CHAPTER 4

AN $M^X/G/1$ RETRIAL QUEUE WITH TWO PHASE SERVICE SUBJECT TO ACTIVE SERVER BREAKDOWNS AND TWO TYPES OF REPAIR

4.1 INTRODUCTION

This paper analyses a single server batch arrival retrial queue with active breakdowns, two types of repair and second optional service. Most of the classical queueing systems assume a reliable machine or server; however, in practice we often meet cases where the servers may fail and can be repaired. Such queueing systems with repairable service station have been studied by many authors (Avi–Itzhak and Naor 1963, Li et al 2005, Tang 1997, Yue and Cao 1997). Recently, there has been a fast development in the literature on retrial queues, there are only a few works taking into account both the retrial phenomenon and unreliability of the server. For related literature, readers can find the main results and methods about unreliable retrial queues in Aissani (1993, 1994), Aissani and Artalejo (1998), Atencia et al (2006, 2008), Wang et al (2001) and Yang and Li (1994).

The breakdowns may be active or passive according to whether the failures occur in a working or idle period of the server. Besides, failures can take place after a random amount of service time or just before starting the service. Recently, Atencia et al (2006) have analyzed $M/G/1$ retrial queue with active breakdowns and exponential server lifetime but not considering second optional service and bulk arrival. In practice, it is realistic that the
repair time of server during the presence of patient customer and the repair
time of the server while the customer (impatient customer) joining the orbit
due to failure, are different. Thus this chapter concerns about studying the
unreliability of the server and the impatience problems under two different
types of repair.

A possible application of bulk arrival retrial queueing system with
two phases of heterogeneous service under active server breakdowns and two
types of repair is given as follows:

The queueing system under consideration has an intrinsic interest to
model some situations in packet switching network. Consider a computer
network which consists of a group of processors connected with a central
transmission unit (CTU). If a processor wishes to send a message it first
sends the message to the CTU. If the transmission medium is available, the
CTU sends immediately message; otherwise, the message will be stored in a
buffer and the messages in CTU must retry for the transmission some time
later. It is noted that most papers on retrial queues assume that the server is
available on a permanent basis. In practice, however, these assumptions are
apparently unrealistic. CTU may well be subject to lengthy and unpredictable
breakdowns like scheduled backups and unpredictable failures, while
transmitting the message. If CTU is subject to unpredictable breakdowns
(not so lengthy) while transmitting the message, CTU gives the priority to
transmit that message after being repaired. In the case of lengthy
breakdowns, the messages in buffer must retry for the transmission some time
later after being repaired. The above situation can be modeled as a batch
arrival single server retrial queueing system with active breakdowns, two
types of repair time and second optional service.

The primary focus of the proposed model is to realize an extensive
analysis of this system from both the queueing and reliability points of view.
Analytical treatment of this model is obtained by supplementary variable technique. The steady state orbit size and system size distribution are found. Also, other performance measures are obtained. Reliability measures of this model are also discussed. Numerical illustrations are also provided.

4.2 MATHEMATICAL MODEL

If the server is busy at the arrival epoch, then all calls join the orbit, whereas if the server is free, then one of the arriving calls begins its service and the others form sources of repeated calls in order to seek service again after a random amount of time. The time between two successive repeated attempts of each call in orbit is assumed to be exponentially distributed with rate \( \nu \). The server provides preliminary first essential service (FES) and may break down while serving customers. When the server fails it is sent to repair immediately. It supposed that the server lifetime follows an exponential law with rate \( \gamma > 0 \), i.e., the server fails after an exponential time with mean \( \gamma^{-1} \). The customer who was being served during the server failure chooses, with probability \( q \), to enter the orbit (impatient customer) and, with complementary probability \( p \), to remain in the server for repair in order to conclude his remaining service (patient customer). Both service and repair times are assumed to have a general distribution. It is natural to consider that the repair time of server during the presence of patient customer and the repair time of the server while the customer (impatient customer) joining the orbit due to failure, are different. Therefore, a persistent customer will receive the service first and in this case, the customer has a certain priority in the service. On successful completion of FES, the customer may opt for second optional service (SOS) with probability \( \alpha \).

Let \( S_i(x) \) \( (s_i(x)) \) \( \left[ \mathcal{S}_i(\theta) \right][S_i^0(x)] \) be the cumulative distribution function (probability density function) \{Laplace–Stieltjes transform(LST)\)
[remaining service time] of FES. \( R_1(y) (\tilde{r}_1(y))\overline{R}_1(s) [R_1^0(y)] \) be cumulative distribution function (probability density function) \{Laplace-Stieltjes transform\} [remaining repair time] of repair time of the server during the presence of patient customer. \( R_2(y) (\tilde{r}_2(y))\overline{R}_2(s) [R_2^0(y)] \) be cumulative distribution function (probability density function) \{Laplace-Stieltjes transform\} [remaining repair time] of repair time of the server while the customer (impatient customer) joining the orbit due to failure. \( S_2(x) (\tilde{s}_2(x))\overline{S}_2(s) [S_2^0(x)] \) be the cumulative distribution function (probability density function) \{Laplace – Stieltjes transform\} [remaining service time] of SOS. \( N(t) \) denotes the number of customers in the orbit at time \( t \).

The server state is denoted as,

\[
C(t) = \begin{cases} 
0 & \text{if the server is idle} \\
1 & \text{if the server is busy with FES} \\
2 & \text{if the server is down with a customer with the server} \\
3 & \text{if the server is down without a customer in the server} \\
4 & \text{if the server is busy with SOS}
\end{cases}
\]

The state of the system at time \( t \) can be described by the Markov process

\[
K(t) = \left\{ C(t), N(t), S_1^0(t), R_1^0(t), R_2^0(t), S_2^0(t) \right\}; t \geq 0.
\]

Now the system state probabilities are defined as follows

\[
P_{0,n}(t) = P\{C(t) = 0; N(t) = n\} \quad n \geq 0
\]

\[
P_{1,n}(x,t)dx = P\{C(t) = 1; N(t) = n; x < S_1^0(t) \leq x + dx\} \quad n \geq 0, x \geq 0
\]

\[
P_{2,n}(x,y,t)dy = P\{C(t) = 2; N(t) = n; x = S_1^0(t); y < R_1^0(t) \leq y + dy\} \quad n \geq 0; x, y \geq 0
\]

\[
P_{3,n}(y,t)dy = P\{C(t) = 3; N(t) = n; y < R_2^0(t) \leq y + dy\} \quad n \geq 1, y \geq 0
\]

\[
P_{4,n}(x,t)dx = P\{C(t) = 4; N(t) = n; x < S_2^0(t) \leq x + dx\} \quad n \geq 0, x \geq 0
\]
4.3 SYSTEM SIZE DISTRIBUTION

The following equations are obtained for the queueing system, using supplementary variable technique.

\[ P_{0,j}(t + \Delta t) = P_{0,j}(t)(1 - \lambda \Delta t - jv \Delta t) + P_{4,j}(0,t) \Delta t + (1 - \alpha)P_{1,j}(0,t) \Delta t \\
+ (1 - \delta_{0,j})P_{3,j}(0,t) \Delta t \quad j \geq 0 \]

\[ P_{1,j}(x - \Delta t, y + \Delta t) = P_{1,j}(x,t)(1 - \lambda \Delta t - j\gamma \Delta t) + \lambda \Delta t \sum_{k=1}^{j+1} g_k P_{0,j-k+1}(t) s_j(x) \\
+ (j + 1) \nu P_{0,j+1}(t) s_1(x) \Delta t + \lambda \Delta t \sum_{k=1}^{j} g_k P_{1,j-k}(x,t) + P_{2,j}(x,0,t) \Delta t \quad j \geq 0 \]

\[ P_{2,j}(x, y - \Delta t, t + \Delta t) = P_{2,j}(x, y,t)(1 - \lambda \Delta t) + p \gamma P_{j,j}(x,t) r_j(y) \Delta t \\
+ \lambda \Delta t \sum_{k=1}^{j} g_k P_{2,j-k}(x, y,t) \quad j \geq 0 \]

\[ P_{3,j}(y - \Delta t, t + \Delta t) = P_{3,j}(y,t)(1 - \lambda \Delta t) + q \gamma r_j(y) \left[ \int_0^{\infty} P_{1,j-1}(x,t) dx \right] \Delta t \\
+ \lambda \Delta t \sum_{k=1}^{j} g_k P_{3,j-k}(y, t) \quad j \geq 1 \]

\[ P_{4,j}(x - \Delta t, t + \Delta t) = P_{4,j}(x,t)(1 - \lambda \Delta t) + \alpha P_{1,j}(0,t) s_2(x) \Delta t \\
+ \lambda \Delta t \sum_{k=1}^{j} g_k P_{4,j-k}(x,t) \quad j \geq 0 \]

where \( \delta_{a,b} \) denotes Kronecker’s delta.

The steady state equations of the above equations are,

\[
(\lambda + jv)P_{0,j} = P_{4,j}(0) + (1 - \alpha)P_{1,j}(0) + (1 - \delta_{0,j})P_{3,j}(0) \tag{4.1}
\]

\[
-\frac{d}{dx} P_{1,j}(x) = -(\lambda + \gamma)P_{1,j}(x) + \lambda \sum_{k=1}^{j+1} g_k P_{0,j-k+1} s_j(x) + (j + 1) \nu P_{0,j+1} s_1(x) \\
+ \sum_{k=1}^{j} g_k P_{1,j-k}(x) + P_{2,j}(x,0) \tag{4.2}
\]
\[- \frac{\partial}{\partial y} P_{2,j}(x, y) = -\lambda P_{2,j}(x, y) + \gamma \, p \, P_{1,j}(x) \, r_1(y) + \sum_{k=1}^{j} g_k P_{2,j-k}(x, y) \quad (4.3)\]

\[- \frac{d}{dy} P_{3,j}(y) = -\lambda P_{3,j}(y) + q \, \gamma \left[ \int P_{1,j-1}(x) \, dx \right] r_2(y) + \lambda \sum_{k=1}^{j} g_k P_{3,j-k}(y) \quad (4.4)\]

\[- \frac{d}{dx} P_{4,j}(x) = -\lambda P_{4,j}(x) + \alpha P_{1,j}(0) s_2(x) + \lambda \sum_{k=1}^{j} g_k P_{4,j-k}(x) \quad (4.5)\]

Assume that \( \text{LST}(P_{i,j}(x)) = \tilde{P}_{i,j}(\theta); \ i = 1, 4; \ \text{LST}(P_{2,j}(x, y)) = \tilde{P}_{2,j}(\theta, s); \ \text{LST}(P_{3,j}(y)) = \tilde{P}_{3,j}(s) \)

Taking LST on (4.2) with respect to remaining service time of FES, then the equation (4.2) becomes,

\[ \theta \, \tilde{P}_{1,j}(\theta) - P_{1,j}(0) = (\lambda + \gamma) \tilde{P}_{1,j}(\theta) - \lambda \sum_{k=1}^{j+1} g_k P_{0,j-k+1} \tilde{S}_1(\theta) \]

\[ - (j + 1) \nu P_{0,j+1} \tilde{S}_1(\theta) - \lambda \sum_{k=1}^{j} g_k \tilde{P}_{1,j-k}(\theta) - \tilde{P}_{2,j}(\theta, 0) \]

\[(4.6)\]

Taking LST on (4.3), first with respect to remaining repair time of the server and then with respect to remaining service time of FES, the equation (4.3) becomes,

\[ s \, \tilde{P}_{2,j}(\theta, s) - \tilde{P}_{2,j}(\theta, 0) = \lambda \tilde{P}_{2,j}(\theta, s) - \gamma \, p \tilde{R}_1(s) \tilde{P}_{1,j}(\theta) - \lambda \sum_{k=1}^{j} g_k \tilde{P}_{2,j-k}(\theta, s) \quad (4.7)\]

Taking LST on (4.4) with respect to remaining repair time of the server, then equation (4.4) becomes,

\[ s \frac{\tilde{P}_{3,j}(s)}{s_{\tilde{P}_{3,j}(s)}} = \lambda \tilde{P}_{3,j}(s) - q \, \gamma \left[ \int P_{1,j-1}(x) \, dx \right] \tilde{R}_2(s) - \lambda \sum_{k=1}^{j} g_k \tilde{P}_{3,j-k}(s) \quad (4.8)\]
Taking LST on (4.5) with respect to remaining service time of SOS, then equation (4.5) becomes,

$$\theta \tilde{P}_{k,j}(s) - P_{4,j}(0) = \lambda \tilde{P}_{k,j}(s) - \alpha P_{1,j}(0) \tilde{S}_2(\theta) - \lambda \sum_{k=1}^{j} g_{k} \tilde{P}_{4,k}(\theta)$$  \hspace{1cm} (4.9)$$

Now, to obtain the steady state probability generating function (PGF) of the number of customers in orbit at an arbitrary time epoch, the following probability generating functions are defined

$$P_{0}(z) = \sum_{j=0}^{\infty} P_{0,j} z^{j}$$

$$\tilde{P}_{1}(z,\theta) = \sum_{j=0}^{\infty} \tilde{P}_{1,j}(\theta) z^{j} \hspace{1cm} P_{1}(z,0) = \sum_{j=0}^{\infty} P_{1,j}(0) z^{j}$$

$$\tilde{P}_{2}(z,\theta,s) = \sum_{j=0}^{\infty} \tilde{P}_{2,j}(\theta,s) z^{j} \hspace{1cm} \tilde{P}_{2}(z,\theta,0) = \sum_{j=0}^{\infty} \tilde{P}_{2,j}(\theta,0) z^{j}$$

$$\tilde{P}_{3}(z,s) = \sum_{j=1}^{\infty} \tilde{P}_{3,j}(s) z^{j} \hspace{1cm} P_{3}(z,0) = \sum_{j=1}^{\infty} P_{3,j}(0) z^{j}$$

$$\tilde{P}_{4}(z,\theta) = \sum_{j=0}^{\infty} \tilde{P}_{4,j}(\theta) z^{j} \hspace{1cm} P_{4}(z,0) = \sum_{j=0}^{\infty} P_{4,j}(0) z^{j}$$  \hspace{1cm} (4.10)$$

Using Equations (4.1), (4.6) – (4.9) and PGF (4.10), the following are derived.

$$\lambda P_{0}(z) + vz P_{0}^{'}(z) = (1-\alpha) P_{1}(z,0) + P_{3}(z,0) + P_{4}(z,0)$$  \hspace{1cm} (4.11)$$

$$(\theta - (\lambda + \gamma) + \lambda X(z)) \tilde{P}_{1}(z,\theta) = P_{1}(z,0) - \lambda \frac{X(z)}{z} P_{0}(z) \tilde{S}_{1}(\theta) - \nu P_{0}^{'}(z) \tilde{S}_{1}(\theta) - \tilde{P}_{2}(z,\theta,0)$$  \hspace{1cm} (4.12)$$

$$(s - \lambda + \lambda X(z)) \tilde{P}_{2}(z,\theta,s) = \tilde{P}_{2}(z,\theta,0) - p \gamma \tilde{P}(z,\theta) R_{1}(s)$$  \hspace{1cm} (4.13)$$

$$(s - \lambda + \lambda X(z)) \tilde{P}_{3}(z,s) = P_{3}(z,0) - q \gamma z \tilde{R}_{2}(s) \tilde{P}_{1}(z,0)$$  \hspace{1cm} (4.14)$$

$$(\theta - \lambda + \lambda X(z)) \tilde{P}_{4}(z,\theta) = P_{4}(z,0) - \alpha P_{1}(z,0) \tilde{S}_{2}(\theta)$$  \hspace{1cm} (4.15)$$
Substituting \( s = \lambda - \lambda X(z) \) in (4.13) and (4.14) and \( \theta = \lambda - \lambda X(z) \) in (4.15), the equations become,

\[
\begin{align*}
\tilde{P}_z(z, \theta, 0) &= p \gamma \tilde{P}_z(z, \theta) \tilde{R}_1(\lambda - \lambda X(z)) \\

P_z(z, 0) &= q \gamma z \tilde{P}_z(z, 0) \tilde{R}_2(\lambda - \lambda X(z)) \\

P_z(z, 0) &= \alpha P_z(z, 0) \tilde{S}_z(\lambda - \lambda X(z))
\end{align*}
\]  

(4.16) (4.17) (4.18)

From equation (4.16), the equation (4.12) becomes,

\[
\left[ \theta - (\lambda + \gamma) + \lambda X(z) + \gamma p \tilde{R}_1(\lambda - \lambda X(z)) \right] \tilde{P}_1(z, \theta) = P_1(z, 0) - \lambda X(z) \frac{P_0(z) \tilde{S}_1(\theta)}{z}
\]  

(4.19)

Substituting \( h(z) = \theta = \lambda + \gamma - \lambda X(z) - \gamma p \tilde{R}_1(\lambda - \lambda X(z)) \) in (4.19), it is obtained as,

\[
P_1(z, 0) = \lambda X(z) \frac{P_0(z) \tilde{S}_1(\theta)}{z}
\]  

(4.20)

and therefore,

\[
\tilde{P}_1(z, 0) = \left( \frac{1 - \tilde{S}_1(h(z))}{h(z)} \right) \left( \lambda \frac{X(z)}{z} P_0(z) + \nu P_0'(z) \right)
\]  

(4.21)

where \( h(z) \) satisfies the following properties

(i) \( h(1) = q \gamma \)

(ii) \( h'(1) = -\lambda E(X)(1 + p \gamma E[R_1]) \)

From the equations (4.11), (4.17), (4.18), (4.20) and (4.21), the following is obtained
\[ P_0(z) = P_0(z) \]
\[
\begin{aligned}
&= \left\{-\frac{\lambda}{v} \left[ h(z) - \left\{ (\alpha \tilde{S}_1(\lambda - \lambda X(z)) + (1-\alpha) h(z) \right\} \tilde{S}_1(h(z)) + \gamma \varphi z \tilde{R}_2(\lambda - \lambda X(z)) \left( 1 - \tilde{S}_1(h(z)) \right) \frac{X(z)}{z} \right] \right. \\
&\left. \quad - q \gamma z \tilde{R}_2(\lambda - \lambda X(z)) (1 - \tilde{S}_1(h(z))) - h(z) \left[ \alpha \tilde{S}_1(\lambda - \lambda X(z)) + (1-\alpha) \tilde{S}_1(h(z)) \right] \right\}
\end{aligned}
\]

On integrating the above equation, \( P_0(z) \) is obtained as,
\[
\begin{aligned}
P_0(z) &= P_0(l) \\
&= \exp \left\{ \frac{\lambda}{v} \left[ h(u) - \left( \alpha \tilde{S}_1(\lambda - \lambda X(u)) + (1-\alpha) h(u) \right) \tilde{S}_1(h(u)) + \gamma \varphi z \tilde{R}_2(\lambda - \lambda X(u)) \left( 1 - \tilde{S}_1(h(u)) \right) \frac{X(u)}{u} \right] \right\} \\
&
\end{aligned}
\]

(4.22)

From the equations (4.21) and (4.22), the marginal generating function of the orbit size when the server is busy with FES is given by,
\[
\tilde{P}_1(z,0) = \frac{\lambda(1-X(z))(1-\tilde{S}_1(h(z)))P_0(z)}{q \gamma z \tilde{R}_2(\lambda - \lambda X(z))(1-\tilde{S}_1(h(z))) - h(z)[\alpha \tilde{S}_1(\lambda - \lambda X(z)) + (1-\alpha) \tilde{S}_1(h(z))]} \\
\]

(4.23)

Substituting the equations (4.21) and (4.16) in (4.13), the marginal generating function of the orbit size when the server is down with a customer waiting in the server is given by
\[
\tilde{P}_2(z,0) = \frac{P \gamma \left( \tilde{R}_1(\lambda - \lambda X(z)) - 1 \right) \left( 1 - \tilde{S}_1(h(z)) \right) \left( \frac{\lambda}{z} \frac{X(z)}{P_0(z)} + v P_0'(z) \right)}{\left( -\lambda + \lambda X(z) \right)}
\]

and from the equation (4.22), we obtained,
\[
\tilde{P}_2(z,0) = \frac{P \gamma (1-\tilde{R}_1(\lambda - \lambda X(z))(1-\tilde{S}_1(h(z)))) P_0(z)}{q \gamma z \tilde{R}_2(\lambda - \lambda X(z))(1-\tilde{S}_1(h(z))) - h(z)[\alpha \tilde{S}_1(\lambda - \lambda X(z)) + (1-\alpha) \tilde{S}_1(h(z))]} \\
\]

(4.24)
Substituting the equations (4.21) and (4.17) in (4.14), the marginal generating function of the orbit size when the server is down without a customer waiting in the server is given by

$$
\tilde{P}_3(z,0) = \frac{q \gamma \left( \tilde{R}_2(\lambda - \lambda X(z)) - 1 \right) \left( 1 - \tilde{S}_1(h(z)) \right) \left( \frac{\lambda}{z} X(z) P_0(z) + vP_0'(z) \right)}{(-\lambda + \lambda X(z))}
$$

Substituting the equation (4.22), we obtain,

$$
\tilde{P}_3(z,0) = \frac{q \gamma (1 - \tilde{R}_2(\lambda - \lambda X(z)))\tilde{S}_1(h(z))P_0(z)}{q \gamma \tilde{R}_2(\lambda - \lambda X(z))(1 - \tilde{S}_1(h(z))) - h(z)(z) - [\alpha \tilde{S}_2(\lambda - \lambda X(z)) + (1 - \alpha)]\tilde{S}_1(h(z))}
$$

(4.25)

Substituting the equations (4.20) and (4.18) in (4.15), The marginal generating function of the orbit size when the server is busy with SOS is given by

$$
\tilde{P}_4(z,0) = \frac{\alpha \left( \tilde{S}_2(\lambda - \lambda X(z)) - 1 \right)\tilde{S}_1(h(z)) \left( \frac{\lambda}{z} X(z) P_0(z) + vP_0'(z) \right)}{(-\lambda + \lambda X(z))}
$$

Substituting the equation (4.22), we obtain

$$
\tilde{P}_4(z,0) = \frac{h(z)\alpha(1 - \tilde{S}_2(\lambda - \lambda X(z)))\tilde{S}_1(h(z))P_0(z)}{q \gamma \tilde{R}_2(\lambda - \lambda X(z))(1 - \tilde{S}_1(h(z))) - h(z)(z) - [\alpha \tilde{S}_2(\lambda - \lambda X(z)) + (1 - \alpha)]\tilde{S}_1(h(z))}
$$

(4.26)

Using the equations (4.22) – (4.26), the probability generating function $P(z)$ of number of customers in orbit and the probability generating function $R(z)$ of the system size at an arbitrary epoch in stationary regime is expressed as follows,
\[ P(z) = P_0(z) + \tilde{P}_1(z,0) + \tilde{P}_2(z,0,0) + \tilde{P}_3(z,0) + \tilde{P}_4(z,0) \]

\[
P(z) = \frac{[\lambda(1-X(z)) + \gamma p(1 - \tilde{R}_1(\lambda - \lambda X(z))) + \gamma q \tilde{S}_1(h(z))]P_0(z)(1-z)}{\gamma \gamma \tilde{R}_2(\lambda - \lambda X(z))(1 - \tilde{S}_1(h(z))) - h(z)(z - [\alpha \tilde{S}_2(\lambda - \lambda X(z)) + (1 - \alpha)]\tilde{S}_1(h(z)))} \tag{4.27} \]

\[ R(z) = P_0(z) + z\tilde{P}_1(z,0) + z\tilde{P}_2(z,0,0) + \tilde{P}_3(z,0) + z\tilde{P}_4(z,0) \]

\[
R(z) = \frac{h(z)(1-z)P_0(z)\tilde{S}_1(h(z))[\alpha \tilde{S}_2(\lambda - \lambda X(z)) + (1 - \alpha)]}{\gamma \gamma \tilde{R}_2(\lambda - \lambda X(z))(1 - \tilde{S}_1(h(z))) - h(z)(z - [\alpha \tilde{S}_2(\lambda - \lambda X(z)) + (1 - \alpha)]\tilde{S}_1(h(z)))} \tag{4.28} \]

### 4.3.1 Remarks

Some interesting steady state probabilities are derived.

1. the probability that the server is idle is

\[
P_0(1) = 1 - \alpha \lambda E[X]E[S_2] - \frac{\lambda E[X](1 + \gamma [pE[R_1] + qE[R_2] - (1 - \tilde{S}_1(q\gamma)])}{q\gamma \tilde{S}_1(q\gamma)} \tag{4.29} \]

2. the probability that the server is busy with FES is

\[
\tilde{P}_1(1,0) = \frac{\lambda E[X](1 - \tilde{S}_1(q\gamma))}{q\gamma \tilde{S}_1(q\gamma)} \tag{4.30} \]

3. the probability that the server is down with a customer in the server is

\[
\tilde{P}_2(1,0,0) = \frac{\lambda pE[X]E[R_1](1 - \tilde{S}_1(q\gamma))}{q\tilde{S}_1(q\gamma)} \tag{4.31} \]
the probability that the server is down without a customer in the server is

\[ \tilde{P}_1(1,0) = \frac{\lambda E[X]E[R_2](1 - \tilde{S}_1(q\gamma))}{\tilde{S}_1(q\gamma)} \]  \hspace{1cm} (4.32)

the probability that the server is busy with SOS is

\[ \tilde{P}_2(1,0) = \alpha \lambda E[X]E[S_2] \]  \hspace{1cm} (4.33)

**Theorem 4.1** If \( T_b \) and \( T_c \) be the length of busy period and busy cycle, then under the steady state conditions, we have

\[
E[T_b] = \exp \left\{ \frac{\lambda}{\nu_0} \left[ \frac{h(u) - \left[ (\alpha \tilde{S}_2(\lambda - \alpha E[X]) + (1 - \alpha))h(u)\tilde{S}_1(h(u)) + \gamma \mu R \tilde{R}_1(\lambda - \lambda X(u)(1 - \tilde{S}_1(h(u)))) \right] X(u)}{\mu} \right] du \right\} \frac{1}{\lambda} \\
= \frac{\lambda [1 - \alpha \lambda E[X]E[S_2] - \frac{E[X](1 + \gamma[pE[R_1] + qE[R_2])]}{\tilde{S}_1(q\gamma)}]}{\mu} \]  \hspace{1cm} (4.34)

and

\[
E[T_c] = \exp \left\{ \frac{\lambda}{\nu_0} \left[ \frac{h(u) - \left[ (\alpha \tilde{S}_2(\lambda - \alpha E[X]) + (1 - \alpha))h(u)\tilde{S}_1(h(u)) + \gamma \mu R \tilde{R}_1(\lambda - \lambda X(u)(1 - \tilde{S}_1(h(u)))) \right] X(u)}{\mu} \right] du \right\} \frac{1}{\lambda} \\
= \frac{\lambda [1 - \alpha \lambda E[X]E[S_2] - \frac{E[X](1 + \gamma[pE[R_1] + qE[R_2])]}{\tilde{S}_1(q\gamma)}]}{\mu} \]  \hspace{1cm} (4.35)

**Proof**

By applying the argument of alternating renewal process, it is known that,
\[
E[T_b] = \frac{1}{\lambda} \left( \frac{1}{p_0} - 1 \right) \quad \text{and} \quad E[T_c] = \frac{1}{\lambda} \left( \frac{1}{p_0} \right)
\]

From the equation (4.22) substituting \( z = 0 \),

\[
p_b = \left( 1 - \alpha \lambda E[X] E[S_2] \right) - \frac{\lambda E[X] (1 + \gamma (pE[R_1] + qE[R_2])) (1 - \tilde{S}_i(q\gamma))}{q\gamma \tilde{S}_i(q\gamma)}
\]

\[
\exp \left[ \frac{\lambda}{v_z} \left[ h(u) - [\alpha \tilde{S}_i(\lambda - \lambda X(u)) + (1 - \alpha)]h(u)\tilde{S}_i(h(u)) + \gamma a q \tilde{R}_z(\lambda - \lambda X(u))(1 - \tilde{S}_i(h(u))) \right] X(u) \right] du
\]

4.4 STOCHASTIC DECOMPOSITION

In this case, to emphasize the dependence of characteristics of the M^Y/G/1 retrial queue with two phase service under active breakdowns and two types of service, denoting the variables \( C = C(t) \) and \( N = N(t) \) as \( C_v \) and \( N_v \) respectively. It is denoted that \( C_\infty \) and \( N_\infty \) are the corresponding variables for the standard M^Y/G/1 queue with service under active breakdowns and two types of service in the steady state. (i.e., when \( \nu \to \infty \) or high rate of retrials, our model behaves as a standard queueing system with batch arrivals and server breakdowns which agrees the intuitive expectations.). The state of the standard M^Y/G/1 queue with service under active breakdowns and two types of service in the steady state \( K_\infty \) which represents the total number of customers in the system rather than by the vector \( (C_\alpha, N_\alpha) \).

This section investigates the stochastic decomposition law. First, we observe the following relationship between generating functions

\[
L_{\nu \to \infty} R(z) = E^z K_\infty
\]

\[
= \left\{ 1 - \alpha \lambda E[X] E[S_2] \right\} - \frac{\lambda E[X] (1 + \gamma (pE[R_1] + qE[R_2])) (1 - \tilde{S}_i(q\gamma))}{q\gamma \tilde{S}_i(q\gamma)} \left[ h(z)(1 - z)\tilde{S}_i(h(z))[\alpha \tilde{S}_i(\lambda - \lambda X(z)) + (1 - \alpha)] \right]
\]

\[
q\gamma \tilde{R}_z(\lambda - \lambda X(z)(1 - \tilde{S}_i(h(z)) - h(z)) \left[ z - \left[ \alpha \tilde{S}_i(\lambda - \lambda X(z)) + (1 - \alpha) \right] \tilde{S}_i(h(z)) \right]
\]
$Ez^K$ is the generating function of the stationary distribution of the number of customers in the $M^{[X]} / G / 1 / \infty$ queueing system with active server breakdowns and two types of repair time and second optional service.

Now introducing a random variable $R_i$ representing the number of customers in orbit given that the server is idle, with the generating function

$$Ez^K = \exp \left( \frac{\lambda}{v} \left[ \frac{h(u) - [\alpha \tilde{S}_2(\lambda - \lambda X(u)) + (1 - \alpha)h(u)\tilde{S}_1(h(u)) + \gamma w_q \tilde{R}_i(\lambda - \lambda X(u))]}{q\gamma u \tilde{R}_i(\lambda - \lambda X(u)) - h(u)(\alpha\tilde{S}_2(\lambda - \lambda X(u)) + (1 - \alpha)\tilde{S}_1(h(u)))} \right] u \right)$$

Thus, the distribution of the random variable $R_i$ is the conditional distribution of the number of sources of repeated calls given that the server is free. It is observed that the vector $(C_i, N_i)$ can be represented as a sum of two independent random vectors as follows: $(C_i, N_i) = (C_{\infty}, N_{\infty}) + (0, R_i)$

In particular, the number of customers in the queueing system of the proposed $M^{X} / G / 1 / \infty$ retrial queueing system with active server breakdowns and two types of repair time and second optional service. $K_i$, can be represented as the sum of two independent random variables as follows, $K_i = K_{\infty} + R_i$. Therefore, the probability generating function of the number of customers in the system $K_i$ can be expressed as,
This is to be noted that the generating function of the system size distribution can be written as
\[ R(z) = \frac{1 - \alpha \lambda E[X] E[S_z]}{\lambda E[X](1 + \gamma [pE[R_1] + qE[R_2]])(1 - \tilde{S}_1(q\gamma)) h(z)(1 - z)\tilde{S}_1(h(z))[\alpha \tilde{S}_2(\lambda - \lambda X(z)) + (1 - \alpha)]}{q\gamma \tilde{S}_1(q\gamma)} \]
where the fraction corresponds to the probability generating function to the system size given that the server is idle. Indeed the above equality provides the stochastic decomposition property for our queueing system in an immediate way. i.e., the number of customers in our system is the sum of two independent random variables: one is the number of customers in the corresponding standard system with batch arrivals and server breakdowns and second optional service, and the other is the number of repeated customers given that the server is idle.

### 4.5 RELIABILITY ANALYSIS

This section discusses some reliability indexes of the queueing system under study, specifically the analysis of availability of the server, the failure frequency of the server.

The server is available when it is either idle or working on a customer. The following results concern the availability of the server.
(1) From the equations (4.21), (4.22) and (4.26), the marginal generating function of the orbit size when the server is available is given by

\[
P_0(z) + \tilde{P}_1(z,0) + \tilde{P}_2(z,0) = \frac{\{(1-S_0(h(z)))[\lambda(1-X(z)) + q\gamma z R_2(\lambda - \lambda X(z)) - h(z)]\} P_0(z)}{q\gamma z R_2(\lambda - \lambda X(z))(1-S_0(h(z)))-h(z)(z-[\alpha S_2(\lambda - \lambda X(z))+(1-\alpha)] S_0(h(z)))}
\]

(4.36)

(2) Substituting \(z=1\) in the equation (4.36), we get the probability that the server is available

\[
P_0(1) + \tilde{P}_1(1,0) + \tilde{P}_2(1,0) = 1 - \frac{\lambda E[X](pE[R_1] + qE[R_2])(1-S_1(q\gamma))}{qS_1(q\gamma)}
\]

(4.37)

(3) From the equation (4.21), the failure frequency of the server is obtained as,

\[
W_f = \sum_{j=0}^{\infty} \int_0^\infty \gamma P_{1,j}(x) dx = \lim_{z \to 1} \frac{\lambda [1 - S_1(q\gamma)]}{qS_1(q\gamma)}
\]

(4.38)

4.6 PERFORMANCE MEASURES

Some useful performance measures of proposed model are listed below:

(a) The mean number of customers in the orbit ‘\(L_Q\)’ is derived using the Equation (4.27)

\[
L_Q = \lim_{z \to 1} \frac{d}{dz} P(z),
\]
Using L’Hospital rule, we have,

\[
L_q = \frac{M_2 M_1 - M_3 M_2}{2M_1^2} \quad (4.39)
\]

where

\[
M_1 = q \beta S_{\beta}(q \beta) \left[ \alpha S_{\gamma} - 1 \right] + \left( 1 - S_{\gamma}(q \beta) \right) \left( \gamma (p R_{r1} + q R_{r2}) + \lambda E(X) \right)
\]

\[
M_2 = q \beta \left[ (R_{r2} + 2R_{r2})(1 - S_{\gamma}(q \beta)) + h'(1) S_{\alpha1}(1 + R_{r2}) - 2h'(1) \alpha S_{\alpha1} S_{\alpha2} + S_{\gamma}(q \beta) S_{\alpha2} \alpha \right]
\]

\[
= -h''(1) \left( 1 - S_{\gamma}(q \beta) \right) - 2h'(1) [1 + h'(1) S_{\alpha1} - S_{\gamma}(q \beta) S_{\alpha2} \alpha]
\]

\[
M_3 = - \beta (1) q \beta S_{\beta}(q \beta)
\]

\[
M_4 = -2 \beta (1) q \beta S_{\beta}(q \beta) + \beta (1) \left( \lambda E(X) + \gamma p R_{r1} + \gamma q h'(1) S_{\alpha1} \right)
\]

\[
R_{\alpha1} = \lambda E(X) E(R_{r1}); \quad R_{\alpha2} = \lambda E(X) E(R_{r2}); \quad S_{\alpha1} = \lambda E(X) E(S_{r1})
\]

\[
R_{\alpha2} = \lambda E(X^2) E(R_{r2}) + \lambda^2 (E(X))^2 E(R_{r2})^2; \quad R_{\alpha2} = \lambda E(X^2) E(R_{r2}) + \lambda^2 (E(X))^2 E(R_{r2})^2
\]

\[
S_{\alpha2} = \lambda E(X^2) E(S_{r2}) + \lambda^2 (E(X))^2 E(S_{r2})^2
\]

\[
S_{11} = \int_0^\infty e^{-\omega t} s_{\alpha1}(t) dt; \quad S_{\beta}(q \beta) = \int_0^\infty e^{-\omega t} S_{\beta}(t) dt
\]

\[
= \frac{P_0'(1) \left[ h'(1) - \left( \frac{\alpha S_{\gamma} + h'(1) S_{\gamma} + h'(1) S_{\gamma} + q R_{r2} (1 - S_{\gamma}(q \beta))}{1 - q E(X)} \right) \right]}{M_1}
\]

\[(b) \quad \text{Mean waiting time in retrial queue}
\]

Using Little’s formula, the mean waiting time in the retrial queue \((W_q)\) is obtained as,

\[
W_q = E(W) = \frac{L_q}{\lambda E(X)} \quad (4.40)
\]
The mean number of customers in the system

\[
L_s = L_0 + \alpha \lambda E[X] E[S_2] + \frac{\lambda E[X](1 + \gamma[pE[R_1] + qE[R_2])}{q\gamma S_1(q\gamma)}
\]

Mean waiting time in the system

\[
W_s = \frac{L_s}{\lambda E(X)}
\]

4.7 PARTICULAR CASE

In this section, a particular case is discussed as follows:

If there is no optional second phase of service and single type of repair time and single arrival, then \(\bar{S}_2(\lambda - \lambda X(z)) = 1\), \(\bar{R}_s(\lambda - \lambda X(z)) = 1\) and \(X(z) = 1\). Substituting this in equation (4.27) is reduced into the PGF of distribution of number of customers in the orbit as,

\[
P(z) = \left[ \frac{\lambda(1-z) + p(1-\bar{R}_s(\lambda - \lambda z) + \gamma q S_1(h(z))}{q\gamma \bar{R}_s(\lambda - \lambda X(z)(1-\bar{S}_1(h(z))) - h(z)[z-\bar{S}_1(h(z)])} \right] P_0(z)(1-z)
\]

Equation (4.43) agrees the PGF of the distribution of number of customers in the orbit of M/G/1 retrial queue with active breakdowns and Bernoulli schedule in the server obtained by Atencia et al (2006).

4.8 SPECIAL CASES

In this section, some interesting special cases are discussed.

Case I: Single server batch arrival retrial queue with Exponential FES time subject to breakdowns, two types of repair and second optional service
If the service time is exponential with probability density function \( s_i(t) = \mu e^{-\mu t} \), where \( \mu \) is the parameter and substituting in (4.27), then the PGF of the orbit size is as follows when \( \tilde{S}_i(h(z)) = \frac{\mu}{\mu + h(z)} \),

\[
P(z) = \frac{\tilde{\lambda}(1 - X(z) + \gamma p(1 - \widetilde{R}_1(\lambda - \lambda X(z)) + \gamma q\frac{\mu}{\mu + h(z)})}{q\gamma\tilde{R}_2(\lambda - \lambda X(z))\left(1 - \frac{\mu}{\mu + h(z)}\right) - h(z)} \left(z - \left[\alpha \tilde{S}_x(\lambda - \lambda X(z)) + (1 - \alpha)\frac{\mu}{\mu + h(z)}\right]\right) P_0(z)(1 - z)
\]

(4.44)

where

\[
P_0(z) = P_i(1)
\]

\[
\exp \left\{ \frac{1}{\gamma} \int_0^z \left[ \left( \alpha \tilde{S}_x(\lambda - \lambda X(u)) + (1 - \alpha)\frac{\mu}{\mu + h(u)} \right) X(u) \right]_u \right\}
\]

Case II: Single server batch arrival retrial queue with two types of Erlangian repair time and second optional service.

If two repair times are assumed as k-Erlang with probability density function, \( r_i(x) = \left( \frac{k u_i}{(k - 1)!} \right)^{x-1} e^{-u_i x} \); \( i = 1, 2 \), \( k > 0 \); where \( u_i \) is the parameter, then the equation (4.27), gives the PGF of the retrial queue size distribution as follows when \( \widetilde{R}_i(\lambda - \lambda X(z)) = \left( \frac{u_i k}{u_i k + \lambda (1 - X(z))} \right)^k \); \( i = 1, 2 \)

\[
P(z) = \frac{\tilde{\lambda}(1 - X(z) + \gamma p\left( \frac{u_i k}{u_i k + \lambda (1 - X(z))} \right)^k + \gamma q\tilde{S}_i(h(z))}{q\gamma\left( \frac{u_i k}{u_i k + \lambda (1 - X(z))} \right)^k (1 - \tilde{S}_i(h(z)) - h(z)) \left(z - \left[\alpha \tilde{S}_x(\lambda - \lambda X(z)) + (1 - \alpha)\tilde{S}_1(h(z))\right]\right) P_0(z)(1 - z)}
\]

(4.45)
where

\[ P_{\alpha}(z) = P_{\alpha}(1) \exp \frac{1}{\sqrt{\lambda}} \left[ \frac{h(u) - \left( \alpha \tilde{T}_1(\lambda - \lambda X(u)) + (1 - \alpha) \tilde{T}_2(h(u)) \right)}{\lambda(u X(u) + (1 - \alpha) \tilde{T}_2(h(u)))} \right] \]

**Case III:** Single server batch arrival retrial queue with two types of hyper Exponential repair time and second optional service.

Considering the case of Hyper Exponential repair time random variable, the pdf of Hyper Exponential vacation time is given as follows,

\[ r_i(x) = cu_{i,1}e^{-u_{i,1}x} + (1 - c)u_{i,2}e^{-u_{i,2}x}; i = 1, 2. \]

Hence the PGF of the orbit size is given by,

\[
P(z) = \left[ \frac{\lambda(1 - X(z)) + \gamma p(1 - \frac{cu_{i,1}}{u_{i,1} + \lambda(1 - X(z))} + \frac{(1 - c)u_{i,2}}{u_{i,2} + \lambda(1 - X(z))}) + \eta q \tilde{T}_i(h(z))}{q \gamma \frac{cu_{i,1}}{u_{i,1} + \lambda(1 - X(z))} + \frac{(1 - c)u_{i,2}}{u_{i,2} + \lambda(1 - X(z))}} \right] P_{\alpha}(z)(1 - z) \]

\[
(4.46)
\]

### 4.9 Numerical Results

Consider a computer network which consists of a group of processors connected with a central transmission unit (CTU). Typically, the messages arrive at CTU following Poisson stream. If the transmission medium is available, the CTU immediately sends a message; otherwise the message will be stored in a buffer (retrial group) and the messages in CTU must retry for the transmission some time later. CTU may well be subjected to lengthy and unpredictable breakdowns like scheduled backups and unpredictable failures, while transmitting the messages. If CTU is subjected to unpredictable breakdowns (not so lengthy) while transmitting the message,
CTU gives the priority to transmit that message after being repaired (type-I repair). In the case of lengthy breakdowns, the messages in buffer must retry for the transmission some time later after being repaired (type-II repair). The above situation can be modeled as a batch arrival single server retrial queueing system with active breakdowns, two types of repair time and second optional service. It is important to study the effect of Bernoulli schedule of the server ‘p’ and the effect of failure rate $\gamma$ with mean orbit size $L_Q$ and mean waiting time of a message in orbit W.

In Table 4.1(a) - 4.1(b) and Figure 4.1(a) - 4.1(b) for repair time parameters $r_1=20$ and $r_2=10$, the mean orbit size is compared with varying values of the Bernoulli Schedule probability of the server ‘p’ and with varying failure rate $\gamma$ when First Type of Repair, Second type of Repair and Second optional Service time distribution follow Exponential and Hyper – Exponential, respectively. It is observed that

- the mean buffer size is increased if the failure rate $\gamma$ increases.
- the mean buffer size is decreased if the Bernoulli schedule probability ‘p’ is increased.

In Table 4.2(a) - 4.2(b) and Figure 4.2(a) - 4.2(b), for repair time parameters $r_1=20$ and $r_2=10$, the mean waiting time of a customer in orbit is compared with varying values of the Bernoulli Schedule probability of the server ‘p’ and with varying failure rate $\gamma$ when First Type of Repair, Second type of Repair and Second optional Service time distribution follow Exponential and Hyper – Exponential, respectively. It is observed that

- the mean waiting time of packets in buffer $W_Q$ is increased if the failure rate $\gamma$ increases.
- the buffer size $W_Q$ is decreased if the Bernoulli schedule probability ‘p’ is increased.
In Table 4.3(a) - 4.3(b) and Figure 4.3(a) - 4.3(b), for Bernoulli Schedule probability \( p=0.5 \) and second type of repair time with parameter \( r_2=10 \), the mean number of packets in buffer is compared with varying values of the first type of repair time \( r_1 \) and with varying failure rate ‘\( \gamma \)’ when First Type of Repair, Second type of Repair and Second optional Service time distribution follow Exponential and Hyper – Exponential, respectively. It is observed that

- the mean number of packets in buffer ‘\( L_Q \)’ is increased if the failure rate ‘\( \gamma \)’ increases.
- the mean number of packets in buffer ‘\( L_Q \)’ is decreased if first type of repair time \( r_1 \) is increased.

In Table 4.4(a) - 4.4(b) and Figure 4.4(a) - 4.4(b), for repair time parameters \( r_1=20 \) and \( r_2=10 \), the mean waiting time of a customer in orbit is compared with varying values of the Bernoulli SOS probability of the server ‘\( \alpha \)’ and with varying failure rate ‘\( \gamma \)’ when First Type of Repair, Second type of Repair and Second optional Service time distribution follow Exponential and Hyper – Exponential, respectively. It is observed that

- the buffer size ‘\( L_Q \)’ is increased if the failure rate ‘\( \gamma \)’ increases.
- the buffer size ‘\( L_Q \)’ is increased if the Bernoulli SOS probability ‘\( \alpha \)’ is increased.

In Table 4.5 for repair time parameters \( r_1=20 \) and \( r_2=10 \), the probability of the server availability is compared with varying values of the service rate and failure rate ‘\( \gamma \)’. It is observed that the probability of the server availability is increased as service rate increases, but it is decreased if failure rate is increased. This observation is depicted in Figure 4.5.
Table 4.1(a) Mean Orbit size $L_Q$ when First type and second type of repair time and second optional service time follow Exponential distribution

(failure rate $\gamma$, Bernoulli schedule probability 'p')

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(First type and second type of repair time and second optional service time follow Exponential distribution with repair rate of first type and second type $r_1 = 20$ and $r_2 = 10$, second optional service rate $s_2 = 7$)
Table 4.1(b) Mean Orbit size $L_Q$ when First type and second type of Repair time and second optional service time follow Hyper Exponential

(failure rate $\gamma$, Bernoulli schedule probability ‘p’)

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(First type and second type of Repair time and second optional service time follow Hyper Exponential $\alpha_0=ce^{\gamma}+e^{\gamma}(1-c)$ with repair rate of first type and second type $r_1 = 20$ and $r_2 = 10$, second optional service rate $s_2 = 7$)

18
Table 4.2(a) Mean Waiting Time $W_Q$ when First type and second type of repair time and second optional service time follow Exponential distribution

(failure rate $\gamma$, Bernoulli schedule probability ‘$p$’)

<table>
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(First type and second type of repair time and second optional service time follow Exponential distribution with repair rate of first type and second type $r_1 = 20$ and $r_2 = 10$, second optional service rate $s_2 = 7$)
Table 4.2(b) Mean Waiting Time $W_Q$ when First type and second type of repair time and second optional service time follow Hyper Exponential

(failure rate $\gamma$, Bernoulli schedule probability ‘p’)

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(First type and second type of repair time and second optional service time follow Hyper Exponential $\alpha(\theta) = c v^t + \hat{v} e^{t-\theta}$ with repair rate of first type and second type $r_1 = 20$ and $r_2 = 10$, second optional service rate $s_2 = 7$)
Table 4.3(a) Mean Orbit size $L_q$ when Second type of repair time and second optional service time follow Exponential distribution

(failure rate $\gamma$, first type of repair rate $r_1$)

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(Second type of repair time and second optional service time follow Exponential distribution with repair rate of first type and second type $r_2 = 10$ and second optional service rate $s_2 = 7$)
Table 4.3(b) Mean Orbit size $L_Q$ when Second type of repair time and second optional service time follow Hyper Exponential

(failure rate $\gamma$, first type of repair rate $r_1$)

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(Second type of repair time and second optional service time follow Hyper Exponential $d(x)=ce^{\gamma x}+r_1 e^{\gamma (1-c)}$ with repair rate of first type and second type $r_2 = 10$ and second optional service rate $s_2 = \gamma$)
Table 4.4(a) Mean Orbit size $L_Q$ when First type and second type of repair time and second optional service time follow Exponential distribution

(failure rate $\gamma$, Bernoulli SOS probability ‘$\alpha$’)

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(First type and second type of repair time and second optional service time follow **Exponential distribution** with repair rate of First type and second type $r_1 = 20$ and $r_2 = 10$, second optional service rate $s_2 = 2$.)

8
Table 4.4(b) Mean Orbit size $L_Q$ when First type and second type of repair time and second optional service time follow Hyper Exponential

(failure rate $\gamma$, Bernoulli SOS probability ' $\alpha$ ')

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(First type and second type of repair time and second optional service time follow Hyper Exponential $d(x) = e^{\gamma_1 x} + \gamma_2 e^{\gamma_2 x} (1-x)$ distribution with repair rate of first type and second type $r_1 = 20$ and $r_2 = 10$, second optional service rate $s_2 = 7$)
### Table 4.5 Probability of Server Availability

(failure rate $\gamma$, Bernoulli SOS probability ‘$\alpha$’)

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(First type and Second type of Repair time and second optional service time follow **Exponential** distribution with repair rate of First type and second type $r_1 = 20$ and $r_2 = 10$, second optional service rate $s_2 = 7$)
Figure 4.1 Mean Orbit Size Vs Failure rate when (a) First type and second type of repair time and SOS follow Exponential distribution (b) First type and second type of repair time and SOS follow Hyper Exponential
Figure 4.2  Mean Waiting time Vs Failure rate when (a) First type and second type of repair time and SOS follow Exponential distribution (b) First type and second type of repair time and SOS follow Hyper Exponential
Figure 4.3  Mean Orbit Size Vs Failure rate for various type 1 repair rates (a) SOS and type 2 repair time follow Exponential (b) SOS and type 2 repair time follow Hyper Exponential
Figure 4.4  Mean Orbit Size Vs Bernoulli SOS probability (a) First type and second type of repair time and SOS follow Exponential distribution (b) First type and second type of repair time and SOS follow Hyper Exponential
Figure 4.5 Probability of Server Availability