ABSTRACT
OF THE
THESIS ENTITLED

ON EVOLUTIONARY BEHAVIOUR AND CHAOS MEASURE IN DISCRETE DYNAMICAL SYSTEMS

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ABSTRACT

The studies made in the thesis comprises with the chaotic motion and complexity emerging in various nonlinear dynamical systems during evolution. Real systems are mostly nonlinear and may show chaotic motion when the parameters involving within them assume certain set of critical values. The objective of investigation in the thesis is to detect sets of values of parameters of a system for its regular and chaotic motions and then to find some measure the complexity and dimensionality of chaos. We may call this as chaos measure. The studies made in the thesis are confined to discrete systems only. Evolutionary phenomena of various systems are discussed here through bifurcation leading to chaos. In order to detect regular and chaotic phenomena, in addition to traditional tools, (like time series curves, phase plots, Poincare maps), some more effective tools, such as Lyapunov characteristic exponents (LCE), Topological Entropy, correlation dimension etc. are used here. These are calculated numerically and shown through various graphics plotted as a part of numerical studies. These tools efficiently characterize regular and chaotic behaviour emerging in certain system during evolution. Graphics of numerical calculation displayed here show exciting and interesting results. Application of some recently discovered chaos indicators, Fast Lyapunov indicators (FLI), Smaller Alignment Indices (SALI), and Dynamic Lyapunov Indicator (DLI), have also been employed for specific interest and distinguishing regularity and chaos in higher dimensional systems.

The first chapter is for the general introduction which deals with the various concepts, important definitions with suitable examples and terminologies related with phenomenon of chaos in nonlinear dynamical systems. A brief review of the subject of dynamical systems and its development towards chaos theory has been given here. The concept of chaos measures and recent articles related to this appeared through different articles reviewed here for clear understanding. In this chapter, we have explained some of the chaos measure techniques in brief. We have also explained some basic concepts of nonlinear dynamics which are frequently used in the thesis. Within this section, we have given the definitions of dynamical system, discrete and continuous dynamical system, fixed points and their stability criteria, chaos and some
related concepts. Also, the concept of some chaos indicators like Lyapunov exponent, poincaré surface of section, bifurcation diagram, topological entropy, correlation dimension have been explained with examples. Finally, the definition of some new indicators like Fast Lyapunov Indicators (FLIs), the Smaller Alignment Index (SALI) and the Dynamic Lyapunov Indicator (DLI), have been given in this chapter.

In chapter two, first we have defined and explained in details of the chaos measuring tools Lyapunov exponents, topological entropies and correlation dimensions. Then, we have considered a number of one dimensional discrete systems for our study. For such systems, we have obtained bifurcation diagrams and performed numerical calculations to obtain plots for Lyapunov exponents, topological entropies and correlation curves. From the correlation data set, we have obtained the correlation dimensions for each model and for their chaotic sets by using least square linear fit.

In chapter three, we have considered a number of two dimensional discrete maps for our study. For these maps, we have obtained bifurcation diagrams and performed numerical calculations to obtain plots for Lyapunov exponents and correlation curves. From the correlation data set, we have obtained the correlation dimensions for each model and for their chaotic sets by using least square linear fit.

In the fourth chapter, we have made studies on specific problems on one and two dimensional discrete models of population dynamics of certain insect population. We have investigated these under various feasible conditions within the framework of nonlinear dynamics. Initially we have investigated chaotic motion of Ricker Type model at different parameter values of inherent per capita recruitment rate per census, b and fraction of individuals expected to die during one census period, μ. We have also introduced a period forcing in the Ricker Type model, such that the birth rate oscillates with relative amplitude α, and average b. We have obtained the critical set of parameter values at which calculations have been carried for the bifurcation points, period doubling and chaos after changing one parameter and keeping others fixed. At the same set of critical values, we have calculated the Lyapunov numbers and LCEs and topological entropies for one dimensional maps. Two Species model of population dynamics have been chosen here for studies of two dimensional models.
Results similar to one dimensional Ricker Type model have also been calculated for this. Numerical results shown through bifurcation diagrams and LCEs curves are interesting. We have also used meaningful statistical measures to justify the results obtained through this study. Towards this, we have introduced the correlation dimension which helps to explain the fractal nature of the chaotic system that possibly displaying certain strange attractor. Correlation dimension describes the measure of dimensionality of the space occupied by chaotic attractor of any system having presence of complexity. We have calculated the correlation dimension for each one and two dimensional models and demonstrated the correlation curves through graphics.

In the fifth chapter of the thesis, first we have obtained phase plots for regular and chaotic cases for a number of two dimensional discrete maps considered in chapter three. Then, we have performed numerical calculations to obtain the plots for FLI, SALI and DLI for regular as well as for chaotic cases for all those maps. Some comparison in regard to effectiveness of these indicators, have also been discussed. Results shown through various graphics appear very interesting.