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I hereby declare that the thesis entitled " ***Modelling and Analysis of Perishable Inventory-Queueing Systems*** "submitted to Periyar University in partial fulfillment of the requirements for the award of Degree of **Doctor of Philosophy in Mathematics** is a record of original research work carried out by me under the guidance and supervision of **Dr. S.Padmasekaran,** Assistant Professor, Department of Mathematics, Periyar University Salem-11 and that it has not formed before the basis for the award of any Degree, Diploma, Associate-ship, Fellowship or any other similar titles in this University or any other University or Institutions of higher learning.

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S.JEHOASHAN KINGSLY

Preface

This thesis provides a comprehensive analysis of continuous review inventory systems. Mainly, this thesis is the study of different kinds of Perishable Inventory Queueing Models. More specifically, the limiting values of inventory level distribution, mean reorder rate, mean inventory level, mean waiting time, etc., are derived for the models studied. In addition, adequate emphasis is laid on cost analysis based on stationary approach and varying results are illustrated numerically.

This thesis is divided into five chapters. Chapter 1 is an introduction to some of the literature and ideas surrounding the queueing inventory systems and also it contains a preliminary consideration of the Discrete and Continuous time Markov Chains, Markov Renewal processes, Matrix Geometric Method, Measures of System Performance. Most of the researchers, it was assumed that after the completion of services, the customers may leave the system. However, in day-to-day life, it does not always happen. In some queueing situations all arriving customer require the essential service, whereas a few of them may further demand the subsidiary service provided by the same server immediately after completion of the regular service. For example, in a car showroom, a customer who enters the showroom will receive the essential service (basic service) initially. Once the service of that customer is over in the essential service area, the customer may move to the extra service centre to get the additional services. For instance, if the customer buys a car and wants to fix an audio system to the vehicle (car), the customer may approach the extra service centre to fix the audio system or else the customer may leave directly from essential service area. The features discussed above have inspiration in writing this second Chapter.

Chapter 2 considers a continuous review (s; Q) inventory system at service facility with finite homogeneous sources of demands and retrial. The lifetime of each item is assumed to be exponential. Before delivered items to the demanding customers, first we will give some basic service on the item. It is known as regular or main service. The service may get interrupted according to a Poisson process and it restarts after an exponentially distributed time. If the server is idle at the time of arrival of a customer and the inventory level is positive, then the service begins immediately. After the completion of regular service, a customer may either abandon the system forever or demand for a second service from the same server, which is multi-optional. If any arriving customer finds that the server is busy or inventory level is zero, he either enters into the orbit with probability p or balks (does not enter) with probability 1 - p. The stationary distribution of the number of customers in the system, server status and the inventory level is obtained by matrix method. The Laplace-Stieltjes transform of the waiting time of the tagged customer is derived. Various system performance measures are derived and the total expected cost rate is computed under a suitable cost structure. A numerical illustration is given.

In Chapter 3, considering a Markovian queueing-inventory system with delayed vacation, negative and impatient customers. In this chapter, we discuss two dissimilar forms of

Markov chains based on state space arrangements, for modelling the system. Some important system performance measures in the steady state are derived. Finally, some cost minimization numerical results are presented. The decision maker will provide flexible servers (well trained servers) to meet the customer needs efficiently, whenever the waiting line increases. Flexible servers who are well trained are often used to increase the efficiency of the business. The features discussed above has inspiration in writing this Chapter 4.

Chapter4 focuses on a perishable inventory model with two stations, namely, station 1 and station 2 with two dedicated servers and one flexible server, where service rates are different with respect to stations and servers. By using matrix analysis method, the joint stationary distribution of the number of customers in station 1 and station 2, the status of the servers and the inventory level under steady state is investigated. Some significant measures of the system performance are obtained. Finally, some intrinsic numerical examples are given. An important issue in the queueing-inventory system with two types of customers is the priority assignment problem. For example, in assembly manufacturing system customers with long-term supply contracts have been given high priority than the other ordinary customers. In multi-specialty hospitals patients with serious illness are given high priority than the other patients opting for routine check or else. These real life problems stimulate us to study the queueing-inventory systems with two types of customers and also we have assumed Mixed Priority service policy in the Chapter 5.

Chapter 5 considers a continuous review inventory (s; Q) finite buffer queuing model with two heterogeneous servers and mixed priority service customers. The courses of arrival time and service time of both the customers are all independent exponential processes. One of the two servers is engaged exclusively for high priority customer and the other server serves both the customers along with a mixed priority service scheme. Applying matrix method to obtain the steady state joint distributions of waiting time of mixed priority service customers and the inventory level. Further, the some important measures of system performances in the steady state are derived and finally we conclude with some numerical examples.

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