CHAPTER 3

MODELING OF SINGLE PHASE INDUCTION MOTOR

3.1 GENERAL

More single phase induction motors are in use today than the total of all the other types put together. The single phase induction motor is the least expensive, and the lowest maintenance type of an AC motor. As the induction motor has no brushes and is easy to control, many older DC motors are being replaced by induction motors and the accompanying inverters in industrial applications. This chapter deals with the modeling of a single phase induction motor using the double field revolving theory.

3.2 THEORIES BEHIND THE OPERATION OF SINGLE PHASE INDUCTION MOTOR

There are two important methods of analyzing the single phase induction motor. They are double field revolving theory and cross field theory. The double field revolving theory is more akin to the three phase induction motor theory and easiest compared to the cross field theory. Hence, the double field revolving theory is considered to develop the model of the single phase induction motor.

According to this theory, any alternating quantity can be resolved into two rotating components which rotate in opposite directions, each having magnitude as half of the maximum magnitude of the alternating quantity.
In case of the single phase induction motor, the stator winding produces an alternate magnetic field having the maximum magnitude of \( \Phi_m \). According to this theory, the two components of the stator flux, each having magnitude half of the maximum magnitude of the stator flux are considered. Both these components rotate in opposite directions at the synchronous speed \( N_s \) which are dependent on frequency and stator poles. Let \( \Phi_f \) be the forward component rotating in anticlockwise direction while \( \Phi_b \) be the backward component rotating in clockwise direction. The resultant of these two components at any instant gives the instantaneous value of the stator flux at that instant. So the resultant of these two is the original stator flux. This is shown in Figure 3.1.

![Figure 3.1 Positions of pulsating and rotating fluxes with change in angle (θ)](image)

The set of Figures 3.1 (i-iv) show the resultant sum of the two rotating fluxes or fields, as the time axis (angle) is changing from \( θ = 0 \) (0°) to \( θ = \pi \) (180°). Figure 3.2 shows the alternating or pulsating flux (resultant) varying with the time or angle. The two oppositely directed torques and the resultant torque could be shown effectively with the help of torque speed characteristics shown in Figure 3.3.

![Figure 3.2 Pulsating flux as a function of space angle (θ)](image)
Figure 3.3 Speed torque characteristics of induction motor

The equivalent circuit of the single phase induction motor, based on the double field revolving theory is shown in Figure 3.4.

Figure 3.4 Equivalent circuit of single phase induction motor based on double field revolving theory
3.3 TESTS CONDUCTED ON INDUCTION MOTOR

To determine the parameters of single phase induction motor no-load, blocked rotor and retardation tests have to be conducted on it.

3.3.1 No-load Test

The test is conducted by rotating the motor without load. The input current, voltage and power are measured by connecting the ammeter, voltmeter and wattmeter in the circuit. These readings are denoted as $V_0$, $I_0$ and $W_0$. The expressions for no-load power and no-load powerfactor can be expressed as given in Equations (3.1) and (3.2)

$$W_0 = V_0 I_0 \cos \phi_b$$  \hspace{1cm} (3.1)

$$\cos \phi_b = \frac{W_0}{V_0 I_0} = \text{no-load powerfactor}$$  \hspace{1cm} (3.2)

The motor speed on no-load is almost equal to its synchronous speed; hence, for practical purposes the slip can be assumed to be zero. Hence $R_2/2\delta$ becomes $\infty$ and acts as an open circuit in the equivalent circuit. Hence, for the forward rotor circuit, the branch $R_2/2\delta + j X_2/2$ is eliminated, while for a backward rotor circuit, the term $R_2/2(2- \delta)$ tends to be $R_2/4$. Thus $X_0/2$ is much higher than the impedance $R_2/4 + j X_2/2$. Hence, it can be assumed that no current can flow through $X_0/2$ and that the branch can be eliminated.

The Voltage across $X_0/2$ ($V_{ab}$) is expressed as given in Equations (3.3) and (3.4).

$$V_{ab} = V_0 - I_0 \left[ R_l + \left( \frac{R_2}{4} \right) + j \left( X_1 + \frac{X_2}{2} \right) \right]$$  \hspace{1cm} (3.3)
\[ V_{ab} = \frac{I_o X_0}{2} \]  

(3.4)

Hence, Magnetizing inductive reactance of the stator is expressed as given in Equation (3.5).

\[ X_0 = \frac{2V_{ab}}{I_0} \]  

(3.5)

3.3.2 Blocked Rotor Test

In the blocked rotor test, the rotor is held fixed so that it will not rotate. A reduced voltage is applied to limit the short circuit current. This voltage is adjusted with the help of an autotransformer so that the rated current flows through the main winding. The input voltage, current and power is measured by connecting the voltmeter, ammeter and wattmeter respectively. These readings are denoted as \( V_{sc}, I_{sc}, W_{sc} \).

Now, as the rotor is blocked, the slip \( s=1 \); so, the magnetizing reactance \( X_0/2 \) is much higher than the rotor impedance and it can be neglected as connected in parallel with the rotor. The expressions for power and powerfactor at the blocked rotor condition, can be expressed as given in Equations (3.6) and (3.7)

\[ W_{sc} = V_{sc} I_{sc} \cos \phi_{sc} \]  

(3.6)

\[ \cos \phi_{sc} = \frac{W_{sc}}{V_{sc} I_{sc}} = \text{blocked rotor powerfactor} \]  

(3.7)

The rotor resistance and reactance referred to the stator, can be determined from the Equations (3.8) to (3.9).
\[ Z_{eq} = \frac{V_{sc}}{I_{sc}} \]  \hspace{1cm} (3.8)

\[ R_{eq} = \frac{W_{sc}}{I_{sc}^2} \]  \hspace{1cm} (3.9)

\[ R_{eq} = R_1 + R_2 \]  \hspace{1cm} (3.10)

\[ R_2 = R_{eq} - R_1 = \text{rotor resistance referred to the stator} \]  \hspace{1cm} (3.11)

\[ X_{eq} = \sqrt{\left(Z_{eq}^2 - R_{eq}^2\right)} \]  \hspace{1cm} (3.12)

Assuming \( X_1 = X_2 \)

\[ X_2 = \frac{X_{eq}}{2} = \text{rotor reactance referred to the stator}. \]  \hspace{1cm} (3.13)

### 3.3.3 Retardation Test

This method is generally employed for shunt generators and shunt motors. From this method we can get the stray losses. Thus, if armature and shunt copper losses at any given load current are known, then the efficiency of a machine can be easily estimated.

The machine is run at a speed, which is slightly above its normal speed. The supply to the motor is cut off while the field is kept excited. The armature consequently slows down and its kinetic energy is used in supplying the rotational or stray losses that include iron, friction and winding losses. When the supply is cut off from the motor, the speed decreases. The time required for a definite decrease in speed is noted with the help of a stopwatch and the average value is found. From this test, the values of the moment of inertia and viscous friction are determined.
3.4 MODEL OF SINGLE PHASE INDUCTION MOTOR BASED ON DOUBLE FIELD REVOLVING THEORY

The double revolving field theory can be effectively used to obtain the equivalent circuit of a single phase induction motor. This method consists of determining the parameters of both the field at any given slip. When the two fields are known, the torque produced by each can be obtained. The difference between these two torques is the net torque acting on the rotor. In general, the magnitude of the output voltage due to the forward field is 90% to 95% of the applied voltage. When the effect of the backward field is neglected, accuracy will be less. Hence, to obtain accurate speed responses, the effect of the backward field is also considered. The equivalent circuit of single phase induction motor in S domain is shown in Figure 3.5.

![Equivalent circuit of single phase induction motor in S domain](image)

Figure 3.5 Equivalent circuit of single phase induction motor in S domain
The basic equations, which govern the model of the induction motor, can be derived from the equivalent circuit.

Current flowing through the stator is expressed as

\[ I(s) = \frac{(V_i(s) - V_o(s))}{(R_t + sL_t)} \]  \hspace{1cm} (3.14)

### 3.4.1 Model of Forward Field

Current flowing through the stator can be expressed as

\[ I(s) = I_1(s) + I_2(s) \]  \hspace{1cm} (3.15)

If the rotor current referred to the stator is taken as \( I_2 \), then the iron-loss and magnetizing component of the no-load current can be expressed as

\[ I_1(s) = I(s) - I_2(s) \]  \hspace{1cm} (3.16)

The forward field voltage can be obtained from the expression

\[ V_f(s) = V_o(s) - V_b(s) = I_1(s) * \left[ \frac{sR_tL_t}{(R_t + sL_t)} \right] \]  \hspace{1cm} (3.17)

It can be rewritten as

\[ V_f(s) = V_o(s) - V_b(s) = I_1(s) * \left[ R_t - \frac{R_t^2}{(R_t + sL_t)} \right] \]  \hspace{1cm} (3.18)

Voltage across the rotor inductance is expressed as

\[ V_f(s) - I_2(s) * \left[ \frac{R_t}{2s} \right] \]  \hspace{1cm} (3.19)
The rotor current referred to the stator can be expressed as

\[
I_2(s) = \frac{[V_f(s) - V_i(s)]}{sL_3}
\]  
(3.20)

Where,

\[
V_i(s) = I_2(s) * R_i
\]

(3.21)

Airgap power developed by the motor is given by the expression

\[
P_{gf}(s) = I_2(s)^2 * \frac{R_i}{2s}
\]

(3.22)

### 3.4.2 Model of Backward Field

Current flowing through the stator can be expressed as

\[
I(s) = I_3(s) + I_4(s)
\]

(3.23)

If the rotor current referred to the stator is taken as \(I_4\), then the iron-loss and magnetizing component of the no-load current can be expressed as

\[
I_3(s) = I(s) - I_4(s)
\]

(3.24)

The backward field voltage can be obtained from the expression

\[
V_b(s) = I_3(s) * \left\{ \frac{sR_4L_4}{(R_4 + sL_4)} \right\}
\]

(3.25)

It can be rewritten as

\[
V_b(s) = I_3(s) * \left\{ R_4 - \left[ \frac{R_4^2}{(R_4 + sL_4)} \right] \right\}
\]

(3.26)
Voltage across the rotor inductance is expressed as

\[ V_b(s) - I_4(s) \cdot \left( \frac{R_2}{2(2 - \$)} \right) \]  \hspace{1cm} (3.27)

The rotor current referred to the stator can be expressed as

\[ I_4(s) = \frac{[V_2(s) - V_x(s)]}{sL_3} \]  \hspace{1cm} (3.28)

Where,

\[ V_2(s) = I_4(s) \cdot R_5 \]  \hspace{1cm} (3.29)

Airgap power developed by the motor is given by the expression

\[ P_{gb}(s) = I_4(s)^2 \cdot \frac{R_2}{2(2 - \$)} \]  \hspace{1cm} (3.30)

Torque developed by the motor is given by the expression

\[ T(s) = \frac{P_{gf}(s) - P_{gb}(s)}{2\pi n_s} \]  \hspace{1cm} (3.31)

The load balance equation is given by

\[ \omega(s) = \frac{(T(s) - T_L(s))}{J_S + B} \]  \hspace{1cm} (3.32)

By using the Equations (3.14) to (3.32), the simulink model of the single phase induction motor can be obtained as shown in Figure 3.6.
Figure 3.6 Generalized simulink model of single phase induction motor
3.5 CONCLUSION

The simulink model of the single-phase induction motor is developed using the double field revolving theory and the same is used for the simulation of speed control and energy saving systems. An analysis and model of the single phase induction motor are presented in this chapter.