Chapter 5

Neutral current induced $\pi^0$ production and neutrino magnetic moment

5.1 Introduction

Recently lot of activities in measuring the neutral current induced pion production event have come up like the experiments performed at K2K [103, 104] and MiniBooNE [105, 106, 107], as well as the proposed experiments at T2K [108, 109] and NOνA [110]. The importance of observing neutral current $\pi^0$ events was first emphasized by Vissani and Smirnov [161], for the analysis of atmospheric neutrino oscillation experiments. It was proposed by them that the single pion events in the energy region of 0.5-1.5GeV can be used to distinguish between the different types of oscillations, where a neutral current (NC) induced event contains a $\pi^0$, while a charged current (CC) induced event contains a $\pi^+$ or $\pi^-$. The $\pi^0$ neutral current event is detected by its decay into two photons giving rise to two diffuse e-like rings, while a $\nu_e(\bar{\nu}_e)$ induced charged current event is detected as one diffuse ring due to $e^-$ or $e^+$, and one sharp ring due to $\pi^+$ or $\pi^-$, and a $\nu_\mu(\bar{\nu}_\mu)$ induced charged current event is detected as two sharp rings, one from $\mu^-$ or $\mu^+$, and another due to $\pi^+$ or $\pi^-$. In the case of atmospheric neutrinos $\nu_\tau$ events may be ignored in comparison to $\nu_e$ and $\nu_\mu$ events due to small flux. The two ratios, NC event to $\nu_e$ CC event i.e. $R = \frac{N_{\pi^0}}{N_{e-like}}$ and NC event to $\nu_\mu$ CC event i.e. $R = \frac{N_{\pi^0}}{N_{\mu-like}}$ are free from the uncertainties of the theoretical flux calculations and therefore can give an important information on the neutrino flavor oscillations. Total rate of $\pi^0$ events in the absence of oscillations equals to

$$N_{\pi^0} = N_{\pi^0}^{NC} + N_{\pi^0}^{CC\mu} + N_{\pi^0}^{CCe}$$

and the relative partial contributions can be estimated as [161]:

$$N_{\pi^0}^{NC} : N_{\pi^0}^{CC\mu} : N_{\pi^0}^{CCe} = 0.80 : 0.18 : 0.02$$
and the rate of electron like events without oscillations

\[ N^0_e = N^{CCe}_e + N^{NC}_e + N^{CC\mu}_e, \]

where \( N^{CCe}_e \) is the contribution from \( \nu_e \) charged current reaction, \( N^{NC}_e \) is the contribution from neutral current reaction and \( N^{CC\mu}_e \) is the contribution from \( \nu_\mu \) charged current reaction. Their relative contributions at the Super-Kamiokande site for the Sub-GeV events were estimated to be

\[ N^{CCe}_e : N^{NC}_e : N^{CC\mu}_e = 0.90 : 0.08 : 0.02. \]

The ratio \( R^\pi_0/e = \frac{\frac{N^0_e}{e^{-}}}{\frac{N^{NC}_e}{e^{-}}} \) is expected to be 1 for \( \nu_\mu \rightarrow \nu_\tau \) oscillations. Any deviation of \( R^\pi_0/e \) from 1 implies \( \nu_\mu \) oscillation into other flavors except \( \nu_\tau \).

Furthermore, it has been recently pointed out that the neutral current \( \pi^0 \) production in neutrino interactions plays an important role in the background studies of \( \nu_\mu \rightarrow \nu_\tau \) oscillations in the appearance mode as well as in discriminating between \( \nu_\mu \rightarrow \nu_\tau \) and \( \nu_\mu \rightarrow \nu_s \) modes [98, 99, 100, 101]. This process can also help to distinguish between production of \( \nu_\tau \) and \( \bar{\nu}_\tau \) in some oscillation scenarios at neutrino energies much below the \( \tau \) production threshold but above the pion threshold [102].

The neutral pions can also be produced by electromagnetic interactions if \( \nu(\bar{\nu}) \) have diagonal and/or transition magnetic moments. This process would in principle contribute additional events to the neutral current reaction and would modify the energy and angular distributions of the neutral pions, which may be observed in future experiments. It is thus possible, in principle, to get information about the magnetic moment of neutrinos (antineutrinos) from studying neutral current induced \( \pi^0 \) production from nucleons and nuclei. While the minimal extensions of standard model predict very tiny diagonal magnetic moments [162], there are models of electroweak interactions which predict enhanced transition magnetic moment [111]. The present limits for the magnetic moment of neutrinos come from neutrino - electron scattering for \( \nu_\mu \) and from \( e^+ e^- \rightarrow \nu \bar{\nu} \gamma \) for \( \nu_\tau \). These limits for \( \nu_\mu \) and \( \nu_\tau \) magnetic moments are \( \mu_{\nu_\mu} < 6.8 \times 10^{-10} \mu_B \) and \( \mu_{\nu_\tau} < 3.9 \times 10^{-7} \mu_B \) [163].

The data from neutrino oscillation experiments from Sudbury Neutrino Observatory and Super-Kamiokande have also been analyzed to obtain improved limits on the neutrino magnetic moments for \( \nu_\mu \) and \( \nu_\tau \) [164, 165, 166, 167, 168]. A recent analysis of the Borexino experiment claims to improve these limits on the magnetic moments of \( \nu_\mu \) and \( \nu_\tau \) by 3 orders of magnitude [169].

We would like to explore the possibility of obtaining new bounds on the neutrino magnetic moment using high statistics data on neutral current induced \( \pi^0 \) production from nucleons and nuclei in future experiments. Such a possibility was earlier discussed by Kang et al. [170] using first result on \( \pi^0 \) production from Super-Kamiokande experiment on atmospheric neutrinos [171].
5.2. FORMALISM

We study in this chapter, the $\pi^0$ production induced by weak neutral current and magnetic moment interaction of neutrinos and antineutrinos in the energy region of few GeV, relevant for K2K, MiniBooNE, T2K and NOνA experiments.

In further sections, we give the formalism and present our results for the total cross section $\sigma$, $Q^2$ distribution ($\frac{d\sigma}{dQ^2}$), momentum distribution ($\frac{d\sigma}{dp_{\pi^0}}$) and angular distribution ($\frac{d\sigma}{d\cos\theta_{\pi^0q}}$) for $\pi^0$ production from nucleons and nuclei induced by weak neutral current as well as neutrino magnetic induced $\pi^0$ production process.

5.2 Formalism

In the energy region of one GeV relevant for atmospheric neutrinos and present accelerator neutrino experiments the dominant process of pion production is through the excitation of $\Delta$ resonance and its subsequent decay to pions, i.e.

$$\nu N \rightarrow \nu \Delta \rightarrow \nu N \pi^0.$$  (5.1)

The differential scattering cross section for the neutral current induced reaction on free proton, i.e. $\nu(k) + p(p) \rightarrow \nu(p_l) + p(p') + \pi^0(k_{\pi})$ shown in Fig.(5.1) is given by

$$d\sigma = \frac{(2\pi)^4\delta^4(p_i - p_f)}{4p \cdot k} \prod_{j=1}^{3} \frac{d^3p_j}{(2\pi)^32E_j} |M_{fi}|^2,$$  (5.2)

where $p_i(=k+p)$ and $p_f(=\sum_{j=1}^{3}p_j)$ are the four momenta of the initial and final states, respectively. The transition matrix element $M_{fi}$ is written using

$$L^W = \frac{G_F}{\sqrt{2}} l_\alpha j^\alpha.$$  (5.3)
for the weak $ZN\Delta$ interaction and

$$L_{\pi N\Delta} = \frac{f_{\pi N\Delta}}{m_\pi} \bar{\Psi}_\mu \tilde{T}^\mu (\partial_\mu \tilde{\phi}) \Psi + h.c.$$  \hspace{1cm} (5.4)

for the strong $\pi N\Delta$ interaction. $\Psi_\mu$ is a Rarita-Schwinger field for spin-$\frac{3}{2}$ particle, $\tilde{T}^\mu$ is the isospin transition operator, $\tilde{\phi}$ is the pion field.

The matrix element of the leptonic current $l_\alpha$ and the hadronic current $j^\alpha$ are defined as

$$<k|l_\alpha|p_l> = \bar{u}(p_l)\gamma_\alpha (1 - \gamma^5) u(k),$$  \hspace{1cm} (5.5)

and

$$<\Delta(P)|j^\alpha|p> = \bar{\Psi}_\beta (P) O_\beta^\alpha u(p)$$  \hspace{1cm} (5.6)

$u(p)$ is the Dirac spinor for the proton.

$O_\beta^\alpha = (1 - 2\sin^2\theta_W)O_V^\beta_\alpha + O_A^\beta_\alpha$ for the neutral current process where $O_V^\beta_\alpha$ and $O_A^\beta_\alpha$ are given by Eqs.(2.94) and (2.95) respectively. $\theta_W$ is the Weinberg angle ($\sin^2\theta_W = 0.23122$), $q(k' - k)$ is the four momentum transfer and $M$ is the mass of the nucleon.

Using Eqs.(5.3)-(5.6) the matrix element for the process $\nu + p \rightarrow \nu + p + \pi^0$ in the $\Delta$ dominance model is written as

$$M_{fi} = \sqrt{\frac{2}{3}} \sqrt{\frac{2}{3}} G_F \frac{f_{\pi N\Delta}}{m_\pi} \bar{u}(p') k_\pi^\sigma P_{\sigma\lambda} O_\lambda^\alpha l_\alpha u(p)$$  \hspace{1cm} (5.7)

where $\sqrt{\frac{2}{3}}$ has appeared because of the isospin factor coming at the vertex $\Delta^+ \rightarrow p\pi^0$.

$G_F (= 1.16637 \times 10^{-5} GeV^{-2})$ is the Fermi coupling constant.

$P_{\sigma\lambda}$ is the $\Delta$ propagator in momentum space(Eq.2.103) where $P_{\sigma\lambda}$ is the spin-3/2 projection operator given by

$$P_{\sigma\lambda} = \sum_{\text{spins}} \psi_\sigma^\lambda \bar{\psi}_\lambda = (P + M_\Delta) \left( g_\sigma^\lambda + \frac{2 P_{\sigma\lambda}}{3 M_\Delta^2} + \frac{1}{3} \frac{P_{\sigma\lambda} - P_{\gamma\lambda}}{M_\Delta} - \frac{1}{3} \gamma_\sigma^\gamma \gamma_\lambda^\lambda \right)$$  \hspace{1cm} (5.8)

and the delta decay width $\Gamma$ is taken to be an energy dependent P-wave decay width.

If the reaction shown in Eq.(5.1) is induced by neutrino magnetic moment then the matrix element given by Eq.(5.7) would modify to

$$M_{fi} = \sqrt{\frac{2}{3}} \sqrt{\frac{2}{3}} G_F \frac{f_{\pi N\Delta}}{m_\pi} \bar{u}(p') k_\pi^\sigma P_{\sigma\lambda} O_\lambda^\alpha l_\alpha^\text{em} u(p)$$  \hspace{1cm} (5.9)

where

$$l_\alpha^\text{em} = \mu_{\nu\nu}^{eff} \bar{u}(p_\nu) \sigma_{\alpha\beta} \frac{q_{\beta}}{q^2} q_\beta u(k)$$  \hspace{1cm} (5.10)
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Figure 5.2: Total scattering cross section for the reaction $\nu + p \rightarrow \nu + p + \pi^0$ induced by weak neutral current and the magnetic moment induced processes. For neutrino magnetic moment we have taken $\mu_\nu = 6.8 \times 10^{-10} \mu_B$.

Figure 5.3: $Q^2$ distribution for the weak neutral current and the magnetic moment induced processes at $E_\nu = 1\text{GeV}$. For neutrino magnetic moment we have taken $\mu_\nu = 6.8 \times 10^{-10} \mu_B$. 

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Figure 5.4: Pion momentum distribution for the weak neutral current and the magnetic moment induced processes at $E_\nu = 1\, GeV$. For neutrino magnetic moment we have taken $\mu_\nu = 6.8 \times 10^{-10} \mu_B$.

Figure 5.5: Pion angular distribution for the weak neutral current and the magnetic moment induced processes at $E_\nu = 1\, GeV$. For neutrino magnetic moment we have taken $\mu_\nu = 6.8 \times 10^{-10} \mu_B$. 
with $\mu_{\text{eff}}^\nu$ as the effective magnetic moment of the neutrino, which is given in terms of the magnetic moments of the mass eigen states and oscillation probabilities that depend upon the specific oscillation models used for analyzing the neutrino oscillation experiments [164, 165, 166, 167, 168] and $\sigma^{\alpha\beta} = \frac{i}{2}[\gamma^\alpha, \gamma^\beta]$.

For our numerical calculations we have taken the Lalakulich et al. [69] parametrization for the $N - \Delta$ transition form factors.

The differential scattering cross section $\frac{d^5\sigma}{dQ^2d\Omega dE_F}$ is calculated using Eq.(5.2), and is written as

$$\frac{d^5\sigma}{dQ^2d\Omega dE_F} = \frac{1}{(4\pi)^{\frac{3}{2}}} \frac{\pi}{E_{\nu}E_{\ell}} |\vec{p}| |\vec{k}_\pi| EM_{\nu} E_{\pi} \left(1 - \frac{|p_{\pi}|}{|k_\pi|} \cos(\theta_{\pi q})\right)^2 \Sigma \Sigma |M_{fi}|^2$$

(5.11)

where $|\vec{k}_\pi|$ is the pion momentum. Similarly we get an expression for the pion distribution using Eq.(5.2). The cross section for neutrino scattering on free nucleons has to be modified when the nucleons is bound within the nucleus. A detailed description of these is given in Chapter 2, where we applied it to charged current neutrino nucleus scattering. Here we only outline the main features. In the nuclear medium the properties of $\Delta$ like its mass and decay width are modified due to nuclear effect, which in the present calculation are taken into account by using a modified mass $M_\Delta \rightarrow M_\Delta + \text{Re} \Sigma_\Delta$ and modified width $\Gamma_\Delta \rightarrow \tilde{\Gamma}_\Delta - 2 \text{Im} \Sigma_\Delta$ where $\tilde{\Gamma}_\Delta$ is reduced width of $\Delta$ due to Pauli blocking of nucleons in the $\Delta \rightarrow N\pi$ decay and $\Sigma_\Delta$ is the self energy of $\Delta$ calculated in the nuclear many body theory using local density approximation which have been discussed in chapter 2. The expressions of $\text{Re} \Sigma_\Delta$ and $\text{Im} \Sigma_\Delta$ are taken from Ref.[90], [91]. The pions produced in this process are scattered and absorbed in the nuclear medium. This is treated in a Monte Carlo simulation which have been taken from the Ref.[93].

## 5.3 Results and Discussions

### A. $\pi^0$ production induced by weak neutral current and neutrino magnetic moment process in free nucleon

The numerical results for the total cross section $\sigma$, the differential cross sections $\frac{d\sigma}{dQ^2}$, $\frac{d\sigma}{d\cos\theta_{\pi q}}$ and $\frac{d\sigma}{dp_{\pi}}$ for the neutral current production of $\pi^0$ induced by neutrinos(antineutrinos) are presented in Figs.(5.2)-(5.5) along with the contributions of the electromagnetic production induced by the neutrino(antineutrino) magnetic moment. For the neutral current production, the numerical values of the vector and axial vector form factors given in Eqs.(2.98) & (2.100) have been used while for the electromagnetic production the numerical values of the vector form factors given in Eq.(2.98) along with the neutrino magnetic moment $\mu_{\nu}^\text{eff} = 6.8 \times 10^{-10} \mu_B$ is used. A momentum dependent strong form factor with $f_{\pi N\Delta}(m_{\pi}^2) = 2.12$ [172] has been used in numerical calculations.
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We have shown in Fig.(5.2), the total cross section $\sigma$, for the neutral current induced $\nu(\bar{\nu})$ production of $\pi^0$. The present results are in agreement with the results of Leitner et al.[173] and also with the results of Hernandez et al.[174] if their values for the parameter $C_A^5(0)$ and $M_A$ are used but are in disagreement with results of Kang et al.[170] who find a smaller value for $\sigma$. We also show in this figure, the total cross section $\sigma$ for electromagnetic production of $\pi^0$ induced by neutrino magnetic moment which is in agreement with the results of Kang et al.[170] if neutrino magnetic moment $\mu_{\nu_{\tau}} = 6 \times 10^{-9} \mu_B$ as used by them is taken. We see in Fig.(5.2) that with the present limits on the magnetic moment of $\nu_{\tau}$, the electromagnetic production of $\pi^0$ is $10^{-3}$ times smaller than the neutral current induced $\pi^0$ production. It is, therefore, not feasible to improve the present limit on neutrino magnetic moment from $\pi^0$ production cross section measurements as earlier expected from the work of Kang et al. [170].

Neutral current reactions are flavor blind and if one takes $\mu_{\nu_{\tau}} = 3.9 \times 10^{-7} \mu_B$ [163], then the cross section contribution coming from the magnetic moment induced $\pi^0$ production process becomes two order of magnitude larger than the contribution coming from the weak neutral current induced $\pi^0$ production process.

In Figs.(5.3)-(5.5), we also show the differential cross sections $\frac{d\sigma}{dQ^2}$, $\frac{d\sigma}{dcos\theta_{\pi q}}$ and $\frac{d\sigma}{dp_{\pi}}$ for the neutral current induced $\pi^0$ production by $\nu$ and $\bar{\nu}$ as well as the $\pi^0$ production induced by magnetic moment of $\nu_{\tau}$. The present experiments at MiniBooNE [105] see neutral current induced $\pi^0$ events of the order of $2.8 \times 10^4$ which can be further increased by an order of magnitude at T2K and NO\nuA. These pions are produced on nuclear targets like $^{12}C$. In the case of nuclear targets, there are incoherent as well as coherent production of $\pi^0$ which have different angular and momentum distributions. An analysis of these experiments in order to study the neutrino magnetic moment would require an understanding of nuclear effect in the incoherent and coherent production of $\pi^0$ induced by the neutral currents as well as by the neutrino magnetic moment on nuclear targets in the energy region of 1 GeV.

B. $\pi^0$ production induced by weak neutral current process in ($^{16}O$) nuclei

The numerical results for the total cross section $\sigma$, the differential cross sections $\frac{d\sigma}{dQ^2}$, $\frac{d\sigma}{dcos\theta_{\pi q}}$ and $\frac{d\sigma}{dp_{\pi}}$ inside the nucleus for the neutral current production of $\pi^0$ induced by neutrinos are presented in Figs.(5.6)-(5.9).

In Fig.(5.6), we have shown the results for the pion momentum distribution for the weak neutral current process induced by neutrino in $^{16}O$ nuclei. The reduction in the pion momentum distribution while incorporating nuclear medium effect is around 40\% in the peak region and around 30\% at $p_{\pi} = 420$ MeV. When pion absorption effect is also incorporated with medium effect then the further reduction in the differential cross section is around 15-16\% in the peak region and around 18\% at $p_{\pi} = 420$ MeV\%.

The results for pion angular distribution in the case of weak neutral current process induced by neutrino in $^{16}O$ nuclei are shown in Fig.(5.7). The reduction in the angular distribution with medium effect is around 40\% at the forward angles and is around 35\% at $cos\theta_{\pi q} = 0.7$. Taking
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Figure 5.6: Pion momentum distribution for the weak neutral current induced processes at $E_\nu = 1GeV$ in $^{16}O$.

Figure 5.7: Pion angular distribution for the weak neutral current induced processes at $E_\nu = 1GeV$ in $^{16}O$. 
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Figure 5.8: $Q^2$ distribution for the weak neutral current induced processes at $E_\nu = 1 GeV$ in $^{16}O$.

Figure 5.9: Total scattering cross section for the reaction $\nu + p \rightarrow \nu + p + \pi^0$ induced by the weak neutral current induced processes in $^{16}O$. 
pion absorption effect into account, along with medium effect there is a further reduction of around 15% in the peak region and around 16% at $cos\theta_{\pi q} = 0.7$.

In Fig.(5.8), the result for the $Q^2$ distribution for the weak neutral current process induced by neutrino in $^{16}O$ nuclei, is shown. The reduction in the distribution while incorporating nuclear medium effect into account is around 35% in the peak region and around 34% at $Q^2=0.2\text{GeV}^2$. When pion absorption effect is also incorporated with medium effect then reduction in the differential scattering cross section is around 16%.

The results for the total scattering cross section are shown in Fig.(5.9). The reduction in the total scattering cross section with nuclear medium effect is around 50% at $E_\nu = 0.4\text{GeV}$, 35% at $E_\nu = 1.0\text{GeV}$ which becomes 23% at $E_\nu = 3\text{GeV}$. With nuclear medium and pion absorption effect taking into account there is further reduction, which is around 12% at $E_\nu = 0.4\text{GeV}$, 16% at $E_\nu = 1.0\text{GeV}$ and 10% at $E_\nu = 3\text{GeV}$.

C. $\pi^0$ production induced by neutrino magnetic moment process in ($^{16}O$) nuclei

The results for the neutrino magnetic moment induced $\pi^0$ production in nuclei, are presented in Figs.(5.10) - (5.13).

In Fig.(5.10), we have shown the results for the pion momentum distribution for the neutral current process induced by neutrino magnetic moment, in $^{16}O$ nuclei. The reduction in the pion momentum distribution while incorporating nuclear medium effect into account is around 42% in the peak region(=200MeV) and around 25% at $p_\pi= 400\text{MeV}$. When pion absorption effect is also incorporated with medium effect then the further reduction in the differential cross section is around 15-16% in the peak region and around 20% at $p_\pi=400\text{MeV}$.

The results for pion angular distribution in the case of weak neutral current process induced by neutrino magnetic moment in $^{16}O$ nuclei are shown in Fig.(5.11). The reduction in the angular distribution with medium effect in the peak region(= 0.95) is around 25% and at $cos\theta_{\pi q} = 0.7$, the reduction is around 28%. Taking pion absorption effect into account, along with medium effect there is a further reduction of around 15%. In Fig.(5.12), the results for the $Q^2$ distribution for the magnetic moment induced process in $^{16}O$ nuclei, are shown. The reduction in the distribution while incorporating nuclear medium effect into account is around 36% in the peak region(=0.05GeV$^2$) and around 40% at $Q^2=0.5\text{GeV}^2$. When pion absorption effect is also incorporated with medium effect then reduction in the differential scattering cross section is around 16% in the peak region and around 17% at $Q^2=0.2\text{GeV}^2$.

The results for the total scattering cross section for the magnetic moment induced process are shown in Fig.(5.13). The reduction in the total scattering cross section with nuclear medium effect is around 45% at $E_\nu = 0.4\text{GeV}$, 35% at $E_\nu = 1.0\text{GeV}$ which becomes 24% at $E_\nu = 3\text{GeV}$. With nuclear medium and pion absorption effect taking into account this reduction is around 10% at $E_\nu = 0.4\text{GeV}$, 15% at $E_\nu = 1.0\text{GeV}$ and 10% at $E_\nu = 3\text{GeV}$.

We have also studied the ratio of cross section induced by magnetic moment induced process.
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Figure 5.10: Pion momentum distribution for the magnetic moment induced processes at $E_\nu = 1\text{GeV}$ in $^{16}O$. For neutrino magnetic moment we have taken $\mu_\nu = 6.8 \times 10^{-10} \mu_B$.

Figure 5.11: Pion angular distribution for the magnetic moment induced processes at $E_\nu = 1\text{GeV}$ in $^{16}O$. For neutrino magnetic moment we have taken $\mu_\nu = 6.8 \times 10^{-10} \mu_B$. 
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Figure 5.12: $Q^2$ distribution for the magnetic moment induced processes at $E_\nu = 1\text{GeV}$ in $^{16}O$. For neutrino magnetic moment we have taken $\mu_\nu = 6.8 \times 10^{-10} \mu_B$.

Figure 5.13: Total scattering cross section for the reaction $\nu + p \rightarrow \nu + p + \pi^0$ induced by the magnetic moment induced processes in $^{16}O$. For neutrino magnetic moment we have taken $\mu_\nu = 6.8 \times 10^{-10} \mu_B$. 
to the cross section induced by weak neutral current induced process on the free proton target as well as in the $^{16}$O nucleus. We find that if we take $\mu_{\nu_\mu} = 6.8 \times 10^{-10} \mu_B$, then this ratio at $E_\nu = 0.4\text{GeV}$ is equal to $0.4736 \times 10^{-3}$, which becomes $0.5542 \times 10^{-3}$ at $E_\nu = 1\text{GeV}$ and $0.7 \times 10^{-3}$ at $E_\nu = 2\text{GeV}$. While inside the nucleus this ratio becomes two orders of magnitude smaller than that of the free case i.e. this ratio is $0.55 \times 10^{-2}$ at $E_\nu = 0.4\text{GeV}$, $0.658 \times 10^{-2}$ at $E_\nu = 1\text{GeV}$ and $0.83 \times 10^{-2}$ at $E_\nu = 2\text{GeV}$. However, if we take the present limit on the $\nu_\tau$-magnetic moment i.e. $\mu_{\nu_\tau} = 3.9 \times 10^{-7} \mu_B$, then this ratio of cross section induced by magnetic moment induced process becomes two orders of magnitude larger than the neutral current $Z^0$-induced $\pi^0$ production cross section on the free proton target. Similar is the change inside the nucleus. If we take latest limit put by Borexino experiment on $\mu_\nu$ i.e. $\mu_\nu < 8.4 \times 10^{-11} \mu_B$, this ratio becomes almost negligible. Thus an upper limit on neutrino magnetic moment may be obtained in the proposed experiments at T2K and NO$\nu$A, where the $\pi^0$ production events are going to be observed.