

Declaration

I hereby declare that the thesis entitled “ *Modelling and Analysis of Perishable Inventory-Queueing Systems* ”submitted to Periyar University in partial fulfillment of the requirements for the award of Degree of **Doctor of Philosophy in Mathematics** is a record of original research work carried out by me under the guidance and supervision of **Dr. S.Padmasekaran**, Assistant Professor, Department of Mathematics, Periyar University Salem-11 and that it has not formed before the basis for the award of any Degree, Diploma, Associate-ship, Fellowship or any other similar titles in this University or any other University or Institutions of higher learning.

Signature of the Candidate
(S.Jehoashan Kingsly)

Place: Salem

Date:



PERIYAR UNIVERSITY, SALEM-636011

DECLARATION BY THE CANDIDATE

I here by declare that the thesis entitled “Modelling and Analysis of Perishable Inventory Queueing Systems” submitted by me for the award of Ph.D degree in “Mathematics” is my original contribution and it is not plagiarized or copied from any other thesis/books/any other copy right materials

Signature of the Candidate

CERTIFICATE BY THE SUPERVISOR

I here by declare that the candidate **Mr.S.JEHOASHAN KINGSLY**, has carried out the Ph.D Programme under my supervision during the period December 2015to July 2019 and the thesis entitled “Modelling and Analysis of Perishable Inventory Queueing Systems” submitted by him is verified and it is not plagiarized or copied from any other thesis/books/any other copy right materials.

Signature of the Supervisor

CERTIFICATE BY THE LIBRARIAN

It is certified that the thesis the thesis entitled “Modelling and Analysis of Perishable Inventory Queueing Systems” submitted by the candidate **Mr.S.JEHOASHAN KINGSLY** under the supervision of **Dr.S.Padmasekaran**,Assistant professor,Department of Mathematics,Periyar University,Salem-11.Tamil Nadu is verified for plagiarism through the software and the thesis is with in the permissible limits of plagiarism rules and the percentage of plagiarism of the thesis is found to be **Sixteen percentage(16%)**.

Signature of the Librarian



PERIYAR UNIVERSITY, SALEM-636011

CERTIFICATE BY THE R&D COORDINATOR

It is certified that the thesis the thesis entitled “Modelling and Analysis of Perishable Inventory Queueing Systems” submitted by the candidate **Mr.S.JEHOASHAN KINGSLY** under the supervision of **Dr.S.Padmasekaran**,Assistant professor,Department of Mathematics,Periyar University,Salem-11.Tamil Nadu is verified for plagiarism through the software and the thesis is within the permissible limit.

Signature of the R&D Coordinator

Urkund Analysis Result

Analysed Document: Kingsly.pdf (D54255364)
Submitted: 7/1/2019 11:40:00 AM
Submitted By: muruganchinnaraj@gmail.com
Significance: 16 %

Sources included in the report:

<http://klavogonki.ru/vocs/87500/>
<https://www.acronymfinder.com/C%2526I.html>
<https://acronyms.thefreedictionary.com/C%2526I>
https://en.wikipedia.org/wiki/Independent_and_identically_distributed_random_variables
d6c30a0f-ab27-49d3-bbcf-9b68c79a7c0d
f0374e0c-ac0b-4eba-85be-b251c2db56d4
500f85d4-f967-4ae5-abf5-b502fe2251ad
d289d09a-1b78-4f5a-b648-f8ff41c06d93
0cb8e110-bff5-433b-934d-ee51f0652e14
3717ac7a-9629-4d15-a994-4aabdc3a03ad
45e14a17-1642-49dd-b700-549b1278ed02

Instances where selected sources appear:

115

Certificate of Genuineness of the Publication

This is to certify that Ph.D. candidate **Mr. S. JEHOASHAN KINGSLY** working under my supervision has published a research articles in **UGC/SCOPUS** listed Journal named **1. Mathematical Modelling and Geometry** Vol. 4(3), Page No. 11-30, 2016 published by Dr. S. Padmasekaran and S. Jehoashan Kingsly, **2. ORiON**, Vol. No. 33 (2), Page No. 105-135, 2017 Published by Dr. S. Padmasekaran and S. Jehoashan Kingsly, **3. International Journal of Applied and Computational Mathematics**, DOI :10.1007/s40819-019-0605-3. 2019, **Springer**. Published by Dr. S. Padmasekaran and S. Jehoashan Kingsly, **4. International Journal of Applied Engineering Research**, ISSN 0973-4562 Vol.14 (6) Page No.1212-1219, 2019, Published by Dr. S. Padmasekaran and S. Jehoashan Kingsly. The contents of the publication incorporate part of the results presented in his thesis.

Research Supervisor

Countersigned

Head of the Department

Acknowledgements

First of all, I wish to express my gratitude to the *Almighty* for having showered his blessings by giving me the strength and determination to complete this research work successfully.

I am very glad to record my deep sense of gratitude and profound thanks to my supervisor **Dr. S. Padmasekaran**, Assistant Professor, Department of Mathematics, Salem for his friendly and expert guidance, readiness and willingness to help through his continuous support over the years to complete this work.

I wish to record my sincere thanks to **Dr. C. Selvaraj**, Professor and Head, Department of Mathematics, Periyar University, Salem for his constant encouragement. I would like to express my thanks to **Dr. A. Muthusamy**, Professor, **Dr. P. Prakash**, **Dr. V. Muthulakshmi**, Associate Professors, **Dr. S. Karthikeyan**, **Dr. M. Sambath**, Assistant Professors, Department of Mathematics, Periyar University, Salem for the valuable support during the period of my research work.

I would like to extend my heartfelt gratitude to **Dr. G. Ranganath**, Principal, Adhiyamaan College of Engineering, Hosur for his encouragement and guidance to carry out my research work. I would like to convey my thanks to all my colleagues and friends in the Department of Mathematics, Adhiyamaan College of Engineering, Hosur for their valuable support.

I place my thanks to my beloved family members, my parents **Pastor. D. A. Stephen** and **Mrs. Nargis Stephen**, my wife **Mrs. Mercy Kingsly**, My Sisters **Mrs. Beryl Joshva**, **Ms. S. Christine Blossomly** and My Brother **Mr. S. Daniel Rejoiceson** for being the motivating force behind the completion of the thesis. Also, they extended their warmth and affection in all stages of my academic career. I owe my gratitude to them for the successful completion of this research.

S.JEHOASHAN KINGSLY

Preface

This thesis provides a comprehensive analysis of continuous review inventory systems. Mainly, this thesis is the study of different kinds of Perishable Inventory Queueing Models. More specifically, the limiting values of inventory level distribution, mean reorder rate, mean inventory level, mean waiting time, etc., are derived for the models studied. In addition, adequate emphasis is laid on cost analysis based on stationary approach and varying results are illustrated numerically.

This thesis is divided into five chapters. Chapter 1 is an introduction to some of the literature and ideas surrounding the queueing inventory systems and also it contains a preliminary consideration of the Discrete and Continuous time Markov Chains, Markov Renewal processes, Matrix Geometric Method, Measures of System Performance. Most of the researchers, it was assumed that after the completion of services, the customers may leave the system. However, in day-to-day life, it does not always happen. In some queueing situations all arriving customer require the essential service, whereas a few of them may further demand the subsidiary service provided by the same server immediately after completion of the regular service. For example, in a car showroom, a customer who enters the showroom will receive the essential service (basic service) initially. Once the service of that customer is over in the essential service area, the customer may move to the extra service centre to get the additional services. For instance, if the customer buys a car and wants to fix an audio system to the vehicle (car), the customer may approach the extra service centre to fix the audio system or else the customer may leave directly from essential service area. The features discussed above have inspiration in writing this second Chapter.

Chapter 2 considers a continuous review (s; Q) inventory system at service facility with finite homogeneous sources of demands and retrial. The lifetime of each item is assumed to be exponential. Before delivered items to the demanding customers, first we will give some basic service on the item. It is known as regular or main service. The service may get interrupted according to a Poisson process and it restarts after an exponentially distributed time. If the server is idle at the time of arrival of a customer and the inventory level is positive, then the service begins immediately. After the completion of regular service, a customer may either abandon the system forever or demand for a second service from the same server, which is multi-optional. If any arriving customer finds that the server is busy or inventory level is zero, he either enters into the orbit with probability p or balks (does not enter) with probability $1 - p$. The stationary distribution of the number of customers in the system, server status and the inventory level is obtained by matrix method. The Laplace-Stieltjes transform of the waiting time of the tagged customer is derived. Various system performance measures are derived and the total expected cost rate is computed under a suitable cost structure. A numerical illustration is given.

In Chapter 3, considering a Markovian queueing-inventory system with delayed vacation, negative and impatient customers. In this chapter, we discuss two dissimilar forms of

Markov chains based on state space arrangements, for modelling the system. Some important system performance measures in the steady state are derived. Finally, some cost minimization numerical results are presented. The decision maker will provide flexible servers (well trained servers) to meet the customer needs efficiently, whenever the waiting line increases. Flexible servers who are well trained are often used to increase the efficiency of the business. The features discussed above has inspiration in writing this Chapter 4.

Chapter 4 focuses on a perishable inventory model with two stations, namely, station 1 and station 2 with two dedicated servers and one flexible server, where service rates are different with respect to stations and servers. By using matrix analysis method, the joint stationary distribution of the number of customers in station 1 and station 2, the status of the servers and the inventory level under steady state is investigated. Some significant measures of the system performance are obtained. Finally, some intrinsic numerical examples are given. An important issue in the queueing-inventory system with two types of customers is the priority assignment problem. For example, in assembly manufacturing system customers with long-term supply contracts have been given high priority than the other ordinary customers. In multi-specialty hospitals patients with serious illness are given high priority than the other patients opting for routine check or else. These real life problems stimulate us to study the queueing-inventory systems with two types of customers and also we have assumed Mixed Priority service policy in the Chapter 5.

Chapter 5 considers a continuous review inventory $(s; Q)$ finite buffer queueing model with two heterogeneous servers and mixed priority service customers. The courses of arrival time and service time of both the customers are all independent exponential processes. One of the two servers is engaged exclusively for high priority customer and the other server serves both the customers along with a mixed priority service scheme. Applying matrix method to obtain the steady state joint distributions of waiting time of mixed priority service customers and the inventory level. Further, the some important measures of system performances in the steady state are derived and finally we conclude with some numerical examples.

Contents

1	Introduction and Preliminary	1
1.1	Inventory Systems	2
1.1.1	Single commodity inventory system	3
1.1.2	Perishable inventory systems	8
1.2	Service Facility Inventory Systems	10
1.3	Scope of the Thesis	11
1.4	Preliminary	15
1.4.1	Stochastic Processes	15
1.4.2	Markov Chains	16
1.4.3	Some Important Characteristic of the States and of Chain	17
1.4.4	Continuous-time Markov Chains	21
1.4.5	Markov renewal processes and Semi-Markov processes .	25
1.4.6	Markov renewal equations	26
1.4.7	Semi-regenerative Processes (SRP)	28
1.4.8	Matrix-Geometric Methods	29
1.4.9	Measures of System Performance	32
2	A Queueing-Inventory System with J-Additional Options for Service and Finite Source	34
2.1	Introduction	35

2.2	Mathematical Model	38
2.3	Mathematical Solution of the Model	40
2.4	Waiting Time analysis of an orbital customer	48
2.5	System performance measures	53
2.5.1	Average on-hand Inventory Level	54
2.5.2	Mean Reorder Rate	54
2.5.3	Mean Perishable Rate	55
2.5.4	Average Number of Customers in the Orbit	55
2.5.5	Average Number of Customers Lost to the System	55
2.5.6	Effective Interruption Rate	55
2.5.7	Effective Repair Rate	56
2.5.8	The Overall Retrial Rate	56
2.5.9	The Successful Retrial Rate	56
2.5.10	The Fraction of Successful Rate of Retrial	56
2.5.11	The Probability that Server and System are Idle	56
2.5.12	The Probability that Server is Idle, But the System is Not Empty	57
2.5.13	Probability that Server is Busy (FES or SOS)	57
2.5.14	Probability that Server is under Repair	57
2.5.15	The Effective Rate at Which Arriving Customers are Lost on Seeing an Interrupted Server	57
2.5.16	The Effective Rate at Which Arriving Customers are Lost Finding the Inventory Level as Zero	58
2.6	Cost Analysis and sensitivity investigation	58

3 A Stochastic Inventory Model with Two Queues and a Flexible Server **73**

3.1	Introduction	74
3.2	Mathematical Model and Notations	76
3.2.1	Notations:	78
3.3	Analysis of the Model	78
3.4	System performance measures	96
3.4.1	Expected inventory level(η_I):	97
3.4.2	Expected reorder rate(η_R):	97
3.4.3	Expected perishable rate(η_P):	97
3.4.4	Expected number of customers in station 1(η_{S1}):	98
3.4.5	Expected number of customers in station 2(η_{S2}):	98
3.4.6	Expected number of customer not entering station 1(η_{L_1}):	98
3.4.7	Expected number of customers entering station 1($\eta_{L_{e1}}$):	99
3.4.8	Expected waiting time of the customers in station 1($E[W]$):	100
3.4.9	Expected number of customer not entering station 2(η_{L_2}):	100
3.4.10	Expected number of customers entering station 2($\eta_{L_{e2}}$):	101
3.4.11	Expected waiting time of the customers in station 2($E[F]$):	101
3.5	Cost analysis and numerical illustration	101
3.5.1	Numerical illustration	101
3.6	Concluding Remarks	114
4	A perishable inventory-queueing model with delayed vacation, negative and impatient customers	115
4.1	Introduction	116
4.2	Mathematical Model	117
4.3	Analysis of the Model	119
4.3.1	Linear equations for the steady state distribution	120
4.3.2	An Alternative Markov Chain Representation	126

4.4	System characteristics	130
4.5	Cost Analysis and Sensitivity Investigation	131
4.5.1	Sensitivity Investigation	132
4.6	Concluding Remarks	138
5	Two Heterogeneous Servers Queueing-Inventory System with Sharing Finite Buffer and a Flexible Server	139
5.1	Introduction	140
5.2	Model Description	144
5.3	Mathematical Analysis of the Model	146
5.3.1	Steady-state Analysis	148
5.4	System performance measures	150
5.4.1	Expected Inventory Level η_I :	150
5.4.2	Expected Reorder Rate η_R :	150
5.4.3	Expected Perishable Rate η_P :	151
5.4.4	Expected Number of low priority Customers in the Queue η_L :	151
5.4.5	Expected Number of High Priority Customers in the Queue η_H :	151
5.4.6	Expected Loss Rate of Low Priority customers η_{LRL} : .	151
5.4.7	Expected Loss Rate of High Priority customers η_{LRH} : .	152
5.4.8	Expected number of High Priority customers Enter into the Queue η_{HE} :	152
5.4.9	Expected number of Low Priority customers Enter into the Queue η_{LE} :	152
5.4.10	Expected Waiting Time of Low Priority customers η_{WL} : .	152
5.4.11	Expected Waiting Time of High Priority customers η_{WH} : .	153

5.4.12 Fraction of time Server is on Vacation η_{FV} :	153
5.4.13 Expected total cost rate	153
List of Publications	154
References	155

List of Tables

2.1	Total expected cost rate as a function of S and s	63
2.2	Total expected cost rate as a function of s and N	64
2.3	Total expected cost rate as a function of S and N	64
2.4	Variation in optimal values for different values of c_h and c_s	68
2.5	Variation in optimal values for different values of c_h and c_p	69
2.6	Variation in optimal values for different values of c_h and c_w	69
2.7	Variation in optimal values for different values of c_s and c_p	70
2.8	Variation in optimal values for different values of c_s and c_w	70
2.9	Variation in optimal values for different values of c_s and c_l	71
2.10	Variation in optimal values for different values of c_r and c_i	71
3.1	Total expected cost rate as a function of S and s	104
3.2	Effect of c_s and c_h on the optimal cost rate	105
3.3	Effect of c_s and c_{w1} on the optimal cost rate	105
3.4	Effect of c_h and c_{w1} on the optimal cost rate	106
3.5	Effect of c_p and c_s on the optimal cost rate	106
3.6	Effect of c_{w1} and c_p on the optimal cost rate	107
3.7	Effect of c_{l1} and c_{l2} on the optimal cost rate	107
4.1	Effect of c_h, c_s, c_p, c_w on the expected total cost	133
4.2	Effect of c_r, c_l, c_n on the expected total cost	133

List of Figures

2.1	Dynamics of the queueing - inventory system with finite source	38
2.2	A three dimensional plot of the cost function	59
2.3	λ vs β on TC	60
2.4	β vs μ_0 on TC	61
2.5	β vs γ on TC.	62
2.6	β vs θ on TC.	64
2.7	β vs μ_1, μ_2, μ_3 on TC.	65
2.8	β vs μ_1, μ_2, μ_3 on TC.	65
2.9	β vs service cases on TC.	66
2.10	β vs service cases on TC.	66
2.11	β vs service cases on TC.	67
2.12	β vs μ_0, μ_1, μ_2 and μ_3 on η_{CO}	67
2.13	N vs $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ on η_{CO}	68
2.14	N vs $\eta_0, \eta_1, \eta_2, \eta_3$ on η_{CO}	72
3.1	Graphical representation of the model	78
3.2	Convexity of the total cost for various combinations of S and s	102
3.3	TC vs μ_2 for different values of μ_4	108
3.4	TC vs μ_1 for different values of μ_3	108
3.5	TC vs μ_1 for different values of γ	109
3.6	TC vs λ for different values of β	109

3.7	TC vs η for different values of λ	110
3.8	TC vs η for different values of γ	110
3.9	TC vs η for different values of β	111
3.10	$E[W]$ vs β for different values of γ	111
3.11	$E[W]$ vs λ for different values of μ_1	112
3.12	$E[W]$ vs μ_3 for different values of η	112
3.13	$E[F]$ vs μ_2 for different values of μ_4	113
4.1	ETC versus λ	134
4.2	ETC versus γ	134
4.3	ETC versus μ	135
4.4	ETC versus θ	135
4.5	ETC versus β	136
4.6	ETC versus η	136
4.7	ETC versus S	137
4.8	ETC versus N	137