CHAPTER 3

DESIGN OF FAST OUTPUT SAMPLING FEEDBACK CONTROLLER FOR INTERVAL MODEL

3.1 INTRODUCTION

Numerous control techniques based on system states and outputs are applied to piezoelectric actuated structural control problems and the performance of controllers are demonstrated for the objectives like shape control, vibration control, etc... To address the uncertainties in the model, robust control techniques such as $H_{\infty}$ control by Pota et al (1993), Variable Structure Control (VSC) by Gao et al(1995), and Quantitative Feedback Theory (QFT) by Chois et al (1999) have been designed and applied to smart structure control. The state feedback controller design for an uncertain system represented in interval form can be seen in Marcia et al (2005). Variations in system parameters due to uncertainties may result in the performance deterioration. Uncertainties in modeling of structures are often considered to ensure that control system is robust with respect to response error. The vibration control problem of an uncertain system is approximated by a deterministic one. In this chapter, uncertain parameters are described by the interval model and a fast output sampling feedback controller is designed and implemented using interval arithmetic.

This chapter is organized as follows: Review of simultaneous fast output sampling feedback control law is presented in Section 3.2. The controller
design is presented in Section 3.3. The experimental implementation is presented in section 3.4. Conclusions are given in section 3.5.

3.2 REVIEW OF SIMULTANEOUS FAST OUTPUT SAMPLING (FOS) FEEDBACK CONTROL

To design an FOS feedback controller for an uncertain system represented in interval form, a simultaneous FOS feedback control design is adapted by considering lower and upper bound model. A brief review of simultaneous FOS feedback controller proposed by Werner et al (1995) is presented.

Consider the problem of simultaneously stabilizing the collection of systems $S = \{A_i, B_i, c_i^T\}$, defined by

$$\dot{x} = A_i x + B_i u; \quad y = c_i^T x, \quad i = 1 \ldots M. \quad (3.1)$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p$ and $A_i, B_i$ and $c_i^T$ are constant matrices of appropriate dimensions. It is assumed that each $(A_i, B_i)$ and $(A_i, C_i)$ is controllable and observable respectively. The objective is to design, if possible, a controller which performs well for each of these models. The following sampled data control law is used to achieve this objective. Choose an effective sampling time $\tau$, let $\Delta = \frac{\tau}{N}$, and consider

$$u(t) = \begin{bmatrix} L_0 & L_1 & \ldots & L_{N-1} \end{bmatrix} \begin{bmatrix} y(k\tau - \tau) \\ y(k\tau - \tau + \Delta) \\ \vdots \\ y(k\tau - \Delta) \end{bmatrix} = Ly_k \quad (3.2)$$
for $k\tau \leq t < (k+1)\tau$, where the matrix blocks $L_j$ represents output feedback gains, and the notation $L_y_k$ has been introduced for convenience. Note that $\frac{1}{\tau}$ is the rate at which the loop is closed, whereas output samples are taken at the N-times faster rate $\frac{1}{\Delta}$.

To show how a fast output sampling feedback controller (3.2) can be designed to realize a given sampled-data state feedback gain for the system (3.1), we construct a fictitious, lifted system for which equation (3.2) can be interpreted as static output feedback. Let $(\Phi_\tau, \Gamma_\tau, C)$ denote the system (3.1) sampled at rate $\frac{1}{\tau}$, i.e., $\Phi_\tau = e^{A\tau}$. $\Gamma_\tau = \int_0^\tau e^{A\sigma}d\sigma B$, and $(\Phi, \Gamma, C)$ denote the system (3.1) sampled at rate $\frac{1}{\Delta}$. Consider the discrete-time system having at time $t = k\tau$, the input $u_k = u(k\tau)$, states $x_k = x(k\tau)$ and output $y_k$. Then we have

$$x_{k+1} = \Phi_\tau x_k + \Gamma_\tau u_k \quad y_{k+1} = C_0 x_k + D_0 u_k$$

(3.3)

where

$$C_0 = \begin{bmatrix} c \\ c \Phi \\ \vdots \\ c \Phi^{N-1} \end{bmatrix}, \quad D_0 = \begin{bmatrix} 0 \\ c \Gamma \\ \vdots \\ c \sum_{j=0}^{N-2} \Phi^j \Gamma \end{bmatrix}.$$
Next assume that a state feedback gain $F$ has been designed such that $(\Phi_\tau + \Gamma_\tau F)$ has no eigenvalues at origin. For this state feedback one can define the fictitious measurement matrix

$$C(F, N) = (C_0 + D_0 F)(\Phi_\tau + \Gamma_\tau F)^{-1} \quad (3.4)$$

This satisfies the fictitious measurement equation

$$y_k = Cx_k,$$

Then the feedback law (3.2) can be interpreted as static output feedback

$$u_k = Ly_k$$

For the system (3.3) with measurement matrix $C$.

For $L$ realize the effect of $F$, it must satisfy

$$x_{k+1} = (\Phi_\tau + \Gamma_\tau F)x_k = (\Phi_\tau + \Gamma_\tau LC)x_k \quad (3.5)$$
i.e.,

$$LC = F \quad (3.6)$$

For $N \geq \nu$, generically $C$ has full column rank, so that any state feedback gain can be realized by a fast output sampling gain $L$. If the initial state is unknown, there will be an error, $\Delta u_k = u_k - Fx_k$ in constructing a control signal under state feedback. One can verify that the closed loop dynamics are governed by:

$$
\begin{bmatrix}
    x_{k+1} \\
    \Delta u_{k+1}
\end{bmatrix}
= 
\begin{bmatrix}
    (\Phi_\tau + \Gamma_\tau F) & \Gamma_\tau \\
    LD_0 - F \Gamma
\end{bmatrix}
\begin{bmatrix}
    x_k \\
    \Delta u_k
\end{bmatrix}
\tag{3.6}
$$
The system in equation (3.3) is stable, if and only if, $F$ stabilizes $(\Phi_\tau, \Gamma_\tau)$ and matrix $(LD_0 \cdot F \Gamma_\tau)$ has all its eigenvalues inside the unit circle.

### 3.3 CONTROLLER DESIGN

To design a simultaneous FOS feedback controller for an interval system given in equation (2.9), the lower and upper bound models in equation (2.10) and (2.11) are considered. The model in equation (2.10) and (2.11) are discretized at a rate $1/\tau$, where $\tau=0.01$ second. Let $(\Phi_\tau, \Gamma_\tau, C^\tau)$ and $(\Phi^\tau, \Gamma^\tau, C^\tau)$ be the discrete time system of $(A, B, C)$ and $(\overline{A}, \overline{B}, \overline{C})$.

$$
\Phi_\tau = \begin{bmatrix} 1.7288 & 0.9601 \\ -0.9006 & 0.1029 \end{bmatrix}, \quad \Gamma_\tau = \begin{bmatrix} -0.0026 \\ 0.0027 \end{bmatrix}; \quad (3.7)
$$

$$
\Phi^\tau = \begin{bmatrix} 1.8631 & 1.1019 \\ -1.0952 & -0.1143 \end{bmatrix}, \quad \Gamma^\tau = \begin{bmatrix} -0.0026 \\ 0.0027 \end{bmatrix}. \quad (3.8)
$$

By using Linear Quadratic Regulator (LQR) method, a stabilizing state feedback controller is designed for lower and upper bound systems, such that the eigenvalues of $(\Phi_\tau + \Gamma_\tau F)$ and $(\Phi^\tau + \Gamma^\tau F)$ are not at origin. The state feedback gain designed for lower bound system is $F = [-86.7624, -25.7514]$ and for upper bound system is $\overline{F} = [-69.3261, -10.2034]$.

The open loop response and response with state feedback gain for lower and upper bound system obtained in simulation is shown in Figure 3.1.
Figure 3.1 Response to Excitation at First Natural Frequency with State Feedback Gain

(a) --- Uncontrolled and — Controlled for lower bound system
(b) --- Uncontrolled and — Controlled for upper bound system

To realize the state feedback gain $F$ & $\tilde{F}$ by an single output feedback gain $L$ for the system in equation (3.7) and (3.8), we solve the
equation (3.5) subject to the following facts. The closed loop eigen values are the eigen values of the matrix in equation (3.6) and for the fast decay, the eigen values of \((\mathbf{L} \mathbf{D}_0 \cdot \mathbf{F} \Gamma_\tau)\) should be as close to the origin as possible. The gain matrix \(\mathbf{L}\) may have elements with large magnitude. On implementation, this leads to the amplification of high frequency noise and causes the deterioration of plant performance. To determine the gain \(\mathbf{L}\) with these constraints, an approximation \(\mathbf{L} \mathbf{C} = [\mathbf{F} \bar{\mathbf{F}}]\) solved instead of solving equation (3.5), but it can have considerable effect on the two problems described above.

The LMI approach proposed in Werner (1995), is used to solve the equation \(\mathbf{L} \mathbf{C} = \bar{\mathbf{F}}\) by constructing \(\mathbf{C}\) and \(\bar{\mathbf{F}}\) matrices as explained in section 3.2 with \(\mathbf{N} = 4\). Fictitious measurement matrix for lower and upper bound systems are defined from equation (3.4) as

\[
\begin{align*}
\mathbf{C} (\bar{\mathbf{F}} , \mathbf{N}) &= (\mathbf{C}_0 + \mathbf{D}_0 \bar{\mathbf{F}} ) (\Phi_\tau + \Gamma_\tau \bar{\mathbf{F}})^{-1} & (3.9) \\
\widetilde{\mathbf{C}} (\bar{\mathbf{F}} , \mathbf{N}) &= (\widetilde{\mathbf{C}}_0, + \mathbf{D}_0, \bar{\mathbf{F}}_0) (\Phi_\tau + \Gamma_\tau \bar{\mathbf{F}})^{-1} & (3.10)
\end{align*}
\]

where

\[
\mathbf{C}_0 = \begin{bmatrix}
\mathbf{C}_0^T \\
\mathbf{C}_1^T \Phi_1 \\
\mathbf{C}_2^T \Phi_2 \\
\mathbf{C}_3^T \Phi_3
\end{bmatrix} = \begin{bmatrix}
1.000 & 0 \\
1.7288 & 0.9061 \\
2.1728 & 1.6598 \\
2.2615 & 2.1396
\end{bmatrix},
\]

\[
\mathbf{D}_0 = \begin{bmatrix}
0 \\
\mathbf{C}_1^T \Gamma_1 \\
\mathbf{C}_2^T \Gamma_2 \\
\mathbf{C}_3^T \Gamma_3
\end{bmatrix} = \begin{bmatrix}
0 \\
-0.0026 \\
-0.0048 \\
-0.0061
\end{bmatrix},
\]
\[
\bar{C}_0 = \begin{bmatrix}
\bar{C}_0^T \\
\bar{C}_1^T \Phi_1 \\
\bar{C}_2^T \Phi_2 \\
\bar{C}_3^T \Phi_3
\end{bmatrix} = \begin{bmatrix}
1.000 & 0 \\
1.8631 & 1.1019 \\
2.2644 & 1.9271 \\
2.1082 & 2.2749
\end{bmatrix},
\]

\[
\bar{D}_0 = \begin{bmatrix}
0 \\
\bar{C}_1^T \Gamma_1 \\
\bar{C}_2^T \Gamma_2 \\
\bar{C}_3^T \Gamma_3
\end{bmatrix} = \begin{bmatrix}
0 \\
-0.0026 \\
-0.0045 \\
-0.0052
\end{bmatrix},
\]

\[
\bar{C} = \begin{bmatrix}
-4.7255 & -5.1016 & -3.4403 & -3.2182 \\
-2.6237 & -3.6224 & -2.1877 & -2.9175 \\
-0.8662 & -1.9532 & -0.6629 & -1.6496
\end{bmatrix};
\]

\[
\tilde{C} = [C \quad \bar{C}]
\]

\[
\tilde{F} = [F \quad \bar{F}]
\]

\[
\tilde{F} = [-86.7624, -25.7514 \quad -69.3261, -10.2034]
\]

The LMI approach proposed in Herbert Werner, (1998) is used to design \( L \) such that, the spectral norms of \( L \) and \( (LD_0 - F \Gamma) \), as well as, the distance between \( LC_1 \) and \([F \quad F] \) can be controlled. Let \( \rho_1, \rho_2, \rho_3 \) represent upper bounds on the spectral norms of \( L, (LD_0 - F \Gamma) \) and \( (LC - F) \) respectively, and consider the following system of LMIs:

\[
\begin{bmatrix}
-\rho_1^2 I \\
L^T
\end{bmatrix} < 0
\]
Here, the three objectives have been expressed by upper bounds on matrix norms and each should be as small as possible. The value $\rho_1$ small means low noise sensitivity, $\rho_2$ small means fast decay of estimation error and $\rho_3$ small means that fast output sampling controller with gain $L$ is a good approximation of the state feedback gain. The LMI in equation (3.11) are solved with $\rho_1 = 100, \rho_2 = 0.2000, \rho_3 = 2$ by minimizing the linear objective under LMI constraints using the solver mincx() in the LMI control tool box in MATLAB. The fast output feedback gain obtained as explained is

$$L = [35.9275, 11.0757, -18.4293, -46.6727]$$

The open loop response, response with fast output feedback gain and the control signal for Nominal, Lower and upper bound systems are shown in Figures 3.2, 3.3 and 3.4.
Figure 3.2  Response to Excitation at First Natural Frequency with Simultaneous FOS for Nominal System
(a) ---Uncontrolled and — Controlled
(b) Control Signal
Figure 3.3  Response to Excitation at First Natural Frequency with Simultaneous FOS for Lower Bound System

(a) ---Uncontrolled and ---- Controlled

(b)  Control Signal
Figure 3.4  Response to Excitation at First Natural Frequency with Simultaneous FOS for Upper Bound System

(a) Uncontrolled and Controlled

(b) Control Signal
3.4 EXPERIMENTAL EVALUATION

The photograph of the experimental set up developed to measure and control the vibration of cantilever beam is shown in Figure 3.5. The uncertainties are introduced by increasing and decreasing the length of the beam in order to get the lower and upper bound model apart from a nominal plant model. The simultaneous fast output sampling feedback controller designed in section 3.3 is used to suppress the vibration of Nominal system, Lower bound system and Upper bound system. To assess the performance of the control, a sinusoidal excitation at first natural frequency of beam with amplitude of 10V (p-p) is applied. As required by fast output sampling feedback control, the sensor output is sampled at 0.0025 sec ($\tau$) through ADC port of dSPACE and MATLAB/simulink. The control signal is updated and applied to the control actuator at the sampling interval of 0.01 sec ($\Delta=\tau/N$) through DAC port of dSPACE system. The controller is implemented by developing a real time simulink model using MATLAB RTW in simulink. The open loop response, closed loop response and control signal acquired from dSPACE control desk are shown in Figures 3.6, 3.7 and 3.8 respectively. The controlled and uncontrolled frequency responses acquired using digital storage oscilloscope (Agilent 54621A) is shown in Figure 3.9.

![Figure 3.5 Photograph of Experimental Set-Up](image-url)
Figure 3.6  Response to Excitation at First Natural Frequency for the Nominal System (Experimental)

(a) --- Uncontrolled and —— Controlled

(b) Control Signal
Figure 3.7  Response to Excitation at First Natural Frequency with Increased in Beam Length (Experimental)

(a) --- Uncontrolled and — Controlled

(b)  Control Signal
Figure 3.8  Response to Excitation at First Natural Frequency with Decreased in Beam Length (Experimental)

(a) ---Uncontrolled and —— Controlled

(b) Control Signal
Figure 3.9 Frequency Response ---- Uncontrolled; ____ Controlled

(a) Nominal beam
(b) Lower bound system (c) Upper bound system
3.5 CONCLUSION

The fast output sampling feedback controller is designed for piezo actuated cantilever beam by representing the uncertainties in interval. The model represented in interval form has lower and upper bound model along with nominal model. Hence, the controller is designed using simultaneous fast output sampling feedback control approach. The simulation and experimental results show that the controller suppresses the amplitude of vibration of all three plants. To show the vibration suppression in frequency domain, the system is excited with the frequency ranging from 0 – 15Hz. The frequency response acquired during controller implementation for nominal, lower and upper bound systems are shown in figure 3.9, and the average reduction in amplitude of vibration is found to be 27.5dB, which is 90% in comparison to open loop response.