CHAPTER 4

INVESTIGATION ON SCALE EFFICIENCY AND MEASURE OF MOST PRODUCTIVE SCALE SIZE

4.1 INTRODUCTION

Policy recommendations concerning optimal scale of production/ operation units may have serious implications for the restructuring of a sector. The issue of economies of scale is not limited to manufacturing industries but also to regulated or state-owned industries, like electricity, water, telecom, etc. In electricity distribution, the debate has been lively about the minimum efficient scale and the potential for increased productivity by further exploitation of economies of scale. From a policy point of view, examination of scale properties of production technologies and scale efficiency of production units are, therefore, paramount.

This chapter introduces the concept and the significance of calculating the scale efficiency and the Most Productive Scale Size (MPSS). The CCR efficiency is the overall efficiency which takes account of the scale efficiency. For the DMU's which are scale inefficient, it is an indirect measure that they are not operating on the Most Productive Scale Size. If the present scale of operation of the DMU does not lead to 100% scale efficiency, then the scale size of every inefficient DMU to be operated is identified by this calculation. The calculation of MPSS in a multiple input/output environment is complex. Banker (1984), has suggested a mathematical model for calculating the MPSS in a multiple input/output environment. The result of the MPSS also gives the information on RTS. This value is compared against the RTS calculated from the basic CCR model and it is validated for its coherence.

4.2 METHODOLOGY

The standard reaction to information about scale properties in a regulatory context is that activities exhibiting increasing returns to scale should be expanded, and that activities showing decreasing returns to scale should be contracted. The returns to scale, or scale elasticity, is a measurement of the increase in output relative to a proportional increase in all inputs, evaluated as marginal changes at a point on the production function. In a multi-output setting the increase in a single output is the most naturally generalized to a proportional increase in all outputs.

When the inputs are changed with a factor μ, the resulting proportional expansion, $\beta = \beta(\mu, y, x)$, the output is found by solving $F(\beta(\mu, y, x)y, \mu x) = 0$. The scale elasticity, e, as a function of outputs and inputs, is defined for a differentiable function as the marginal change in the output expansion factor by a marginal change in the input expansion factor over the average ratio:

$$\varepsilon(y, x) = \frac{\partial \beta(\mu, y, x)}{\partial \mu} \frac{\mu}{\beta} \tag{4.1}$$

A formula for calculating the scale elasticity is obtained by differentiating (4.1) with respect to the input-scaling factor. Evaluating the derivatives, without loss of generality at $\beta = \mu = 1$ and solving for the scale elasticity yields:

$$\frac{\partial \beta(y, x)}{\partial \mu} \equiv \varepsilon(y, x) = -\frac{\sum_{n=1}^{N} \left(\partial F(y, x) \big/ \partial x_n \right) x_n}{\sum_{m=1}^{M} \left(\partial F(y, x) \big/ \partial y_m \right) y_m} \tag{4.2}$$

4.3 SCALE EFFICIENCY

It is interesting to investigate the sources of inefficiency that a DMU might have. Are they caused by the inefficient operation of the DMU itself or by the disadvantageous conditions under which the DMU is operating?

For this purpose, comparisons of the (input-oriented) CCR and BCC scores deserve consideration. The CCR model assumes the constant returns-to-scale production possibility set, i.e. it is postulated that the radial expansion and reduction of all observed DMUs and their non-negative combinations are possible and hence the CCR score is called *global technical* efficiency. On the other hand, the BCC model assumes that convex combinations of the observed DMUs form the production possibility set and the BCC score is called *local pure technical* efficiency. If a DMU is fully efficient (100%) in both the CCR and BCC scores, it is operating in the *most productive scale*. If a DMU has full BCC efficiency but a low CCR score, then it is operating locally efficiently but not globally efficiently due to the scale size of the DMU. Thus, it is reasonable to characterize the *scale efficiency* of a DMU by the ratio of the two scores.

Based on the CCR and BCC scores, *scale efficiency* is defined as follows:

Scale Efficiency: Let the OCR and BCC scores of a DMU be θ^*_{CCR} and θ^*_{BCC} respectively.

The scale efficiency is defined by

$$SE = \frac{\theta^*_{CCR}}{\theta^*_{BCC}} \tag{4.3}$$

SE is not greater than one. For a BCC-efficient DMU with CRS characteristics, i.e., in the most productive scale size, its scale efficiency is one. The CCR score is called the (global) *technical* efficiency (TE), since it takes no account of scale effect as distinguished from PTE. On the other hand, BCC expresses the (local) *pure technical* efficiency (PTE) under variable returns-to-scale circumstances. Using these concepts, a decomposition of efficiency is defined as

$$\theta^*_{CCR} = \theta^*_{BCC} \text{ x SE} \tag{4.4}$$

or,

[Technical Eff. (TE)] = [Pure Technical Eff. (PTE)] x [Scale Eff. (SE)]

This decomposition, which is unique, depicts the sources of inefficiency, i.e. whether it is caused by inefficient operation (PTE) or by disadvantageous conditions displayed by the scale efficiency (SE) or by both.

The SE represents the proportion of inputs that can be further reduced after pure technical inefficiency is eliminated if scale adjustments are possible. It varies form zero to one. If the SE of the target DMU is equal to 1 then it is said to be operating at constant returns to scale. If SE is less than 1, then the target DMU is scale inefficient and there is potential input savings through the adjustment of its operational scale; whether the scale inefficient DMU should be either downsized or expanded depends on its current operating scale.

4.4 MOST PRODUCTIVE SCALE SIZE

For the DMU's which are scale inefficient, it is an indirect measure that they are not operating on the Most Productive Scale Size. If the present scale of operation of the DMU does not lead to 100% scale efficiency, then the scale size of every inefficient DMU to be operated will be identified by the calculation of Most Productive Scale Size (MPSS).

To understand R.D. Banker's concept of Most Productive Scale Size (MPSS) assume the following expression $(x_o\alpha, y_o\beta)$. Here (x_o, y_o) are vectors with components corresponding to coordinates of the point being evaluated and (α, β) are scalars representing expansion or contraction factors according to whether $\alpha, \beta > 1$ o r $\alpha, \beta < 1$ are applied to the inputs and outputs represented by these coordinates.

To measure the scale which is dimensionless, the following model is utilized,

$$\text{Max } \beta/\alpha$$

Subject to

$$\sum_{j=1}^{n} x_{ij}\lambda_j \leq \alpha x_{io} \quad i = 1,2,\ldots.. m$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \geq \beta y_{ro} \quad r = 1,2,\ldots\ldots s \tag{4.5}$$

$$\lambda_j, s_i^-, s_r^+ \geq 0, \forall i, j, r \quad j = 1,2\ldots\ldots.n$$

$$\sum \lambda_j = 1 \quad \beta, \alpha > 0$$

This is a fractional programming problem which is transformed to an ordinary linear programming problem to obtain solutions $x_o^* = \alpha x_o, y_o^* = \beta y_o$ which is associated with points which are MPSS by means of the following theorem.

Theorem: A necessary condition for a DMUo with output and input vectors y_o and x_o to be at MPSS is $\beta^*/\alpha^* = \beta/\alpha = 1$ in (4.5), in which case $\alpha^* = \beta^*$ and returns to scale will be constant.

In an actual situation, the DMUs will not always operate on IRS. Up to some point on the frontier curve thy will operate on IRS. After that they will operate on DRS. The critical point where the frontier changes from IRS to DRS is known as the Most Productive Scale Size.

Identifying the Most Productive Scale Size is complex for any DMU when dealing with multiple inputs and multiple outputs. Banker (1984) has proved that MPSS for a given inefficient firm can be obtained using the following relationship:

$$\left(X_{i,m}^{MPSS}, Y_{i,m}^{MPSS}\right) = \left[\theta_m^* \frac{X_{im}}{\sum_{n=1}^{N} \lambda_{nm}^*}, \frac{Y_{jm}}{\sum_{n=1}^{N} \lambda_{nm}^*}\right] \tag{4.6}$$

4.5 RESULTS AND DISCUSSION

4.5.1 Scale Efficiency Analysis

The results of the TE, PTE and SE are given in Table 4.1. The analyses of the above efficiency scores provide possible directions of improvement for the TE for each utility whether they have to modify the scale of operation and/or to improve the pure technical efficiency in order to increase the overall efficiency.

Table 4.1 Scale efficiency analysis

Sl.No.	State owned electric utility	TE	PTE	SE
Northern Region				
6	Delhi	100	100	1
9	Haryana	89.08	91.17	0.97
10	Himachal Pradesh	60.94	63.81	0.95
11	Jammu and Kashmir	60.61	100	0.60
22	Punjab	93.77	96.91	0.96
23	Rajasthan	78.86	81.77	0.96
27	Uttar Pradesh	87.97	92.92	0.94
28	Uttaranchal	39.16	57.18	0.68
Eastern Region				
4	Bihar	69.8	96.55	0.72
12	Jharkhand	98.37	100	0.98
21	Orissa	65.32	68.13	0.95
24	Sikkim	37.19	100	0.37
29	West Bengal	89	90.99	0.97
Western Region				
5	Chattisgarh	73.46	79.85	0.91
7	Goa	74.76	87.33	0.85
8	**Gujarat**	**100**	**100**	**1**
15	Madhya Pradesh	70.86	77.53	0.91
16	**Maharashtra**	**100**	**100**	**1**
Southern Region				
1	**Andhra Pradesh**	**100**	**100**	**1**
13	Karnataka	97.47	97.69	0.99
14	**Kerala**	**100**	**100**	**1**
25	Tamil Nadu	99.79	100	0.99
North Eastern Region				
2	Arunachal Pradesh	29.68	60.125	0.49
3	Assam	51.04	60.32	0.84
17	**Manipur**	**100**	**100**	**1**
18	Meghalaya	43.55	59.04	0.73
19	Mizoram	51.65	87.92	0.58
20	Nagaland	86.28	100	0.86
26	Tripura	51.71	73.98	0.69

The SOEU that has the scale efficiency of less than 1 is generally known as scale inefficient. This implies that the inefficiency of the SOEU's is primarily due to the scale inefficiency. For example, Goa has PTE 87.3 % and SE of 85% which is less than the PTE. Therefore, this DMU can enhance their operation scales to improve the TE because they showed IRS in the CCR model. On the other hand, Chattisgarh has PTE score of 79.9% and SE score of 91%. Therefore it should decrease its operation scale because it shows DRS in the CCR model.

4.5.2 MPSS Analysis

Table 4.2 enumerates the MPSS for the inefficient DMUs. This table also includes the λ values of the concerned DMUs and the RTS based on the λ value. It is observed that, the results of the RTS evaluated from the measure of ROE coincide with the results obtained from the MPSS evaluation. For example, DMU 4 (Bihar) has CCR efficiency score of 69.8% with its λ value of 0.664 for which the RTS is identified as IRS. For this DMU, the operating points for the various parameters are given as (1286.8 MW, 132126 km, 33.7%, 1.25 Lakhs and 3730.54 GWh). This actually means that, for the DMU to become as efficient as its Peer unit, it has to increase its scale of operation. But how well it has to operate is evaluated by using MPSS. For the same DMU, the MPSS calculated are (1352.66 Mw, 138888.47 km, 35.42%, 1.88 Lakhs and 5614.58GWh). The DMU has to increase the Installed capacity from 1286.8 MW to 1352.66 MW, circuit length from 132126 km to 138888.47 km, Number of consumers from 1.25 millions to 1.88 millions and quantity of energy supplied form 3730.54 GWh to 5614.58GWh.

Because of increased nature of all the parameters, the losses also increased from 33.7% to 35.42. The feasibility of increasing the parameter as calculated in the actual field depends on the feasibility study of actual implementation. Similarly, for the DMU 9 (Haryana), its CRS efficiency score is 89% with it's λ of 1.307. The RTS for this DMU is DRS i.e., it allocated more resources and produces less output. Therefore, they should decrease the scale of operation to become efficient which is also validated from the MPSS values evaluated.

Table 4.2 MPSS For the scale inefficient units

State Owned Electric Utilities	λ Value	RTS	Installed Capacity (MW)	Circuit km	% T&D Losses	Number of consumers (millions)	Quantity of Energy Supplied (GWh)
Arunachal Pradesh	0.0152	IRS	179.4 [3496.9]	14216 [277101.06]	83.7 [1631.49]	0.113 [7.419]	125.01 [8207.51]
Assam	0.429	IRS	1130.9 [1343.22]	78612 [93371.10]	56.7 [67.35]	1.177 [2.74]	1920.38 [4468.76]
Bihar	0.664	IRS	1286.8 [1352.66]	132126 [138888.47]	33.7 [35.42]	1.25 [1.88]	3730.34 [5614.58]
Chattisgarh	1.429	DRS	1722 [884.73]	120208 [61760.54]	69.0 [35.45]	2.213 [1.55]	5420.83 [3791.18]
Goa	0.297	IRS	470.7 [1183.03]	14274 [35875.44]	78.7 [197.80]	0.396 [1.3311]	1376.66 [4627.78]
Haryana	1.307	DRS	3839.4 [2615.21]	177461 [120877.75]	59.7 [40.66]	3.917 [2.99]	12915.72 [9875.06]
Himachal Pradesh	0.307	IRS	1803.2 [3572.61]	75315 [149218.77]	90.7 [179.01]	1.646 [5.35]	2736.92 [8898.47]
Jammu and Kashmir	0.4578	IRS	1646.1 [2179.20	41774 [55302.89]	31.2 [41.30]	1.0 [2.18]	3534.2 [7718.4]
Jharkand	0.917	IRS	2077.1 [2242.92]	49074 [52991.81]	37.5 [40.49]	0.653 [0.72]	7774.11 [8533.70]
Karnataka	1.079	DRS	7784.3 [7021.22]	597639 [539053.35]	69.0 [62.24]	12.889 [11.94]	23143.17 [21437.96]
Madhya Pradesh	0.724	IRS	6685 [6538.76}	582757 [570008.90]	58.5 [57.22]	6.492 [8.96]	15907.83 [21957.07]
Meghalaya	0.318	IRS	288.2 [393.96]	15657 [21402.45]	60.7 [82.97]	0.168 [0.53]	797.02 [2501.18]
Mizoram	0.0172	IRS	116.8 [3496.9]	14798 [443040.46]	61.6 [1844.25]	0.128 [7.419]	129.9 [7529.12]
Nagaland	0.025	IRS	102.7 [3496.9]	10675 [363480.11]	44.5 [1515.2]	0.188 [7.419]	136.25 [5376.80]

Table 4.2 (Continued)

State Owned Electric Utilities	λ Value	RTS	Installed Capacity (MW)	Circuit km	% T&D Losses	Number of consumers (millions)	Quantity of Energy Supplied (GWh)
Orissa	0.9084	IRS	3023.3 [2174.28]	100464 [72246.10]	56.4 [40.55]	2.149 [2.365]	7157.48 [7878.90]
Punjab	1.623	DRS	6135.3 [3543.5]	287520 [166060.13]	74.5 [43.02]	5.836 [3.59]	22125.3 [13626.80]
Rajasthan	0.87	IRS	5427.6 [4915.88]	441724 [400072.88]	55.6 [50.35]	5.845 [6.712]	14691.24 [16871.67]
Sikkim	0.048	IRS	116.1 [881.43]	5156 [39144.30]	33.8 [256.60]	0.06 [1.22]	182.24 [3719.65]
Tamilnadu	1.083	DRS	12343.3 [11367.87]	625692 [576242.38]	79.4 [73.12]	16.133 [14.88]	39240.21 [36213.96]
Tripura	0.079	IRS	244.5 [1572.28]	14238 [91558.96]	85.2 [547.88]	0.228 [2.835]	414.26 [5151.2]
Uttar Pradesh	0.707	IRS	8864.6 [11023.58]	494417 [614833.16]	56.7 [70.50]	8.806 [12.44]	26659.62 [37685.36]
Uttaranchal	0.445	IRS	1968.9 [1732.5]	65583 [57709.04]	56.5 [49.7]	0.961 [2.159]	2662.15 [5981.39]
West Bengal	1.27	DRS	5559.9 [39.24.00]	188789 [133241.27]	67.1 [47.35]	4.727 [3.721]	17815.87 [14027.12]

The values in the parenthesis show the MPSS.

It is observed that the DMU 9 (Haryana) should decrease the scale of operation from 3839.4 MW to 2615.21 MW for installed capacity, from 177461 km 120877.75 km for the circuit length, from 3.917 millions to 2.99 millions in number of consumers and from 12915.72 GWh to 9875.06 GWh in quantity of energy supplied. Because of the reduction in all the parameters, the derived A T & C losses also became reduced from 59.7% to 40.66 %.

4.6 SUMMARY

In this chapter the current scale of operation has been discussed in the first part and the results have been tabulated. From the SE analysis, it is observed that, most of the DMUs are not operating on the optimal scale. The scale of operation should either increase or decrease. The amount by which the individual has to increase/decrease is calculated by MPSS. The results of MPSS are compared with $\Sigma\lambda$ values for their validity.