CHAPTER 2

SERIES RESONANT CONVERTER SYSTEM

2.1 INTRODUCTION

Recent advances in modern power semiconductor device technologies have led to the wide utilization of power converters in a large number of applications and have opened up a host of new converter topologies for many new applications. The most visible gain in industrial and commercial products occurs in the area of power inverters, which convert a dc voltage into a single or poly phase ac voltage at a desired amplitude and frequency. Technological advances in these areas have arisen primarily from improvements in semiconductor power devices, with insulated gate bipolar transistors (IGBT) leading the market today for medium power applications. IGBTs feature many desirable properties including a MOS input gate, high switching speed, low conduction voltage drop, high current carrying capability, and a high degree of robustness. Devices have drawn closer to the 'ideal switch', with typical voltage ratings of 600 - 1700 volts, on-state voltage of 1.7 - 2.0 volts at currents up to 1000 amperes, and switching speeds of 200 - 500 ns.

In hard switching, the IGBTs switch on and switch off rapidly; So, switching losses occur during device turn-on and turn-off due to the transient existence of both voltage across, and current in the device. These stresses require significant device de-rating for switching frequencies in excess of 5 - 6 kHz, thus increasing the system cost. In addition, IGBT losses are further increased at turn-on by the charge stored in the complementary
switch's anti-parallel diode and at turn off by the energy trapped in the parasitic inductance of the IGBT package and device interconnections.

One technique that has promise in obtaining improved system performance is soft switching. Soft switching converters constrain the switching of power devices to time intervals when the voltage across the device, or the current through it is nearly zero. This significantly reduces the device switching losses, and hence allows higher switching frequencies and wider control bandwidths, while simultaneously lowering dv/dt and electromagnetic interference (EMI) problems. The use of soft switching in induction heating for industrial applications is fairly common and widespread.

Resonant inverters are an attractive choice in low-power high-frequency induction heating applications because they can be used as power factor correctors without additional power stages. The circuit of a resonant converter consists of a capacitor, an inductor, and resistance. The inductive reactance and the capacitive reactance are summarized in the following formulae along with the size of the impedance.

\[ X_L = j\omega L = j2\pi fL \]
\[ X_C = j\omega C = 1/(2\pi fC) \]
\[ |Z| = \sqrt{R^2 + (\omega L - (1/\omega C))^2} \]

At the resonant frequency, the inductive and capacitive reactance become the same; i.e., the voltage of the power source and the current in the circuit stay at the same level. The current in the circuit reaches its peak when the source frequency becomes identical to the resonance frequency.

\[ 2\pi fL = 1/(2\pi fC) \iff f_0 = 1/(2\pi \sqrt{LC}) \text{ [Hz]} \]
Two schemes for the resonant converter are generally used: a series resonant circuit and a parallel resonant circuit.

2.1.1 Series Resonant Circuit

The work coil is made to resonate at the intended operating frequency by means of a capacitor placed in series with it. This causes the current through the work coil to be sinusoidal. The series resonance also magnifies the voltage across the work coil, far higher than the output voltage of the inverter alone. The inverter sees a sinusoidal load current but it must carry the full current that flows in the work coil. For this reason the work coil often consists of many turns of wire with only a few amps or tens of amps flowing.

This arrangement is commonly used in appliances like rice cookers where the power level is low, and the inverter is located next to the object to be heated. The main drawbacks of the series resonant arrangement are that the inverter must carry the same current that flows in the work coil. In addition to this the series resonant action can become very pronounced if there is no work piece present to damp the circuit. The tank capacitor is typically rated for a high voltage because of the resonant voltage rise experienced in the series-tuned resonant circuit.

2.2 SERIES RESONANT CONVERTER

The ordinary circuit of an AC-AC converter for induction heating typically includes a controlled rectifier and a frequency controlled current source or a voltage source inverter. It is a well known fact that the input rectifier does not ensure a sine wave input current, and is characterized by low power. Recently, many studies of high power factor rectifiers with a single switch have been made. These schemes are also characterized by a
close to sine wave input current. Various schemes of the AC-AC converter for induction heating is described. However, the inverting circuit is constructed by traditional mode with four controlled switches.

In the scheme (Figure 2.1) of the AC-AC converter there are two main advantages: It is characterized by a high power factor and a sine wave input current, and on the other hand, the inverter circuit is constructed with a single controlled switch, which serves as a high-frequency generator for induction heating. The resonant circuit in the output produces the high frequency output required by the load.

![Figure 2.1 Circuit Diagram](image)

2.3 PRINCIPLE OF OPERATION

The operating principles of the circuit are illustrated by Figure 2.2 and the theoretical waveforms are shown in Figure 2.3. If the switching frequency is much higher than the input line frequency and in the analysis we arbitrarily choose the time interval where $\text{Vin} > 0$. 

Figure 2.2 Equivalent Circuits

(a) Mode I (t₀-t₁)

(b) Mode II (t₁-t₂)

(c) Mode III (t₂-t₃)
2.3.1 Interval 1: $t_0 < t < t_1$

The equivalent circuit is shown in Figure 2.2a. Four diodes $D_1$-$D_4$ and the switch $S$ are off. In this interval the capacitor $C$ charges up practically linearly at a rate and a polarity corresponding to the instantaneous input voltage $V_{in}$.

2.3.2 Interval 2: $t_1 < t < t_2$

The equivalent circuit is shown in Figure 2.2b. Two diodes $D_1$, $D_3$ and the switch $S$ are on. In this interval the capacitor $C$ is discharged via the circuit $C$-$D_1$-$S$-$L$-load-$D_3$. This interval ends when the capacitor voltage reduces to zero.
2.3.3 Interval 3: \( t_2 < t < t_3 \)

The equivalent circuit is shown in Figure 2.2c. All the diodes and the switch S are on. In this interval the switch current through switch S flows via two parallel bridge branches. This interval ends when this switch current decreases to zero. At this moment the switch turns off and the process starts from the beginning.

2.4 OPERATIONAL ANALYSIS

The analysis of the circuit operation is based on the commonly accepted assumption that all circuit components are ideal. The approximate analytical calculations are based on two additional assumptions; the switch current can be approximated by a semi-sinusoidal, and the load power is determined by the first harmonic of the load voltage. The equations are normalized using the following base quantities: base voltage \( V_a = V_{in} \); base impedance \( R_B = \sqrt{(L_o/C_D)} \); base current \( I_B = V_B / R_B \); base power \( P_B = V_B I_B \); base frequency \( \omega_B = 1 / \sqrt{L_o C_o} \); base time \( T_B = 2\pi / \omega_B \). The optimal range of the normalized parameters was chosen. The criteria of this choice were the reasonable levels of the following quantities; maximal normalized value of the switch current \( (i_{sw, max}^* = I_{sw, max}^* / I_B = 6-10) \) and maximum normalized value of the switch voltage \( (V_{sw, max}^* = V_{sw, max} / V_B = 4-5) \). To provide these values it is necessary to choose the following ranges of the normalized circuit parameters:

\[
\frac{L_1}{L_o} = 0.1 - 0.2 \div \frac{\omega_s^*}{\omega_B} = \frac{1/\sqrt{L C}}{\omega_B} = 3-5 \; ; \; \omega_s^* = 1.1 - 1.9
\]
The evaluation of the relationship between the input and output voltages $M = V_o/V_{in}$ is based on the following statements. We take into account the fact that the average charge of the input capacitor $C$ is constant; therefore, the average current through this capacitor during the full period is zero. In addition, we assume that the input current during the switch period is constant. Then in accordance with Figure 2.4 we write

\[
\int_{0}^{\frac{2\pi}{2D}} i_c d(\omega_s t) = \int_{\frac{2\pi}{2D}}^{2\pi} i_w d(\omega_s t)
\]  

(2.1)

where $D_1 = T_{1(0)} / 2\pi$ and $D=(T_1+T_2)\omega_s / 2\pi$, we consider that the capacitor discharge current can be accepted as a half-wave of the sinusoid

\[
i_c = I_{c,\text{max}} \sin \left( \frac{\omega_s t}{2D} \right)
\]

(2.2)
\[ i_{\text{in}} + i_{c,\text{max}} \sin \left( \frac{\omega_s t}{2D_i} \right) = I_{\text{sw, max}} \sin \left( \frac{\omega_s t}{2D} \right) \]  
\tag{2.3}

Or

\[ \int_0^{2eD_i} \left( i_{\text{in}} - I_{\text{sw, max}} \sin \left( \frac{\omega_s t}{2D} \right) \right) d(\omega_s t) = \int_0^{2eD_i} i_{\text{in}} d(\omega_s t) \]  
\tag{2.4}

Now we can obtain the following relationship:

\[ A_i = \frac{I_{\text{sw, max}}}{i_{\text{in}}} = \frac{\pi}{D} \cdot \frac{(1 - D + D_i)}{1 - \cos \left( \frac{\pi D_i}{D} \right)} \]  
\tag{2.5}

The amplitude of the first harmonic of the switch current \( I_{\text{sw,1,max}} \) as a function of its amplitude and the duty cycle is given by the following expression:

\[ I_{\text{sw,1,max}} = \sqrt{a_i^2 + b_i^2} \]  
\tag{2.6}

where in our case (Figure 2.5a)

\[ a_i = \frac{1}{\pi} \int_0^{2\pi D} I_{\text{sw, max}} \sin \left( \frac{\omega_s t}{2D} \right) \cos \omega_s t d(\omega_s t) \]  
\tag{2.7}

\[ b_i = \frac{1}{\pi} \int_0^{2\pi D} I_{\text{sw, max}} \sin \left( \frac{\omega_s t}{2D} \right) \sin \omega_s t d(\omega_s t) \]  
\tag{2.8}

As a result, the expression for \( I_{\text{sw,1,max}} \) is obtained

\[ A_2 = \frac{I_{\text{sw,1,max}}}{I_{\text{sw, max}}} = \frac{2D}{\pi \left( 1 - 4D^2 \right)} \cos (2\pi D) \]  
\tag{2.9}

Now we may define the relationship between the first harmonic of the switch current and the first harmonic of the load resistive current. Obviously
\[ I_{R.1.\text{max}} = I_{\text{sw.1.\text{max}}} \frac{1}{|Y_o| R_o} \]  

(2.10)

where

\[
|Y_o| = \sqrt{\left( \frac{1}{R_o} \right)^2 + \left( \omega_s C_o - \frac{1}{\omega_s L_o} \right)^2}
\]

\[ = \frac{1}{R_o} \sqrt{1 + \left( \omega_s R_o C_o - \frac{R_o}{\omega_o L_o} \right)^2} \]  

(2.11)

Substituting eqn. 2.11 into eqn. 2.10 obtains

\[ I_{R.1.\text{max}} = I_{\text{sw.1.\text{max}}} \frac{1}{\sqrt{1 + \left( \omega_s R_o C_o - \frac{R_o}{\omega_o L_o} \right)^2}} \]  

(2.12)

and taking into account the normalized quantities, we get

\[ A_3 = \frac{I_{R.1.\text{max}}}{I_{\text{sw.1.\text{max}}}} = \frac{1}{\sqrt{1 + R_o^2 \left( \omega_o - \frac{1}{\omega_o} \right)^2}} \]  

(2.13)

Furthermore we suppose that the input and output power are equal

\[ P_{\text{in}} = P_{\text{out}} \]

and taking into account the following relationships:

\[ P_{\text{in}} = V_{\text{m.r.m.s.}} I_{\text{m.r.m.s.}} = V_{\text{m.r.m.s.}} \frac{I_{\text{in.\text{max}}}}{\sqrt{2}} \]

\[ P_{\text{out}} = V_{\text{o.r.m.s.}} I_{\text{o.r.m.s.}} = V_{\text{o.r.m.s.}} \frac{I_{\text{o.\text{max}}}}{\sqrt{2}} \]
We finally obtain

\[
M_x = \frac{V_{o.r.m.s.}}{V_{i.r.m.s.}} \frac{\sqrt{2}}{A_x A_y A_z}
\]

\[
= \sqrt{2} \frac{1 + R_2^2 (\omega_1 - \frac{1}{\omega_2}) + 1 - \cos \left( \frac{\pi D}{D} \right)}{1.1 \pi (1 - D - D_1)}
\]

(2.14)

---

**Figure 2.5** Determination of factor $A_2$ (a) basic waveform of switch current (b) Dependence of $A2$ against duty cycle $D$
Figure 2.6 Factor $M_g = V_o/V_{in}$ against parameters $R_o^*$ & $\omega_n^*$

Figure 2.7 Duty cycle against parameters $L_r^*$, $\omega_n^*$
This relationship is represented in Figure 2.5 the values of duty cycles \( D_1 \) and \( D \) may be calculated from the plot in Figure 2.7. These parameters may also be found from the approximate polynomial expressions

\[
D_1 = (325.8 - 36.7 \omega_r^* - 33.4 \omega_r^* - 25.4 R_0^* + 2.2 \omega \omega R_0^* + 7.4 R_0^* s^* 10^3)
\]

(2.15)

\[
D = (-88.3 - 445.5 L_r^* - 15.5 \omega_r^* + 175.1 \omega_s^* + 19.3 R_0^* + 725 L_r^* \omega \omega - 10.3 R_0^* s) 10^3
\]

(2.16)

2.5 PARAMETERS OF CONVERTER

2.5.1 Assumptions

Analysis of the inverter is based on the following assumptions.

1. The elements of the series-resonant circuit are passive, linear, time invariant, and do not have parasitic reactive components.

2. The loaded quality factor \( Q_L \) of the series-resonant circuit is high enough so that the current I through the resonant circuit is sinusoidal

The parameters of the series-resonant circuit are defined as follows:

- The resonant frequency

\[
\omega_0 = \frac{1}{\sqrt{LC}}
\]

(2.17)

- The characteristic impedance

\[
Z_0 = \sqrt{\frac{L}{C}} = \omega_0 L = \frac{1}{\omega_0 L}
\]

(2.18)
The loaded quality factor

\[
Q_L = \frac{\omega L}{R} = \frac{1}{\omega C R} = \frac{Z_L}{R} = \sqrt{\frac{L}{C}}
\]  \hspace{1cm} (2.19)

The unloaded quality factor

\[
Q_u = \frac{\omega L}{r} = \frac{1}{\omega C r} = \frac{Z_u}{r}
\]  \hspace{1cm} (2.20)

The loaded quality factor is defined as

\[
Q_L = \frac{2\pi}{\omega} \frac{\text{Total energy stored at resonant frequency}}{\text{Energy dissipated per cycle at resonant frequency}} = \frac{2\pi}{\omega} \frac{W_s}{P_s} = \frac{2\pi}{\omega} \frac{f_s W_s}{P_s + R} = \frac{Q}{P_s}
\]  \hspace{1cm} (2.21)

where \(W_s\) is the total energy in the resonant circuit at the resonant frequency \(f_0 = 1/\omega_0\). \(Q = \omega_0 W_s\) is the reactive power of inductor \(L\) or capacitor \(C\) at the resonant circuit at any frequency is given by frequency \(f_0\). The total energy stored is

\[
W_s(\omega, \tau) = w_L(\omega, \tau) + w_C(\omega, \tau) = \frac{1}{2} L I_m^2 \sin^2(\omega \tau - \psi) + \frac{1}{2} C V_m^2 \cos^2(\omega \tau - \psi - 90^\circ)
\]  \hspace{1cm} (2.22)

For steady-state operation at the resonant frequency \(f_0\) the total instantaneous energy stored in the resonant circuit is constant and equal to the maximum energy stored in the inductor

\[
W_s = W_{L_{\text{max}}} = \frac{1}{2} L I_m^2
\]  \hspace{1cm} (2.23)
Or, using (4.17) in the capacitor

\[ W_s = W_{C_{\text{max}}} = \frac{1}{2} CV_{\text{Cm}}^2 = \frac{1}{2} C \frac{f_m^2}{(\omega C)^2} = \frac{1}{2} f_m^2 = \frac{1}{2} L I_m^2 \]  

(2.24)

Substitution of and into produces

\[ Q_L = \frac{\pi f_o L I_m^2}{P_R} = \frac{\pi f_o C V_{\text{Cm}}^2}{P_R} \]  

(2.25)

The reactive power of the inductor \( f_o \) is \( Q = (1/2) V_{L_m} I_m = (1/2) \omega_o L I_m^2 \) and of the capacitor is \( Q = (1/2) I_m V_{C_m} = (1/2) \omega_o C V_{C_m}^2 \). Thus the quality factor can be defined as a ratio of the inductor or a capacitor to the true power dissipated in the form of heat in R. The total power dissipated in \( R = R_i + r \) is

\[ P_R = \frac{1}{2} R I_m^2 = \frac{1}{2} (R_i + r) I_m^2 \]  

(2.26)

Substitution of (2.23) and (2.26) into (2.21) gives (2.19)

For \( R_i = 0 \)

\[ P_R = P_r = \frac{1}{2} r I_m^2 \]  

(2.27)

And the unloaded quality factor is defined as

\[ Q_o = \frac{\omega_o W_s}{P_r} \]  

(2.28)

Resulting in (4.20) Similarly the quality factor of the inductor is

\[ Q_{L_o} = \frac{\omega_o W_s}{P_{L_o}} = \frac{\omega_o L}{r_L} \]  

(2.29)
And the capacitor is

$$Q_{co} = \frac{\omega_c W_c}{P_{ec}} = \frac{1}{\omega_c C_{ec}}$$  \hspace{1cm} (2.30)

### 2.5.2 Input Impedance

The input impedance of the series-resonant circuit is

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R \left[1 + jQ_L \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right]$$

$$= Z_o \left[\frac{R}{Z_o} + j \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right] = Ze^{j\psi} = R + jX$$  \hspace{1cm} (2.31)

where

$$Z = R \sqrt{1 + Q_L^2 \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2} = Z_o \sqrt{\left(\frac{R}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}$$

$$= Z_o \sqrt{\frac{1}{Q_L^2} \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}$$  \hspace{1cm} (2.32)

$$\psi = \arctan \left[Q_L \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right]$$  \hspace{1cm} (2.33)

$$R = Z \cos \psi$$  \hspace{1cm} (2.34)

$$X = Z \sin \psi$$  \hspace{1cm} (2.35)

Notice that the reactance of the resonant becomes zero at the resonant frequency $f_o$. from (2.33)
\[
\cos \psi = \frac{1}{\sqrt{1 + Q_e^2 \left( \frac{\omega}{\omega_0} - \frac{\omega_e}{\omega} \right)^2}} \tag{2.36}
\]

### 2.5.3 Currents, Voltages and Powers

The input of the series-resonant circuit is a square-wave

\[
v = \begin{cases} 
V_l, & \text{for } 0 < \omega t \leq \pi \\
0, & \text{for } \pi < \omega t \leq 2\pi 
\end{cases} \tag{2.37}
\]

This voltage can be expanded into Fourier series

\[
v = \frac{V_l}{2} + \frac{2V_l}{\pi} \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{2n} \sin(n\omega t)
= V_l \left( \frac{1}{2} - \frac{2}{\pi} \sin \omega t - \frac{2}{3\pi} \sin 3\omega t + \frac{2}{5\pi} \sin 5\omega t + \cdots \right) \tag{2.38}
\]

The fundamental component of voltage \( v \) is

\[
v_{i1} = V_m \sin \omega t \tag{2.39}
\]

Where its amplitude is given by

\[
V_m = \frac{2V_l}{\pi} \approx 0.637V_l \tag{2.40}
\]

This leads to the rms value of voltage \( v_{i1} \)

\[
V_{rms} = \frac{V_m}{\sqrt{2}} \approx 0.45V_l \tag{2.41}
\]

If the operating frequency \( f \) is close to the resonant frequency \( f_\omega \), the impedance of the resonant circuit is very high for higher harmonics and
therefore, the current through the resonant circuit is approximately sinusoidal
equal to the fundamental component.  \( i = I_m \sin(\omega t - \psi) \)

Where from (2.35), (2.36), and (2.40)

\[
I_m = \frac{V_m}{Z} \left( \frac{2V_i}{\pi Z} - \frac{2V_i \cos \psi}{\pi R} \right) = \frac{2V_i}{\pi R} \sqrt{1 - \left( \frac{\omega}{\omega_0} \right)^2} \frac{1}{\pi Z_o \sqrt{\left( \frac{R}{Z_o} \right)^2 + \left( \frac{\omega}{\omega_0} \right)^2}} \]

(2.42)

**Figure 2.8** Normalized amplitude \( I_m Z_o / V_1 \) of the current through the resonant circuit as a function of \( f/f_o \) and \( R/Z_o \).

Figure 2.8 shows a three-dimensional representation of \( I_m Z_o / V_1 \) as a function of \( f/f_o \) and \( R/Z_o \).

The output voltage is also sinusoidal

\[
v_{Ri} = iR_i = V_{Rim} \sin(\omega t - \psi) \]

(2.43)
The input current of the inverter \( i_i \) equal the current through the switch \( S_1 \) and is given by

\[
i_i = \begin{cases} 
I_m \sin(\omega t - \psi), & \text{for } 0 < \omega t \leq \pi \\
0, & \text{for } \pi < \omega t \leq 2\pi 
\end{cases}
\]

(2.44)

Hence from (2.32) (2.35), and (2.36) one obtains the dc component of the input current

\[
I_I = \frac{1}{2\pi} \int_0^{2\pi} i_i d(\omega t) = \frac{I_m}{2\pi} \int_0^\pi \sin(\omega t - \psi) d(\omega t) = \frac{I_m \cos \psi}{\pi} - \frac{V_m \cos \psi}{\pi Z} \\
= \frac{2V_I \cos \psi}{\pi^2 Z} - \frac{2V_d \cos^2 \psi}{\pi^2 R} - \frac{2V_R}{\pi^2 Z^2} \cdot \frac{I_m}{\pi} \\
\left[ 1 + Q_L \left( \frac{\omega}{\omega_o} - \frac{\omega}{\omega} \right)^2 \right] \\
= \frac{2V_I}{\pi^2 R} \left[ 1 + Q_L \left( \frac{\omega}{\omega_o} - \frac{\omega}{\omega} \right)^2 \right] 
\]

(2.45)

At \( f = f_o \),

\[
I_I = \frac{I_m}{\pi} \approx \frac{V_I}{5R} 
\]

(2.46)

The DC input power can be expressed as

\[
P_I = I_I V_I = \frac{2V_I^2 \cos^2 \psi}{\pi^2 R} = \frac{2V_I^2}{\pi^2 R} \left[ 1 + Q_L \left( \frac{\omega}{\omega_o} - \frac{\omega}{\omega} \right)^2 \right] \\
= \frac{2V_I^2}{\pi^2 Z_o^2} \left[ \left( \frac{R}{Z_o} \right)^2 + \left( \frac{\omega}{\omega_o} - \frac{\omega}{\omega} \right)^2 \right] 
\]

(2.47)

At \( f = f_o \),

\[ P_i = \frac{2V_i^2}{\pi^2 R} \approx \frac{V_i^2}{5R} \quad (2.48) \]

Using (2.42), one arrives at the output power

\[ P_{ri} = \frac{I_i^2 R_i}{2} \frac{2V_i^2 R_i \cos^2 \phi'}{\pi^2 R^2} = \frac{2V_i^2 R_i}{\pi^2 R^2 \left[ 1 + Q_l^2 \left( \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right)^2 \right]} \]

\[ = \frac{2V_i^2 R}{\pi^2 Z_o^2 \left( \frac{R}{Z_o} \right)^2 + \left( \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right)^2} \quad (2.49) \]

At \( f = f_o \),

\[ P_{ri} = \frac{2V_i^2 R_i}{\pi^2 R^2} \approx \frac{V_i^2 R_i}{5R^2} \quad (2.50) \]

Figure 2.9 depicts \( P_{ri} Z_o^2 / (V_o^2 R_i) \) as a function of \( f = f_o \) and \( R/Z_o \).

The normalized output power \( P_{ri} Z_o^2 / (V_o^2 R_i) \) is plotted as a function of \( f/f_o \) at different values of \( R/Z_o \) in Fig 2.10. The maximum output power occurs at the resonant frequency \( f_o \) and low total resistance \( R_i \).

![Figure 2.9](image)

**Figure 2.9** Normalized output power \( P_{ri} Z_o^2 / V_o^2 R_i \) as a function of \( f = f_o \) and \( R/Z_o = 1/Q_L \).
2.5.4 Current and Voltage Stresses

The peak voltage across each switch is equal to the dc input voltage

\[ V_{SM} = V_I \]  \hspace{1cm} (2.51)

![Normalized output power](image)

**Figure 2.10** Normalized output power \( P_o V_o^2 R_i \) as function of \( f/f_o \) at fixed values of \( R/Z_o = 1/Q \).

The maximum value of the switch peak currents and the maximum amplitude of the current through the resonant occurs at \( f = f_o \). Hence from (2.42)

\[ I_{SM} = I_{mr} = \frac{2V_I}{\pi R} \]  \hspace{1cm} (2.52)

The amplitude of the voltage across the capacitor C is obtained from (2.42)

\[ V_{Cm} = \frac{I_{m_0}}{\omega C} = \frac{2V_I}{\pi} \left( \frac{\omega}{\omega_o} \right) \left( \sqrt{\frac{R}{Z_o}} + \frac{\omega - \omega_o}{\omega_o} \right) \]  \hspace{1cm} (2.53)

A three-dimensional representation of \( V_{Cm}/V_I \) is shown in Figure 2.7. Figure 2.8 depicts plots of \( V_{Cm}/V_I \) as a function of \( f/f_o \) at fixed values of \( R/Z_o \).
Likewise, the amplitude of the voltage across the inductor \( L \) is expressed as

\[
V_{LM} = \omega LI_m = \frac{2V_I}{\pi} \frac{\left( \frac{\omega}{\omega_o} \right)}{\sqrt{\frac{R}{Z_o} + \left( \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right)}^2}
\]  

(2.54)

Figure 2.11 shows \( V_{LM}/V_I \) as a function of \( f/f_o \) and \( R/Z_o \). Plots of \( V_{LM}/V_I \) against \( f/f_o \) at constant values of \( R/Z_o \)

At \( f = f_o \),

\[
V_{CM(max)} = V_{LM(max)} = Z_o I_m = Q_L V_M = \frac{2V_I Q_L}{\pi}
\]  

(2.55)

Figure 2.11 Normalized amplitude \( V_{LM}/V_I \) of the voltage across the resonance inductor \( L \) as a function of \( f/f_o \) and \( R/Z_o = I/Q_I \)

The maximum voltage stresses of the resonant components occur at the resonant frequency \( f \approx f_o \), a maximum dc input voltage \( V_I = V_{I_{max}} \), and a maximum loaded quality factor \( Q_L \). Actually the maximum value of \( V_{LM} \) occurs slightly above \( f_o \) and of \( V_{CM} \) slightly below \( f_o \). However this effect is negligible for practical purpose.
At the resonant frequency $f = f_0$, the amplitudes of the voltages across the resonant inductor and resonant capacitor are $Q_L$ times higher than the amplitude $V_m$ of the fundamental component of the voltage at the input of the resonant circuit which is equal to the amplitude of the output voltage $V_{Rim}$.

### 2.6 EFFICIENCY

#### 2.6.1 Conduction Losses

The conduction loss for the power MOSFET is

$$\frac{P_{r_{DS}}}{4} = \frac{I_m^2 r_{DS}}{4}.$$  \hfill (2.56)

For the resonant inductor is

$$P_{rL} = \frac{I_m^2 r_L}{2}.$$  \hfill (2.57)

And for the resonant capacitor is

$$P_{rC} = \frac{I_m^2 r_C}{2}.$$  \hfill (2.58)

Hence the conduction power loss in both transistors and resonant circuit is

$$P_r = 2P_{r_{DS}} + P_{r_{L}} + P_{r_{C}} = \frac{I_m^2 (r_{DS} + r_L + r_C)}{2} + \frac{I_m^2 r}{2}.$$  \hfill (2.59)

Neglecting switching and the gate-drive losses one obtains the efficiency of the inverter determined by the conduction losses only

$$\eta_{in} = \frac{P_{in}}{P_i} = \frac{P_{r_{ds}}}{P_{r_{ds}} + P_{r_{L}} + P_{r_{C}}} \frac{R_t}{R_t + r} \frac{1}{1 + \frac{r}{R_t + r}} \frac{1 - \frac{r}{R_t + r}}{Q_L Q_o} = 1 - \frac{1}{1 + \frac{Q_L}{Q_o}}.$$  \hfill (2.60)
Note that in order to achieve high efficiency the ratio of the load resistance $R_l$ to the parasitic resistance $r$ must be high.

### 2.6.2 Turn-On Switching Loss

For the operation below resonance the turn-off switching loss is zero; however, there is turn-on switching loss. This loss is associated with charging and discharging the output capacitances of the MOSFETs. The diode junction capacitance of the MOSFETs. The diode junction capacitance is

$$
c_j(V_D) = \frac{C_{j0}}{1 - \frac{V_D}{V_B}} = \frac{C_{j0} V_B^m}{(V_B - V_D)^m}, \text{ for } V_D \leq V_B
$$

where $C_{j0}$ is the junction capacitance at $V_D = 0$ and $m$ is the grading coefficient; $m = 1/2$ for junctions and $m = 1/3$ for graded junctions. The barrier potential is

$$
V_B = V_T m \left( \frac{N_A N_D}{n_i^2} \right),
$$

where $n_i$ is the intrinsic carrier density ($1.5 \times 10^{10}$ cm$^{-3}$ for silicon at 20°C), $N_A$ is the acceptor concentration, and $N_D$ is the donor concentration. The thermal voltage is

$$
V_T = \frac{kT}{q} = \frac{T}{11,609} \text{ (V)},
$$

where $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann’s constant, $q = 1.602 \times 10^{-19}$ C is the charge per electron, and $T$ is the absolute temperature in K. For $p^+n$ diodes, a typical value of the acceptor concentration is $N_A = 10^{16}$ cm$^{-3}$ and a typical value of the donor of the acceptor concentration is $N_D = 10^{14}$ cm$^{-3}$ which gives $V_B = 0.57$ V. The zero-voltage junction capacitance is given by
\[ C_{j0} = A \left( \frac{\varepsilon \varepsilon_0 q}{2V_B \left( \frac{1}{N_D} + \frac{1}{N_A} \right)} \right) \approx A \left( \frac{\varepsilon \varepsilon_0 q N_D}{2V_B} \right), \quad \text{for} \quad N_D << N_A \]  

(2.64)

where \( A \) is the junction area in cm\(^2\), \( \varepsilon = 11.7 \) for silicon and \( \varepsilon_0 = 8.85 \times 10^{-14} \) (F/cm). Hence, \( C_{j0}/A = 3.1234 \times 10^{-16} \sqrt{N_D} = 1014 \), \( C_{j0}/A \approx 3 \) nF/cm\(^2\). Typical values of \( C_{j0} \) are of the order of 1 nF for power diodes.

The MOSFETs drain-source capacitance \( C_{ds} \) is the capacitance of the body-drain pn step junction diode. Setting \( v_D = -v_{DS} \) and \( m = 1/2 \), one obtains from (2.63)

\[ C_{ds}(v_{DS}) = \frac{C_{j0}}{\sqrt{1 + \frac{v_{DS}}{V_B}}} = C_{j0} \sqrt{\frac{V_B}{v_{DS} + V_B}}, \quad v_{DS} \geq -V_B. \]  

(2.65)

Hence

\[ \frac{C_{d_{sl}}}{C_{d_{s2}}} = \sqrt{\frac{V_{DS2} + V_B}{V_{DS1} + V_B}} \approx \sqrt{\frac{V_{DS2}}{V_{DS1}}}, \]  

(2.66)

where \( C_{d_{sl}} \) is the drain-source capacitance at \( v_{DS1} \) and \( C_{d_{s2}} \) is the drain-source capacitance at \( v_{DS2} \). Manufacturers of power MOSFETS usually specify the capacitances \( C_{oss} = C_{gd} + C_{ds} \) and \( C_{rss} = C_{gd} \) at \( V_{DS} = 25 \) V, \( V_{GS} = 0 \) V, and \( f = 1 \) MHz. Thus the drain-source capacitances of \( V_{DS} = 25 \) V can be found as \( C_{ds(25V)} = C_{oss} - C_{rss} \). The inter terminal capacitances of MOSFETs are essentially independent of frequency. From (2.68), the drain-source capacitance at the dc voltage \( V_I \) is

\[ C_{ds(V_I)} = C_{ds(25V)} \sqrt{\frac{25 + V_B}{V_I + V_B}} \approx \frac{5C_{ds(25V)}}{\sqrt{V_I}} (F) \]  

(2.67)
The drain-source capacitance at $v_{DS} = 0$ is

$$C_{J0} = C_{dsv_{(25V)}} \frac{25}{V_B} + 1 \approx 6.7C_{dsv_{(25V)}} \quad (2.68)$$

For $V_B = 0.57$ V. Also

$$C_{dsv_{(25V)}} = C_{dsv_{(V_I)}} \frac{V_I + V_B}{V_{DS} + V_B} \approx C_{dsv_{(V_I)}} \frac{V_I}{V_{DS}} \quad (2.69)$$

Using (2.67) and $dQ_J = C_{ds}dV_{DS}$, the charge stored in the drain-source junction capacitance at $v_{DS}$ can be found as

$$Q_J(v_{DS}) = \left[ V_{DS} \right]_{-V_B}^{V_B} \int_{-V_B}^{V_B} C_{ds} \frac{dV_{DS}}{dV_B} + V_B \right] = C_{j0} \sqrt{V_B} \frac{dV_{DS}}{dV_B} + V_B \right] = 2C_{j0} \sqrt{V_{DS} + V_B} \frac{V_{DS} + V_B}{V_B} \quad (2.70)$$

Which by substituting (2.69) at $v_{DS} = V_B$ simplifies to

$$Q_J(V_I) = 2V_I C_{dsv_{(V_I)}} = 10C_{dsv_{(25V)}} \sqrt{V_I} \quad (2.71)$$

Hence the energy transferred from the dc input source $V_I$ to the output capacitance of the upper MOSFET after the upper transistor is turned off is

$$W_I = \left[ V_I \right]_{-V_B}^{V_B} \int_{-V_B}^{V_B} dt = V_I Q_J(V_I)$$

$$= 2V_I^2 C_{dsv_{(V_I)}} = 10V_I^2 C_{dsv_{(25V)}}(W) \quad (2.72)$$
Using \( dW_j = (\frac{1}{2}) Q_j dv_{DS} \) and the energy stored in the drain-source junction capacitance \( C_{ds} \) at \( v_{DS} \) is

\[
W_j(v_{DS}) = \frac{1}{2} \int_{-V_D}^{V_D} Q_j dv_{DS} = C_{j0} \sqrt{V_B} \int_{-V_D}^{V_D} \sqrt{V_{DS} + V_B} \, dv_{DS}
\]

\[
= \frac{2}{3} C_{j0} \sqrt{V_B} (V_{DS} + V_B)^{\frac{3}{2}} = \frac{2}{3} C_{ds}(v_{DS}) v_{DS} (V_B)^{\frac{3}{2}} \approx \frac{2}{3} C_{ds}(v_{DS}) v_{DS}^{\frac{3}{2}}
\]

(2.73)

Hence from (2.69) the energy stored in the drain-source junction capacitance at \( v_{DS} = V_t \) is

\[
W_j(V_t) = \frac{2}{3} C_{ds(V_t)} V_t^2 = \frac{10}{3} C_{ds(2V_t)} \sqrt{V_t^3} (J)
\]

(2.74)

This energy is lost as heat when the transistor turns on and the capacitor is discharged through \( r_{DS} \), resulting in the turn-on switching power loss per transistor

\[
P_{on} = \frac{W_j(V_t)}{T} f W_j(V_t) = \frac{2}{3} f C_{j0} \sqrt{V_B} (V_t + V_B)^{\frac{3}{2}}
\]

\[
= \frac{2}{3} f C_{ds(V_t)} V_t^2 = \frac{10}{3} f C_{ds(2V_t)} \sqrt{V_t^3} (W)
\]

(2.75)

Figure 2.12 illustrates \( C_{ds} \), \( Q_j \), and \( W_j \) as functions of \( v_{DS} \) given by (2.67), (2.72), and (2.73).

Using (2.72) and (2.74) one arrives at the energy lost in the resistances of the charging path during the charging process of the capacitance \( C_{ds} \).

\[
W_{char}(V_t) = W_j(V_t) - W_j(V_t) = \frac{4}{3} C_{ds(V_t)} V_t^2 = \frac{20}{3} C_{ds(2V_t)} \sqrt{V_t^3} (J)
\]

(2.76)
And the corresponding power associated with charging the capacitance $C_{ds}$ is

$$P_{\text{char}} = \frac{W_{\text{char}}(V_f)}{T} = f_0 \frac{4}{3} f C_{ds(V_f)} V_i^2 \frac{20}{3} f C_{ds(25V)} \sqrt{V_i^2} (J) \quad (2.77)$$

Figure 2.12 Plots of $C_{ds}$ of $Q_j$, and $W_j$ versus $V_{DS}$ (a) $C_{ds}$ versus $V_{DS}$. (b) Versus $V_{DS}$. (c) $W_j$, versus $V_{DS}$.
From (2.72) one arrives at the total switching power loss per transistor

\[
P_{sw} = \frac{W(V_j)}{T} + jW_j(V_j) - 2fC_j(V_j) \sqrt{V_B(V_T-V_B)}
\]

\[
= 2fC_{ds(V_j)} V_f^2 = 10fC_{ds(V_f)} V_f^3 (W)
\]  

(2.78)

The switching loss associated with charging and discharging an equivalent linear capacitance \(C_{eq}\) is \(P_{sw} = f_{eq} V_f^2\). Hence

\[
C_{eq} = 2C_{ds(V_f)} = \frac{10C_{ds(V_f)}}{V_f^2}
\]  

(2.79)

### 2.6.3 Turn-off Switching Loss

For the operation above resonance, the turn-on switching loss is zero, but there is a turn-off switching loss. The switch current and voltage waveforms during turn-off for \(f > f_o\) are sketched in Figure 2.13. These waveforms were observed in various Class D experimental circuits. Notice that the voltage \(v_{DS2}\) increases slowly at its low values and much faster at its high values. This is because the MOSFET output capacitance is highly nonlinear, and it is much higher at low voltage \(v_{DS2}\) than at high voltage \(v_{DS2}\). The current that charges this capacitance is approximately constant. The drain-to-source voltage \(v_{DS2}\) during rise time \(t_r\) can be approximated by a parabolic function

\[
v_{DS2} = a(\omega t)^2
\]  

(2.80)

Because \(v_{DS2}(\omega t) = V_f\) one obtains

\[
a = \frac{V_f}{(\omega t_r)^2}
\]  

(2.81)
Hence (2.80) becomes

\[ V_{DS2} = V_I \left( \frac{\omega t}{t_r} \right)^2 \]  

(2.82)

The switch current during rise time \( t_r \) is a portion of a sinusoid and can be approximated by a constant

\[ i_{S2} = I_{off} \]  

(2.83)

The average value of the power loss associated with voltage rise time \( t_r \) is

\[
P_n = \frac{1}{2\pi} \int_0^{2\pi} i_{S2} V_{DS2} d(\omega t) = \frac{V_I I_{off}}{2\pi(\omega t)^2} \int_0^{2\pi} (\omega t)^2 d(\omega t)
\]

\[
= \frac{\omega t V_I I_{off}}{6\pi} = \frac{f t V_I I_{off}}{3} = \frac{t_r V_I I_{off}}{3T}
\]  

(2.84)

Figure 2.13 : Waveforms of \( v_{DS2}, i_{DS2}, \) and \( i_{S2} v_{DS2} \) during turn-off for \( f > f_o \).
The switch current during fall time $t_f$ can be approximated by a ramp function

$$i_{s2} = I_{OFF} \left( 1 - \frac{\omega t}{\omega t_f} \right)$$  \hspace{1cm} (2.85)

And the drain-source voltage is

$$v_{DS2} = V_i$$  \hspace{1cm} (2.86)

Which yields the average value of the power loss associated with current fall time $t_f$

$$P_g = \frac{1}{2\pi} \int_0^{2\pi} i_{s2} v_{DS2} d(\beta t) = \frac{V_i I_{OFF}}{2\pi} \int_0^{\beta t_f} \left( 1 - \frac{\omega t}{\omega t_f} \right) d(\beta t)$$

$$= \frac{\omega t_f V_i I_{OFF}}{4\pi} = \frac{f t_f V_i I_{OFF}}{2} = \frac{t_f V_i I_{OFF}}{2T}$$  \hspace{1cm} (2.87)

Hence the turn-off switching loss is

$$P_{off} = P_r + P_g = f V_i I_{OFF} \left( \frac{t_r}{3} + \frac{t_f}{2} \right)$$  \hspace{1cm} (2.88)

Usually $t_r$ is much longer than $t_f$. The overall power dissipation in the Class D half-bridge inverter is

$$P_T = P_r + 2P_{off} + \frac{rI_m^2}{2} + fV_i I_{OFF} \left( \frac{2t_r}{3} + t_f \right) + 2fQ_g V_{GS_m}$$  \hspace{1cm} (2.89)

Hence the efficiency of the inverter for operation above resonance is

$$\eta_I = \frac{P_{RI}}{P_{RI} + P_T} = \frac{P_{RI}}{P_{RI} + P_r - 2P_{off} + 2P_G}$$  \hspace{1cm} (2.90)
2.7 COMPUTER SIMULATION

The series resonant converter fed induction heater is simulated using MATLAB SIMULINK and the results are presented here. The circuit model of the series converter is shown in Figure 2.14 a. Scopes are connected to measure the output voltage, driving pulses and capacitor voltage.

(a) open loop Circuit

(b) Driving pulses

Figure 2.14 (Continued)
(c) Voltage across $S_1$ ($V_{ds}$)

(d) Current through $S_1$

(e) AC Output Voltage

Figure 2.14 Simulation results

The switching pulses are shown in Figure 2.14b. The voltage and current waveforms of the switch are shown in Figure 2.14c and Figure 2.14d.
respectively. The high frequency AC output of converter is shown in Figure 2.14e.

The closed loop simulink circuit model of the series resonant converter is shown in Figure 2.15. Scopes and displays are connected to measure the output voltage. To control the DC voltage, a controlled rectifier is proposed.

![Figure 2.15 Closed loop controlled AC-AC Converter](image)

A disturbance is given at the input by using two switches. The output voltage is sensed and it is compared with the reference voltage. The error signal is given to the controller. The output of the PI controller controls the pulse width of the MOSFETs. The response of the open loop system is shown in Figure 2.16. It can be seen that the amplitude of output voltage increases due to the increase in the input voltage.
Figure 2.16 Output voltage of open loop system

Figure 2.17 Output voltage of closed loop system

The output voltage of the closed loop system is shown in Figure 2.17. The disturbance is applied at 0.3 secs. The settling time is 0.32 secs. The control circuit takes proper action to reduce the amplitude of output
voltage to the set value. Thus, the closed loop system reduces the steady state error.

2.8 EXPERIMENTAL VERIFICATION

The ATMEL 89C2051 based control circuit is shown in Figure 2.18. The ATMEL microcontroller 89C2051 was used to generate the driving pulses for the MOSFET switches. They are amplified using the driver IC IR2110. The gate signal is connected to port pin P1.0. The various steps involved in the firing pulse generation are shown in Figure 2.19. In the flow chart delay1 reflects the turn-on interval of the switch during which the pulse will be at logic 1, whereas delay2 reflects the turn off interval of the switch during which pulse will be at logic 0.

![Figure 2.18 89C2051 based control circuit](image)
The single-switch series resonant converter was built and tested at 230V. The experimental setup of the converter is shown in Figure 2.20. The circuit parameters are $R_0=60\,\Omega$; $L_0=150\,\mu\text{H}$; $C_0=2.35\,\mu\text{F}$; $L_r=22\,\mu\text{H}$; $L_i=8.0\,\text{mH}$; $C_{in}=0.94\,\mu\text{F}$ and the switching frequency $\omega_S=62\times10^3\,\text{s}^{-1}$, $V_0=200\,\text{V}$, $P_0=5.86\,\text{kW}$. The experimental waveform of the output voltage is shown in Figure 2.21. The input voltage and current waveforms are shown in Figure 2.22. It can be seen that the power factor is close to unity. The output power control was also checked and its dependency on the switching frequency is shown in Figure 2.23. The output power increases with an increase in the switching frequency.
Figure 2.20 Hardware layout

Figure 2.21 Oscillogram of output voltage

Figure 2.22 Oscillogram of input voltage and current
A comparison on the performance of the conventional AC-AC converter and the single switch AC-AC converter is presented in Table 2.1. The results confirm that the single switch converter delivers a better performance than any other converter in achieving better efficiency.

**Table 2.1 Performance comparison**

<table>
<thead>
<tr>
<th>Input voltage in volts</th>
<th>Output voltage in volts</th>
<th>Efficiency in %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conventional converter</td>
<td>Series Resonant converter</td>
</tr>
<tr>
<td>180</td>
<td>129.6</td>
<td>152.4</td>
</tr>
<tr>
<td>200</td>
<td>144.1</td>
<td>169.4</td>
</tr>
<tr>
<td>220</td>
<td>158.2</td>
<td>186.4</td>
</tr>
<tr>
<td>240</td>
<td>173</td>
<td>203.5</td>
</tr>
</tbody>
</table>

When the proposed converter and the previously developed one operate under the condition of maximum output power, the power loss analysis is shown in Table 2.2. The power loss dissipation of the newly
developed converter is comparatively lower than the previously developed one.

Table 2.2 Power loss comparison

<table>
<thead>
<tr>
<th>Devices and components</th>
<th>Power dissipation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>New system</td>
</tr>
<tr>
<td>Switch</td>
<td>49W</td>
</tr>
<tr>
<td>Filter inductor</td>
<td>18W</td>
</tr>
<tr>
<td>IH load</td>
<td>58W</td>
</tr>
<tr>
<td>Bridge rectifier</td>
<td>23W</td>
</tr>
</tbody>
</table>

2.9 CONCLUSION

A series resonant converter circuit fed induction heater is simulated and tested. The converter input current is practically sinusoidal and the power factor is close to unity. The circuit topology is very simple and it includes only one power switch. This switch operates in a soft commutation mode. The converter provides a wide-range power control. This converter has advantages like reduced hardware, reduced stresses and high power density. The simulation and experimental results demonstrate the capability of the proposed converter to develop higher output power than the conventional converter. The experimental results agree closely with the simulation results.