T^2 / (m |q|) a_6 T_\beta, \\
W_\beta &= m_T / m (a_7 + \omega / |q| a_6) T_\beta, \\
W_8 &= m_T / m (a_7 + \omega / |q| a_6) T_8
\end{align*}
$$

(3.18)

where

$$
\begin{align*}
a_1 &= \int \frac{d k \mathcal{F}(k, q, \omega)}{m^2} a_5 = \int d k \mathcal{F}(k, q, \omega) \frac{\epsilon_k k \cos \tau}{m^2}, \\
a_2 &= \int d k \mathcal{F}(k, q, \omega) \frac{k^2}{m^2}, \quad a_6 = \int d k \mathcal{F}(k, q, \omega) \frac{k \cos \tau}{m}, \\
a_3 &= \int d k \mathcal{F}(k, q, \omega) \frac{k^2 \cos^2 \tau}{m^2}, \quad a_7 = \int d k \mathcal{F}(k, q, \omega) \frac{\epsilon_k}{m}, \\
a_4 &= \int d k \mathcal{F}(k, q, \omega) \frac{\epsilon_k}{m^2}.
\end{align*}
$$

$a_j$’s contains all the information in the single particle momentum distributions and energies. For quasielastic scattering the matrix element is given by

$$
\langle k' \lambda' | j^{(+)}_{\mu} | 0 | k \lambda \rangle = i \left( \frac{m^2}{\epsilon_k \epsilon_{k'} \Omega^2} \right)^{\frac{1}{2}} \bar{u}(k' \lambda') \{ F_1 \gamma_\mu + F_2 \sigma_{\mu\alpha} q_\alpha \\
- i F_3 q_\mu \tau z + F_A \gamma_5 \gamma_\mu - i F_F \gamma_5 q_\mu + F_T \gamma_5 \sigma_{\mu\alpha} q_\tau \}_\tau u(k \lambda) \text{ (3.19)}
$$

Using this form of matrix element in Eq. (3.16), $T_j$’s can be written as

$$
\begin{align*}
T_1 &= \frac{1}{2} q^2 \{ F_1 + 2 m F_2 \}^2 + (2 m^2 + \frac{1}{2} q^2) F_F^2, \\
T_2 &= 2 m^2 \{ F_1^2 + q^2 P_F^2 + F_A^2 + q^2 P_F^2 \}, \\
T_\alpha &= - m^2 / q T_1 + \frac{1}{2} T_2 + 2 m F_s \left[ -2 m F_1 + q^2 F_2 + (2 m^2 + \frac{1}{2} q^2) F_s \right] \\
&+ m^2 \left[ 2 m F_A - q^2 F_F \right] \left[ - F_T + 1 / (2 q^2) (2 m F_A - q^2 F_F) \right], \\
T_\beta &= - \frac{1}{2} T_2 + m^2 F_s \left[ 2 m F_1 - q^2 F_2 \right] + m^2 F_T \left[ 2 m F_A - q^2 F_F \right], \\
T_8 &= 2 m^2 F_A \{ F_1 + 2 m F_2 \}.
\end{align*}
$$

(3.20)

3.3.3 Expression used by Gaisser and O’ Connell [36]

In the non-relativistic limit the expression of the differential cross section used by Gaisser and O’ Connell [36] is

$$
\frac{d^2 \sigma}{d \Omega \omega} = C d \sigma_0 \frac{R(q, \omega)}{d \Omega \omega} \text{ (3.21)}
$$
where $l = e^\pm, \mu^\pm$, $C = Z$ or $N$ depending upon $\bar{\nu}$ or $\nu$ and $d\sigma_0/d\Omega_l$ is the fundamental nucleon cross section given by

$$\frac{d\sigma_0(\nu/\bar{\nu})}{d\Omega} = \frac{G^2 k'E_l}{2\pi^2} \left\{ (F_1^2 + F_A^2 + Q^2 F_2^2) \cos^2 \frac{\theta}{2} + 2 \left[ F_A \left[ 1 + \frac{Q^2}{4M^2} \right] + \frac{Q^2}{4M^2}\mu^2 F_1^2 \right] \right\} \times \sin^2 \frac{\theta}{2} \pm \frac{2F_A \mu}{M} F_1 \mu \left[ \frac{Q^2 + \omega^2 \sin^2 \frac{\theta}{2}}{2} \right] \sin \frac{\theta}{2}$$

(3.22)

with $F_1 = (1 + Q^2/0.71GeV^2)^{-1/2}$, $2MF_2 = (\mu - 1)F_1$, $(\mu - 1)=3.71$, $F_A = -1.23F_1$, $GM^2 = 10^{-5}$ with $M$ as nucleon mass, $Q^2 = q^2 - \omega^2$, $\omega = E_\nu - E_l$ and $q = p_\nu - k'$. The factor $R(q, \omega)$ is the nuclear response function which contains the effect of nuclear binding and Fermi motion of the target nucleon and of the Pauli exclusion principle for the recoil nucleon. In the Fermi gas model, the nuclear response function $R(q, \omega)$ is given by

$$R(q, \omega) = \frac{1}{\pi p_F^2} \int \frac{d^3pM^2}{E_NE_N'} \delta(E_N + \omega - E_N') \theta(p_F - |p|) \theta(|p + q| - p_F) \left( \frac{E_l}{1 + 2(E_\nu/M)\sin^2\theta/2} \right)$$

(3.23)

where $p_F$ is the Fermi momentum, $\omega' = \omega - E_B$ with $E_B$ as the average nucleon separation energy. In the free nucleon limit ($p_F \to 0, E_B \to 0$), the nuclear response function becomes

$$R(q, \omega) \to \delta \left[ \frac{E_l}{1 + 2(E_\nu/M)\sin^2\theta/2} \right]$$

(3.24)

### 3.4 Results and Conclusions

#### 3.4.1 Quasielastic scattering

**A. Total Scattering Cross section**

In Figs.3.3(a) and (b), we have compared our results for the $\nu_\mu$ and $\bar{\nu}_\mu$ induced charged current quasielastic lepton production cross sections in $^{16}O$, obtained in the local Fermi gas model with and without the RPA effect, with the results obtained in the Fermi gas model given by Llewellyn Smith [21] and Smith and Moniz [22], which have been used in some of the Monte Carlo generators. We find that our results are fairly in good agreement (within 2%) with their results [21, 22] in the case of $\nu_\mu$ induced process, while in the case of $\bar{\nu}_\mu$ process the results obtained in our local Fermi gas model are in agreement with the results obtained by using Llewellyn Smith’s [21] Fermi gas model, however, the results obtained by Smith and Moniz [22] Fermi gas model is about 3-5% higher. We find that our results in the local FGM with RPA effect agree (not shown here) with the recent calculations performed by Nieves et al. [132], Benhar et al. [136] and Leitner et al. [131],[137].
3.4. RESULTS AND CONCLUSIONS

Figure 3.3: Total scattering cross section ($\sigma$) as a function of neutrino energy for $\nu_\mu$ ($\bar{\nu}_\mu$) induced reaction in $^{16}O$. These results have been presented for the cross sections calculated by using different models.

B. Differential Scattering Cross Section $<\frac{d\sigma}{dQ^2}>$

In this section we shall present the results of the flux averaged differential scattering cross section $<\frac{d\sigma}{dQ^2}>$ as a function of $Q^2$. This has been obtained by integrating $\frac{d\sigma}{dQ^2}$ over the atmospheric neutrino flux given by Honda et al. [96] for the Super-Kamiokande cite. We have parameterized the flux using B-spline function [150] and we have ensured that the $\chi^2$ is very close to zero, therefore, the uncertainty arising due to the parametrization of the atmospheric neutrino flux is negligibly small. The flux averaged differential scattering cross section $<\frac{d\sigma}{dQ^2}>$ is defined as

$$<\frac{d\sigma}{dQ^2}> = \frac{\int_{E_{\nu_{\min}}}^{E_{\nu_{\max}}} \frac{d\sigma}{dQ^2} \phi(E) dE}{\int_{E_{\nu_{\min}}}^{E_{\nu_{\max}}} \phi(E) dE}$$  \hspace{1cm} (3.25)$$

where $\frac{d\sigma}{dQ^2}$ is the differential scattering cross section for the $Q^2$ distribution and $\phi(E)$ is the atmospheric neutrino flux(Fig.3.4) given by Honda et al. [96],[151]. In Figs.3.5(a) and (b), we present the results for the $Q^2$-distribution in the case of charged current quasielastic lepton production process induced by electron and muon neutrino(antineutrino) averaged over the atmospheric neutrino flux. The results have been presented for the $Q^2$-distribution calculated in the local Fermi gas model with and without RPA effect. In the case of $\nu_e$ and $\nu_\mu$ induced processes the results have been shown in Fig.3.5(a). We find that in the case of $\nu_e$ induced process when there is no cut applied on the electron’s energy and momenta, and the differential
cross section is calculated in the local Fermi gas model with RPA effect, the reduction in the cross section is around 42% in the peak region of $Q^2(=0.044\text{GeV}^2)$ and around 30% at $Q^2=0.2\text{GeV}^2$ as compared to the cross section calculated without the RPA effect. For $\bar{\nu}_e$, there is a slight shift in the peak region which is towards low $Q^2(=0.022\text{GeV}^2)$ and the reduction is around 35% which is smaller than in the case of $\nu_e$ induced process and at high $Q^2(=0.1\text{GeV}^2)$ the reduction is around 30% as compared to the cross section calculated without the RPA effect. When a cut on the electron’s energy ($E_e < 1.33\text{GeV}$) and momenta ($p_e \geq 100\text{MeV}$) are applied, there is a small change in the nature of reduction which results in the suppression of lepton events in the peak region of $Q^2$. In the case of $\nu_\mu$ induced process the reduction in the cross section is around 40% in the peak region of $Q^2(=0.06\text{GeV}^2)$ and around 30% at $Q^2=0.2\text{GeV}^2$ as compared to the cross section calculated without the RPA effect. In the case of $\bar{\nu}_\mu$, there is a shift in the peak region which is towards low $Q^2(=0.028\text{GeV}^2)$ and the reduction is around 36% which is smaller than in the case of $\nu_\mu$ induced process and at high $Q^2(=0.2\text{GeV}^2)$ the reduction is around 22% as compared to the cross section calculated without the RPA effect. When a cut on the muon’s energy ($E_\mu < 1.33\text{GeV}$) and momenta ($p_\mu \geq 200\text{MeV}$) are applied, then there is a large change in the nature of reduction. For example, in the case of $\nu_\mu$ induced reaction the reduction due to RPA effect is around 35% in the peak region and becomes 30% around $Q^2 = 0.2\text{GeV}^2$. While in the case of $\bar{\nu}_\mu$ induced reaction this reduction is around 32% in the peak region and becomes 24% around $Q^2 = 0.2\text{GeV}^2$. We find that the reduction in the $Q^2$ distribution when we apply cuts on the lepton's energy and momenta, is around 40% in the peak region which becomes around 15% at $Q^2 = 0.2\text{GeV}^2$ in the case of $\nu_\mu$ induced lepton production calculated in the local
3.4. RESULTS AND CONCLUSIONS

Figure 3.5: $<\frac{d\sigma}{dQ^2}>$ vs $Q^2$ for the quasielastic process induced by electron type (upper panels) and muon type (lower panels) (a) neutrino and (b) antineutrino in $^{16}$O. The results are presented for the $Q^2$-distribution with and without applying cuts on the lepton’s energy and momenta. The dashed (dotted) line is the result in the local FGM with (without) RPA effect and without cuts. The solid (dashed-dotted) line is the result in the local FGM with (without) RPA effect and with cuts ($E_l < 1.33 GeV$, $p_e \geq 100 MeV$ and $p_\mu \geq 200 MeV$).

Fermi gas model with RPA effect, while for the $\bar{\nu}_\mu$ induced lepton production this reduction is around 38% in the peak region which becomes around 3% at $Q^2 = 0.2 GeV^2$. Thus it has been observed that in the case of $\nu_\mu$ and $\bar{\nu}_\mu$ induced processes by applying cuts, the differential cross section flattens when calculated in the local FGM with RPA effect. This would give rise to considerably large reduction in the event rates in comparison to the muon events calculated by using the local Fermi gas model without RPA effect as well as without applying any cuts on the lepton energy and momenta.

To show the dependence of the different flux on the Super-Kamiokande site for the Solar minimum and Solar maximum defined by Kam1997 and Kam2000 by Honda et al. [96], we have obtained the numerical results for the $Q^2$ distribution in the case of charged current quasielastic lepton production process induced by $\nu_\mu$ and $\bar{\nu}_\mu$ reactions in $^{16}$O calculated in the local Fermi gas model with RPA effect. The results for $\nu_\mu$ induced process is shown in
Figure 3.6: $\langle \frac{d\sigma}{dQ^2} \rangle$ vs $Q^2$ for quasielastic process induced by muon type (a) neutrino and (b) antineutrino averaged over the atmospheric neutrino flux for the solar minimum and solar maximum given by Honda et al. [96], [151].

Fig.3.6(a) and for the $\bar{\nu}_\mu$ induced process the results are shown in Fig.3.6(b). We find that these two fluxes result in a very small difference in the $Q^2$ distribution.

C. Total lepton events

Here we are going to present the results for the total number of lepton events for the sub-GeV energy region. These results have been presented for a 22.5kT water fiducial mass for 1489 days, and we have put a cut on the lepton’s energy $E_l < 1.33 GeV$ and momenta of electrons and muons as $p_e > 100MeV$ and $p_\mu > 200MeV$. We have integrated the total scattering cross section $\sigma$ over the atmospheric neutrino flux given by Honda et al. [151] for the Super-Kamiokande cite.

In our Monte Carlo events for predicting the lepton events, we have considered the following channels. In the quasielastic process the contributions to the lepton events have been taken from the channels (i) $\nu n \rightarrow l^- p (in \ ^{16}O)$, (ii) $\bar{\nu}_p \rightarrow l^+ n (in \ ^{16}O)$ and (iii) $\bar{\nu}_p \rightarrow l^+ n$ (on free p due to $H_2$).

The flux averaged cross section is defined as

$$<\sigma> = \int_{E_{\nu_{min}}}^{E_{\nu_{max}}} \sigma(E) \phi(E) dE$$

(3.26)

where $\sigma(E)$ is the total scattering cross section and $\phi(E)$ is the atmospheric neutrino flux.
### 3.4. RESULTS AND CONCLUSIONS

#### Table 3.2: $<\sigma>$ in $\text{cm}^2$ in $\nu_l + \nu_e$ scattering averaged over Kamioka 1997 flux given by Honda et al.\cite{96}, \cite{151} for the Free case, our local Fermi gas model without RPA, Llewellyn Smith Fermi Gas Model (LS FGM)\cite{21} and Gaisser and O’Connell Fermi Gas Model (GO FGM)\cite{36} for different axial masses. Results presented here are without any momentum cut.

<table>
<thead>
<tr>
<th>$&lt;\sigma&gt;$</th>
<th>Free</th>
<th>Without RPA</th>
<th>LS FGM [21]</th>
<th>GO FGM [36]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_A$ 1.05 GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>4.0</td>
<td>3.03</td>
<td>3.18</td>
<td>3.0</td>
</tr>
<tr>
<td>$\bar{\nu}_e$</td>
<td>1.30</td>
<td>0.76</td>
<td>0.84</td>
<td>0.79</td>
</tr>
<tr>
<td>$M_A$ 1.10 GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>4.11</td>
<td>3.13</td>
<td>3.28</td>
<td>3.05</td>
</tr>
<tr>
<td>$\bar{\nu}_e$</td>
<td>1.32</td>
<td>0.78</td>
<td>0.86</td>
<td>0.81</td>
</tr>
<tr>
<td>$M_A$ 1.21 GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>4.32</td>
<td>3.32</td>
<td>3.49</td>
<td>3.24</td>
</tr>
<tr>
<td>$\bar{\nu}_e$</td>
<td>1.38</td>
<td>0.82</td>
<td>0.91</td>
<td>0.86</td>
</tr>
</tbody>
</table>

#### Table 3.3: $<\sigma>$ in $\text{cm}^2$ in $\nu_l + \nu_e$ scattering averaged over Kamioka 1997 flux given by Honda et al.\cite{96}, \cite{151} for the Free and Llewellyn Smith Fermi Gas Model (LS FGM)\cite{21} with different parametrization of form factors at $M_A=1.1$ GeV.

<table>
<thead>
<tr>
<th>$&lt;\sigma&gt;$</th>
<th>BBBA05 [119]</th>
<th>BBA03 [118]</th>
<th>Bosted [120]</th>
<th>Dipole [113]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>4.11</td>
<td>4.17</td>
<td>4.17</td>
<td>4.19</td>
</tr>
<tr>
<td>$\bar{\nu}_e$</td>
<td>1.32</td>
<td>1.35</td>
<td>1.33</td>
<td>1.36</td>
</tr>
<tr>
<td>LS FGM [21]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>3.28</td>
<td>3.34</td>
<td>3.336</td>
<td>3.355</td>
</tr>
<tr>
<td>$\bar{\nu}_e$</td>
<td>0.86</td>
<td>0.89</td>
<td>0.88</td>
<td>0.90</td>
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</table>
### Table 3.4: $<\sigma'>^*_{10^{-38}}\text{cm}^2$ in $\nu_1+^{16}\text{O}$ scattering averaged over Kamioka 1997 flux given by Honda et al. [96], [151] using BBBA05 [119] form factor for different axial mass values.

\[<\sigma' > = \int \phi(E_\nu)\sigma(E_\nu)dE_\nu; \quad p_\nu \geq 100\text{MeV for electron type and } p_\mu \geq 200\text{MeV for muon type neutrinos and antineutrinos.}\]

<table>
<thead>
<tr>
<th>$M_A$</th>
<th>$\nu_e$</th>
<th>$\bar{\nu}_e$</th>
<th>$\nu_\mu$</th>
<th>$\bar{\nu}_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05\text{GeV}</td>
<td>0.16</td>
<td>0.12</td>
<td>0.16</td>
<td>0.12</td>
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<tr>
<td></td>
<td>0.035</td>
<td>0.026</td>
<td>0.034</td>
<td>0.025</td>
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<td></td>
<td>0.28</td>
<td>0.21</td>
<td>0.23</td>
<td>0.18</td>
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<tr>
<td></td>
<td>0.074</td>
<td>0.054</td>
<td>0.064</td>
<td>0.048</td>
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<tr>
<td>1.10\text{GeV}</td>
<td>0.17</td>
<td>0.12</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>0.036</td>
<td>0.027</td>
<td>0.035</td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>0.29</td>
<td>0.22</td>
<td>0.24</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>0.076</td>
<td>0.056</td>
<td>0.066</td>
<td>0.050</td>
</tr>
<tr>
<td>1.21\text{GeV}</td>
<td>0.178</td>
<td>0.13</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>0.038</td>
<td>0.028</td>
<td>0.037</td>
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</tr>
<tr>
<td></td>
<td>0.31</td>
<td>0.23</td>
<td>0.26</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>0.06</td>
<td>0.07</td>
<td>0.05</td>
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</tbody>
</table>

### Table 3.5: Total number of events for a quasielastic process induced by $\nu_\mu$ and $\bar{\nu}_\mu$. †: For reaction on free protons the events would be the same in all the three columns.

<table>
<thead>
<tr>
<th>Quasielastic Process</th>
<th>Free Case</th>
<th>FGM</th>
<th>FGM With RPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>I $\nu_\mu n \rightarrow \mu^- p (in ^{16}\text{O})$</td>
<td>3332</td>
<td>2472</td>
<td>1894</td>
</tr>
<tr>
<td>II $\bar{\nu}_\mu p \rightarrow \mu^+ n (in ^{16}\text{O})$</td>
<td>966</td>
<td>620</td>
<td>461</td>
</tr>
<tr>
<td>III $\nu_\mu p \rightarrow \mu^+ n (on free p due to H_2)$</td>
<td>241</td>
<td>241†</td>
<td>241†</td>
</tr>
<tr>
<td>$\nu_\mu + \bar{\nu}_\mu$</td>
<td>4539</td>
<td>3232</td>
<td>2596</td>
</tr>
</tbody>
</table>
3.4. RESULTS AND CONCLUSIONS

<table>
<thead>
<tr>
<th>Quasielastic Process</th>
<th>Free Case</th>
<th>FGM</th>
<th>FGM with RPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>XII $\nu_e n \rightarrow e^- p$ (in $^{16}O$)</td>
<td>2332</td>
<td>1754</td>
<td>1278</td>
</tr>
<tr>
<td>XIII $\bar{\nu}_e p \rightarrow e^+ n$ (in $^{16}O$)</td>
<td>609</td>
<td>358</td>
<td>266</td>
</tr>
<tr>
<td>XIV $\nu_e p \rightarrow e^+ n$ (on free p due to $H_2$)</td>
<td>152</td>
<td>152</td>
<td>152</td>
</tr>
<tr>
<td>$\nu_e + \bar{\nu}_e$</td>
<td>3093</td>
<td>2264</td>
<td>1696</td>
</tr>
</tbody>
</table>

Table 3.6: Total number of events for a quasielastic process induced by $\nu_e$ and $\bar{\nu}_e$.

In Table-3.2, we have compared our result for the free case and local Fermi gas model without RPA effect with the results for the cross section obtained by using Llewellyn Smith Fermi gas model (LS FGM) and Gaissier O’ Connell Fermi gas model (GO FGM). We find that for charged current quasielastic lepton production process the cross section averaged over Kamioka 1997 flux calculated in local Fermi gas model without RPA effect are in good agreement (within 4-5%) with the results obtained by using Llewellyn Smith Fermi gas model and Gaissier O’ Connell Fermi gas model. When we calculate the cross section averaged over Kamioka 1997 flux in our local Fermi gas model without RPA effect for the $\nu_\mu$ induced process decreases by around 25% as compared to the cross section calculated in free case. While for the $\bar{\nu}_\mu$ induced quasielastic lepton production process the cross section decreases by around 40% as compared to the cross section calculated in free case.

In Table-3.3, results for the cross section averaged over Kamioka 1997 flux given by Honda et al. [96], [151] for the Free case and Llewellyn Smith Fermi gas model (LS FGM) with different parametrization of form factors at $M_A=1.1$GeV are presented. We find that for charged current quasielastic lepton production process the cross section averaged over Kamioka 1997 flux calculated with various parametrization of form factors agree within 1-2%.

In Table-3.4, we present the results for $\nu_l$ and $\bar{\nu}_l$ ($l=e, \mu$) induced cross section averaged over Kamioka 1997 flux given by Honda et al. [96],[151] using different axial dipole mass. We have taken $M_A=1.1$GeV as base in our numerical calculation. For $\nu_\mu$ induced charged current quasielastic lepton production process when we take $M_A=1.05$GeV then there is a decrease of around 3% in the cross section as compared to the cross section calculated with $M_A=1.1$GeV. While if we take $M_A=1.21$GeV then there is an increase of around 6% in the averaged cross section as compared to the cross section calculated with $M_A=1.1$GeV. In the case of $\bar{\nu}_\mu$ induced charged current quasielastic lepton production process when we take $M_A=1.05$GeV then there is a decrease of around 2% in the cross section averaged over Kamioka 1997 flux as compared to the cross section calculated with $M_A=1.1$GeV. While if we take $M_A=1.21$GeV then there is an increase of around 5% in the averaged cross section as compared to the cross section calculated with $M_A=1.1$GeV.

With the incorporation of RPA effect into account the reduction in the cross section averaged over Kamioka 1997 flux is around 25% as compared to the cross section calculated in
Figure 3.7: $<\frac{d\sigma}{dQ^2}>$ vs $Q^2$ for the incoherent process induced by electron type (upper panels) and muon type (lower panels) (a) neutrino and (b) antineutrino. The results are presented for the $Q^2$-distribution with and without cut on the lepton’s energy ($E_l < 1.33\,\text{GeV}$) and momenta for electron (muon) as $p_e \geq 100\,\text{MeV}$ ($p_\mu \geq 200\,\text{MeV}$). The solid (dashed-dotted) line is the result with ME and FSI effect taking into account with (without) cut.

our local Fermi gas model without RPA effect. To compare our result with that of the Super-Kamiokande site we have put a cut on the lepton’s energy $E_\mu < 1.33\,\text{GeV}$ and momenta of electrons and muons as $p_e > 100\,\text{MeV}$ and $p_\mu > 200\,\text{MeV}$ respectively. For $\nu_\mu$ induced charged current quasielastic lepton production process when we take $M_A=1.1\,\text{GeV}$ and apply cut on the muon’s energy ($E_\mu < 1.33\,\text{GeV}$) and momenta ($p_\mu > 200\,\text{MeV}$) then there is a decrease of around 15% in the cross section averaged over Kamioka 1997 flux as compared to the cross section calculated without applying any cut. We find that if the cross section is calculated in the local Fermi gas model with RPA effect and averaged over Kamioka 1997 atmospheric neutrino flux by putting cuts on muon’s energy and momenta the reduction in the event rates is about 18% as compared to the event rate obtained without putting cut on muon’s energy and momenta.

Similar are the results for the $\nu_e (\bar{\nu}_e)$ induced quasielastic lepton production process.

In Table-3.5 and Table-3.6 the lepton event rates have been obtained for the charged current quasielastic lepton production processes induced by $\nu_l$ and $\bar{\nu}_l$ ($l = e, \mu$). We find that in the
### 3.4. RESULTS AND CONCLUSIONS

In the case of quasielastic process when we calculate the lepton events in the local Fermi gas model without RPA effect, there is a reduction of around 30\% in the event rates as compared to the events calculated for the free case. When we incorporate RPA effect in our local Fermi gas model then there is a further reduction of around 20\% in the lepton event rates.

#### 3.4.2 Incoherent Process

The basic reaction for the charged current neutrino(antineutrino) induced CC1\(\pi\) pion production in nuclei, is that a neutrino(antineutrino) interacts with a nucleon N.

In the local density approximation the expression for the total cross section for the neutrino induced charged current 1\(\pi^+\) production is written as (Chapter-2)

\[
\sigma_A(E) = \frac{1}{(4\pi)^5} \int_{r_{min}}^{r_{max}} (\rho_p(r) + \frac{1}{9} \rho_n(r)) d\vec{r} \int_{Q_{min}^2}^{Q_{max}^2} dQ^2 \times \int_{k_{min}'}^{k_{max}'} dk' \int_{-1}^{+1} d\cos\theta_{\pi} \times d\phi \times \frac{\pi |\vec{k}'||\vec{k}|}{ME_p^2 E_i} \frac{1}{E_p + E_{\pi}} \left(1 - \frac{|q|}{|\vec{k}| \cos\theta_{\pi}}\right) \Sigma \Sigma |M_{fi}|^2
\]  

(3.27)
In this section we shall present the results for the flux averaged differential scattering cross section \( \frac{d\sigma}{dQ^2} \) as a function of \( Q^2 \) for the incoherent process.

The \( Q^2 \) distribution in the case of charged current electron and muon neutrino (antineutrino) induced incoherent \( 1\pi^+(1\pi^-) \) production processes have been shown in Figs.3.7(a) and (b). The results have been shown without and with the effect of nuclear medium as well as with the final state interaction of pions taken into account along with the nuclear medium effect. In the case of \( \nu_e \) and \( \nu_{\mu} \) induced processes the results have been shown in Fig.3.7(a). We find that in the case of \( \nu_e \) induced process when the cut is applied on the electron’s energy \( (E_e < 1.33 GeV) \) and momenta \( (p_e \geq 100 MeV) \), and the nuclear medium effect are also taken into account, the reduction in the cross section is around 36% in the peak region of \( Q^2(=0.08 GeV^2) \) and around
3.4. RESULTS AND CONCLUSIONS

<table>
<thead>
<tr>
<th>Process</th>
<th>$\nu_e + \bar{\nu}_e$</th>
<th>$\nu_\mu + \bar{\nu}_\mu$</th>
<th>$R = \frac{\nu_e + \bar{\nu}<em>e}{\nu</em>\mu + \bar{\nu}_\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free case (QE+Inelastic)</td>
<td>4057</td>
<td>6084</td>
<td>0.667</td>
</tr>
<tr>
<td>FGM without RPA+Inelastic with nuclear medium and final state interaction effect</td>
<td>2973</td>
<td>4499</td>
<td>0.66</td>
</tr>
<tr>
<td>FGM with RPA + Inelastic with nuclear medium and final state interaction effect</td>
<td>2405</td>
<td>3762</td>
<td>0.64</td>
</tr>
<tr>
<td>Monte Carlo events</td>
<td>2533.9</td>
<td>3979.7</td>
<td>0.636</td>
</tr>
<tr>
<td>Reported by experiments</td>
<td>3353</td>
<td>3227</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 3.9: Ratio $R = \frac{\nu_e + \bar{\nu}_e}{\nu_\mu + \bar{\nu}_\mu}$.

33% at $Q^2=0.7\text{GeV}^2$ as compared to the cross section calculated without medium effect. When final state interactions effect are also taken into account there is a further reduction of around 15% in the peak region and around 14% at $Q^2=0.7\text{GeV}^2$. For $\bar{\nu}_e$, there is a shift in the peak region which is towards low $Q^2(=0.002\text{GeV}^2)$ and the reduction is around 35% and at high $Q^2 (=0.2\text{GeV}^2)$ the reduction is around 30% as compared to the cross section calculated without medium effect. And when we compare our final results with medium and final state interaction effect taken into account, but with and without cuts on the electron's energy and momenta, it has been found that in the case of $\nu_e$ induced reaction the reduction is around 30% in the peak region and becomes 25% around $Q^2=0.7\text{GeV}^2$. However, in the case of $\bar{\nu}_e$ induced reaction this reduction is around 35% in the peak region and becomes 30% around $Q^2=0.2\text{GeV}^2$.

In the case of $\nu_\mu$, when the differential cross section is calculated by applying cuts on the lepton's energy and momenta, with the nuclear medium effect taken into account, the reduction in the cross section is around 35% in the peak region of $Q^2(\approx 0.1\text{GeV}^2)$ as compared to the cross section calculated without the nuclear medium effect. When pion absorption effect is also taken into account there is further reduction of about 15%. In the case of $\bar{\nu}_\mu$ induced process there is a shift in the peak region which is towards low $Q^2(=0.02\text{GeV}^2)$ and the nature of reduction is almost the same as in the case of $\nu_\mu$ induced process. When there is no cut applied on the muon’s energy and momenta, there is a change in the nature of reduction. For example, in the case of $\nu_\mu$ induced process the reduction in the cross section calculated with the nuclear medium effect taken into account, is around 35% in the peak region of $Q^2$ as compared to the differential cross section calculated without the nuclear medium effect, and when pion absorption effect is also taken into account there is further reduction of about 14%, while for $\bar{\nu}_\mu$ induced process this reduction is around 30% in the peak region and with medium and final state interaction effect taken into account, the further reduction in the differential cross section is around 12%. When we compare our final results with medium and final state interaction effect taken into account, but with and without cuts on the muon’s energy and
momenta, it has been found that in the case of $\nu_\mu$ induced reaction the reduction is around 45% in the peak region and becomes 32% around $Q^2=0.7\text{GeV}^2$. However, in the case of $\bar{\nu}_\mu$ induced reaction this reduction is around 35% in the peak region and becomes 24% around $Q^2=0.2\text{GeV}^2$.

B. Total lepton events

Here we are going to present the results for the total number of lepton events for the sub-GeV energy region for the incoherent process where we have put a cut on the lepton’s energy $E_l<1.33\text{GeV}$ and momenta of electrons and muons as $p_e>100\text{MeV}$ and $p_\mu>200\text{MeV}$.

In the case of incoherent pion production process the various channels contributing to the lepton events are (i) $\nu_l p \rightarrow l^- \Delta^{++}$ (on free $p$ due to $H_2$), (ii) $\nu_l p \rightarrow l^+ \Delta^0$ (on free $p$ due to $H_3$), (iii) $\nu_l^{16}O$ ($l^-$ accompanied by $\pi^0$), (iv) $\bar{\nu}_l^{16}O$ ($l^+$ accompanied with $\pi^0$), (v) $\nu_l^{16}O$ ($l^-$ accompanied with $\pi^+$), (vi) $\bar{\nu}_l^{16}O$ ($l^+$ accompanied with $\pi^-$).

In Table-3.7 and Table-3.8 the lepton event rates have been obtained for the CC1$\pi$ production due to $\nu_l$ and $\bar{\nu}_l$ ($l=e,\mu$)induced reactions in $^{16}O$, as well as the leptons obtained from $\nu_l$ and $\bar{\nu}_l$ ($l=e,\mu$)induced reactions on the free protons and the leptons accompanied with $\pi^0$ in the neutrino(antineutrino) induced processes. We find that in the case of inelastic process the reduction in the event rate is around 25% when we incorporate nuclear medium and final state interaction effect as compared to the events calculated without nuclear medium effect. Therefore, when we calculate lepton event rates with nuclear medium and final state interaction effect in the case of leptons obtained from the inelastic process and the quasielastic lepton production cross section is calculated in the local Fermi gas model with RPA effect along with the quasi-like events, the total event rate reduces by about 40% as compared to the event rates obtained from the inelastic channel without nuclear medium effect and quasielastic lepton events calculated in the local Fermi gas model.

3.4.3 Coherent Process

Total lepton Events

In the case of coherent process the contributions would be made from (i)$\nu_l +^{16}O \rightarrow l^- + \pi^+ +^{16}O$ and (ii)$\bar{\nu}_l +^{16}O \rightarrow l^+ + \pi^- +^{16}O$ processes. We have used Eq.(2.113) along with Eq.(2.114) to calculate the event rates averaged over the atmospheric neutrino flux.

3.5 Comparison with the experimental and Monte Carlo Super-Kamiokande data

In Table-3.9, we have presented the total lepton events from $\nu_l + \bar{\nu}_l$ ($e$ or $\mu$) induced process obtained for the quasielastic and inelastic pion production processes. Here we have compared our final results (leptons obtained from CCQE reaction in local FGM with RPA effect + CC1$\pi$
production with nuclear medium and final state interaction effect) with the experimentally observed lepton events by Super-Kamiokande collaboration and also with the lepton events used in the Monte Carlo analysis of these events by the Super-Kamiokande collaboration.

In the case of muon and electron events we find that with nuclear medium effect the total reduction in the event rate is around 40% as compared to the event rate calculated without the nuclear medium effect.

Thus we find that the nuclear medium effect reduces the lepton event rate and plays a very important role in the study of lepton events in the sub-GeV energy range of atmospheric neutrinos.