CHAPTER 3
POWER LINE MODELING

3.1 Introduction

Several modeling techniques are available in literature to obtain the transfer characteristics of power lines which are discussed in the previous chapter. It is important to note that transfer functions developed in earlier literature do not consider a variety of loads. The transfer function (TF) is dependent on RLCG parameters of a line and on the network topology. The modelling techniques can be classified into two categories: the top-down method and the bottom-up method. In top-down method, the model parameters and structures are retrieved from measurements [14]. In this case the parameters depend on measurement results, the accuracy of model is affected by measurement equipment and measurement methods. In contrast the bottom up approach begins with a theoretical analysis of indoor power line structure [19]. It depends on a detailed understanding of power network topology. This approach requires more computation efforts as compared to the top-down approach, and describes a clear relationship between the network behaviour and model parameters. Moreover this modelling approach is more versatile and flexible since all the parameters are formulated, making it easy to predict the changes in the transfer function, due to changes in the system configuration. Hence the bottom up approach of modeling power lines is adopted in this thesis.

The bottom-up approach can be achieved either in time domain or frequency domain. In frequency domain modelling, the network is regarded as a composition of many cascaded-distributed portions [19]. Then the whole network behaviour can be described based on the transmission matrices or as scattering matrices of the cascaded portions. The main advantage of the frequency-domain modelling is its ability to consider all the signals reflected from the discontinuities regardless of the network complexity. Hence the frequency domain bottom-up approach of power line modeling is adopted in this thesis.
In this chapter the bottom-up approach of channel model has been developed for the indoor power line (AWG (American wire gauge) number 14), with primary and secondary parameters R, L, C, G, $\gamma$ and $Z_0$. Two-port network model has been developed for the mainline, distribution line or the bridge taps with open and load connected. Transmission matrix has been derived for both the conditions of the distribution line. The final transmission matrix of the complete network is the product of transmission matrix of main propagation line and transmission matrix of several distribution branches. Transfer function is obtained for different network topologies with the ABCD parameters of the cascaded transmission matrix.

The transfer function of different line topologies is obtained by considering the test cases similar to the residential power line network viz a plain line, line with single bridge tap, line with 2, 5 & 10 bridge taps. The effect of different residential loads connected to the bridge taps which are inductive are analysed and observed.

To find the transfer function of a particular residential power line network using bottom-up approach, it is essential to know the loop or network topology. Network topology can be estimated either by Double Ended Loop Testing (DELT) or by SELT. The proposed method in this thesis aims at using the blocks of an existing DSL modem; thus eliminating need of any external equipment for measurement. The measurement phase is completed in two steps: first step is when Correlation Time Domain Reflectometry (CTDR) measurements are carried out at different sockets (due to large number of BTs) to estimate part sections and a second step when a Frequency Domain Reflectometry (FDR) measurement is performed for the whole Power line loop. Results have been presented for a number of test loops of varying complexity in terms of bridge taps.

This chapter is organised as follows: section 3.2.1 introduces the indoor power line primary and secondary parameters. The two-port network model is discussed in the section 3.2.2. Transmission matrix for a plain line, open bridge tap and BT with load is derived in the section 3.2.3 & 3.2.4. Transfer function for different line
topologies (test cases) is evaluated in the section 3.4.2, followed by the results and analysis. To obtain the powerline model the knowledge of loop or network topology is a prerequisite. Hence loop topology estimation is discussed in the section 3.3

3.2 Quantitative Modeling of Powerline

When power lines are used to transmit high frequency communication signals, they can be regarded as transmission lines, which guide the transverse electromagnetic (TEM) waves along them [19]. The cable under study in this thesis is the typical single-phase house wiring found in India as shown in Figure 3.1. The cables are made up of stranded copper conductors with PVC insulation. The three cables live, neutral and earth are usually laid inside the metal or PVC conduits that are embedded inside the concrete wall.

![Figure 3.1: Cross-sectional view of residential power line.](image)

Typically, the live and neutral cables are used as the PLC transmission channel, which can be approximated as a close form of the “two-wire transmission line”. According to [31], the two-wire transmission line must be a pair of parallel conducting wires separated by a uniform distance. In the actual installation, the power cables are simply pulled through the conduit and the separation between them is not uniform at all. But, the conduit normally has small cross-sectional area and this limits the variation of the separation between the cables. Hence, the assumption of uniform separation is reasonable.
Based on the above considerations, the paired power cables are regarded as a distributed parameter network, where voltages and currents can vary in magnitude and phase over its length. Hence it can be described by circuit parameters that are distributed over its length that is discussed in next section.

3.2.1 Indoor Power Line Parameters

The power transmission lines are the examples of open lines, wherein the conductors of such lines may be considered parallel and separated by air dielectric. A parallel-wire transmission line is constructed so that the spacing ‘d’ between wires, is large with respect to the radius ‘a’ of a conductor as shown in the figure 3.1. The transmission line model represents the transmission line as an infinite series of two-port elementary components, each representing an infinitely short segment of the transmission line. The model consists of an infinite series of the elements shown in the figure 3.2, and that the values of the components are specified per unit length and may also be functions of frequency. These parameters include R, L, C and G distributed along the line [35]. These quantities can also be known as the primary line constants to distinguish from the secondary line constants derived from them, these being the propagation constant ‘γ’, attenuation constant ‘α,’ phase constant 'β’ and characteristic impedance \( Z_0 \).

![Figure 3.2: Schematic representation of the elementary component of a transmission line.](image)
The primary parameters are defined as

**R**: the distributed resistance of the conductors is represented by a series resistor expressed in Ohms per unit length.

**L**: the distributed inductance (due to the magnetic field around the wires, self-inductance, etc.) is represented by a series inductor, Henries per unit length.

**C**: the capacitance between the two conductors is represented by a shunt capacitor $C$, Farads per unit length.

**G**: the conductance of the dielectric material separating the two conductors is represented by a shunt resistor between the signal wire and the return wire expressed in Siemens per unit length.

The RLCG parameters of a transmission line help defining two secondary parameters useful in characterising voltage and current along the line. These are the propagation constant ‘$\gamma$’ and the characteristic impedance ‘$Z_0$’. According to [31, 35] the resistance, inductance, capacitance and conductance can be obtained from the physical parameters. Diameter and separation distance ‘a’ and ‘d’ are obtained from the table of American Wire Gauge (AWG) 14 as shown in figure 3.1, ‘$\mu$’ is permeability of copper, ‘$\varepsilon$’ is permittivity in free space. The conductivity of conductor ‘$\sigma$’ given by $\sigma=1/\rho$, where ‘$\rho$’ is the resistivity of copper and ‘$\delta$’ is the skin depth, function of frequency. These are calculated from

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$$  \hspace{1cm} (3.1)

Table 3.1 shows the values of the physical constraints discussed above for the power lines. The relations for RLCG for the power lines are given by

$$R = \frac{2}{a} \sqrt{\frac{\mu f}{\pi \sigma}}$$  \hspace{1cm} (3.2)
\[ L = \frac{\mu}{\pi} \cosh^{-1} \left( \frac{d}{a} \right) + \frac{R}{2 \pi f} \]  \hspace{1cm} (3.3)

\[ C = \frac{\pi \varepsilon}{\cosh^{-1} \left( \frac{d}{a} \right)} \]  \hspace{1cm} (3.4)

\[ G = 2 \pi f C \tan \delta \]  \hspace{1cm} (3.5)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>( 4\pi \times 10^{-7} ) H/m</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>( 8.8542 \times 10^{-12} ) m²/N</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( 1.72 \times 10^{-8} ) ( \Omega )/m</td>
</tr>
<tr>
<td>( a )</td>
<td>1.62814 mm</td>
</tr>
<tr>
<td>( d )</td>
<td>3.43 mm</td>
</tr>
</tbody>
</table>

Based on transmission line theory, the propagation constant ‘\( \gamma \)’ and characteristic impedance ‘\( Z_0 \)’ can be written as follows. Both ‘\( \gamma \)’ and ‘\( Z_0 \)’ are characteristic properties of a transmission line, which depend on \( R, L, C, G \) and \( \omega \) but not on the length of the line.

\[ \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \]  \hspace{1cm} (3.6)

For the power lines at high frequencies [31, 35] with the equations for \( R, L, C \) the
propagation constant ‘γ’ is

\[ \gamma = \frac{R}{2} \sqrt{\frac{C}{L}} \left(1 - \frac{R^2}{2\omega^2 L^2}\right) + j\omega \sqrt{\frac{LC}{2\omega^2 L^2}} \left(1 - \frac{R^2}{8\omega^2 L^2}\right) \]  

(3.7)

And the characteristic impedance

\[ Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \]  

(3.8)

Where ‘ω’ is the angular frequency.

Using the above parameters ‘γ’ and ‘Z₀’, and with A, B, C and D parameters of the two-port model the transmission matrix is obtained for a plain line and bridge tap.

### 3.2.2 Two-port Network Model

According to transmission theory, a uniform transmission line can be modelled as a two-port network [35] as shown in figure 3.3.

![Figure 3.3: Two-port network model of power line](image)

In the figure 3.3 the output voltage and current V₂ and I₂ are related to the input voltage and current V₁ and I₁ in terms of ABCD parameters by the following equations [33, 35]
\[ V_1 = AV_2 + BI_2 \quad \text{(3.9)} \]
\[ I_1 = CV_2 + DI_2 \quad \text{(3.10)} \]

The above equation characterizes the network without making any assumptions about the network’s termination. The parameters A, B, C and D are specific to the transfer function of the network and are obtained from primary and secondary line parameters discussed in the previous section.

As shown in Figure 3.3, the relationship between current and voltage at these two ports is

\[
\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = T_m \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \text{(3.11)}
\]

Where \( V_1 \) and \( I_1 \) are input voltage and current. \( V_2 \) and \( I_2 \) are output voltage and current respectively and ‘Tm’ is the transfer matrix.

Considering a typical indoor power line as a uniform transmission line, it is more convenient to represent each junction by a transmission matrix that gives the output quantities in terms of input quantities. With such a representation, the matrix which describes the complete cascade connection may be obtained simply by multiplying the matrices describing each junction together.

Typically, an indoor power line is composed of a main propagation path or the straight line section and several distributed branches or the bridge taps (BT) as shown in Figure 3.4. The bridge taps or the distribution branches are the lines drawn from the straight line sections to connect different residential loads like bulb, fan, computer, mixer, refrigerator etc at the convenient points.
Indoor power line structure is represented by the simplified model as in figure 3.5 with the straight line sections and the bridge taps connected to the load.

Each path has its own transmission matrix. So the indoor power line can be seen as a cascade of various two-port networks [16] for straight line section and the distribution line as shown in figure 3.6.
The effective ABCD parameters for the entire system can be given by a simple matrix multiplication of the individual parameter as given below,

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
= \begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix}
\begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\cdots
\begin{bmatrix}
A_N & B_N \\
C_N & D_N
\end{bmatrix}
\tag{3.12}
\]

\[
T' = T_1 \times T_2 \times \cdots \times T_N = \prod_{i=1}^{N} \begin{bmatrix}
A_i & B_i \\
C_i & D_i
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\tag{3.13}
\]

Where \(T'\) is the product of matrices,

\[
\begin{bmatrix}
V_o \\
I_o
\end{bmatrix}
= T' \begin{bmatrix}
V_N \\
I_N
\end{bmatrix}
\tag{3.14}
\]

Therefore the final transmission matrix of the entire system is the product of transfer matrix of several main propagation lines and transmission matrix of several distribution branches as seen in the equation 3.13.

### 3.2.3 ABCD Parameters of Straight Line Branch

The voltage at a point on the line is the sum of a forward travelling wave at that point and the backward travelling wave. The wave traveling is attenuated exponentially with distance and that exponential depends on the propagation constant [33]. Figure 3.7 shows a twisted pair with forward and backward travelling wave.
The voltage at \( x=0 \) is the sum of a forward and backward traveling wave seen at \( x=L \).

The voltage at \( X = 0 \), as shown in figure 3.7 is given by

\[
V_1(f, 0) = (V^+_{X=L} e^{\gamma(f)L} + V^-_{X=L} e^{-\gamma(f)L})
\]  

(3.15)

Where \((V^+)\) is forward traveling wave and \((V^-)\) is backward traveling wave. The voltage at \( x=0 \) is the sum of forward and backward travelling wave.

The above equation can be written as

\[
V_1(f, 0) = (V^+_{X=L} (e^{\gamma(f)L} + \tau e^{-\gamma(f)L}))
\]  

(3.16)

Where ‘\( \tau \)’ is the reflection coefficient and is given by

\[
\tau_L(f) = \frac{Z_L(f)-Z_0(f)}{Z_L(f)+Z_0(f)}
\]  

(3.17)

Where \( Z_0 \) is the characteristic impedance and \( Z_L \) is the load impedance.

The output voltage \( V_2 \) is the sum of forward and backward wave voltages at \( X=L \), the expression for \( V_2 \) can be written as
\[ V_2 = V_{x=L}^+ + V_{x=L}^- \]  
\[ V_2 = V_{x=L}^+ (1 + \tau_L) \]

Solving for \( V_{x=L}^+ \) gives

\[ V_{x=L}^+ = \frac{V_2}{1 + \tau_L} \]  
(3.19)

Substituting equation (3.19) in equation (3.16)

\[ V_1 = \left( \frac{V_2}{1 + \tau_L} (e^{\gamma f L} + \tau_L e^{-\gamma f L}) \right) \]  
(3.20)

Substituting for \( \tau_L \) from the equation 3.17 in the above equation and simplifying

\[ V_1 = \left[ \frac{e^{\gamma f L} + e^{-\gamma f L}}{2} \right] V_2 + Z_0 \left[ \frac{e^{\gamma f L} - e^{-\gamma f L}}{2} \right] I_2 \]

The above expression can be replaced with cosh and sinh

\[ V_1 = (\cosh(\gamma L)) V_2 + Z_0 (\sinh(\gamma L)) I_2 \]  
(3.21)

This is equal to

\[ V_1 = AV_2 + BI_2 \]  
(3.22)

\[ I_1 = \left[ \frac{\sinh(\gamma L)}{Z_0} \right] V_2 + (\cosh(\gamma L)) I_2 \]  
(3.23)

This is equal to

\[ I_1 = CV_2 + DI_2 \]  
(3.24)

From the equations (3.21) to (3.24), the ABCD parameters for generic uniform
transmission line can be written as

\[
T_m = \begin{bmatrix}
\cosh (\gamma l) & Z_0 \sinh (\gamma l) \\
\frac{1}{Z_0} \sinh (\gamma l) & \cosh (\gamma l)
\end{bmatrix}
\]

(3.25)

Where ‘l’ is the length of the line, ‘\(\gamma\)’ is its propagation constant and ‘\(Z_0\)’ is the characteristic impedance given by equation (3.8).

### 3.2.4 ABCD Parameters of Distribution Branches

For the distributed branch, the ABCD parameters may be obtained as follows,

![Two-port network diagram](image)

**Figure 3.8: A two-port network consisting of parallel impedance**

From the figure 3.8, the voltage at the parallel ports are same, ie \(V_2=V_1\) independent of \(I_2\). The output voltage and current \(V_2\) and \(I_2\) are related to the input voltage and current \(V_1\) and \(I_1\) in terms of ABCD parameters by the following equations, and by applying the above condition,

\[
V_1 = AV_2 + BI_2 = AV_1 + BI_2 \tag{3.26}
\]

\[
I_1 = CV_2 + DI_2 = CV_1 + DI_2 \tag{3.27}
\]

Solving the equation (3.25), yields
A=1 and B=0,

Applying Kirchoff’s Current Law at the junction in the figure 3.8
\[ I_1 = I_z + I_2 \]
\[ I_1 = \frac{V_1}{Z} + I_2 \]

From the above equation and the equation (3.24),
\[ C = \frac{1}{Z} \quad \text{and} \quad D = 1. \]

Thus the matrix for a two-port network consisting only of shunt impedance is given by
\[
T_D = \begin{bmatrix}
1 & 0 \\
\frac{1}{Z} & 1 \\
\end{bmatrix}
\]

viz the transmission matrix of the distribution branch is

\[
\begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{Z_{in, tap}} & 0 \\
1 & 1 \\
\end{bmatrix} = T_D
\] (3.28)

From the equation 3.21 & 3.23

\[ V_1 = \cosh(\gamma L_{tap}) V_2 + Z_{otap} \sinh(\gamma L_{tap}) I_2 \] (3.29)
\[ I_1 = \frac{\sinh(\gamma L_{tap})}{Z_{o, tap}} V_2 + \cosh(\gamma L_{tap}) I_2 \] (3.30)

Where ‘\( L_{tap} \)’ is the length of the bridge tap or distribution line.

Considering the two cases: open and inductive load terminated at the end of distribution line or the BT.
**Case (i):** For bridge tap open circuited at one end,

Viz \( Z_L = \infty \), hence \( I_2 = 0 \).

Substituting \( I_2 = 0 \) in the equations (3.29) & (3.30)

\[
V_1 = \cosh(\gamma L_{\text{tap}}) V_2 \\
I_1 = \frac{\sinh(\gamma L_{\text{tap}})}{Z_{o,\text{tap}}} V_2
\]

Therefore,

\[
Z_{\text{in, tap}} = \frac{V_1}{I_1} = Z_{o,\text{tap}} \coth(\gamma L_{\text{tap}}) \tag{3.31}
\]

Where ‘\( L_{\text{tap}} \)’ is the length of the tap.

**Case (ii):** Bridge tap terminated with an inductive load ‘\( Z_R \),

The input impedance is:

Considering at the sending end, from the equations (3.26) & (3.27)

\[
I_1 = I_2 (\cosh(\gamma L_{\text{tap}}) + \frac{Z_R}{Z_o} \sinh(\gamma L_{\text{tap}})) \tag{3.32}
\]

\[
V_1 = I_2 (Z_R \cosh(\gamma L_{\text{tap}}) + Z_o \sinh(\gamma L_{\text{tap}})) \tag{3.33}
\]

Therefore

\[
Z_{\text{in, tap}} = \frac{V_1}{I_1} = Z_o \left( \frac{Z_R \cosh(\gamma L_{\text{tap}}) + Z_o \sinh(\gamma L_{\text{tap}})}{Z_o \cosh(\gamma L_{\text{tap}}) + Z_R \sinh(\gamma L_{\text{tap}})} \right)
\]

\[
Z_{\text{in, tap}} = Z_o \left( \frac{e^{\gamma L} + e^{-\gamma L} \left( \frac{Z_R - Z_o}{Z_R + Z_o} \right)}{e^{\gamma L} - e^{-\gamma L} \left( \frac{Z_R - Z_o}{Z_R + Z_o} \right)} \right)
\]
Replacing \( e^{\gamma L} = \cosh \gamma L + \sinh \gamma L \) and \( e^{-\gamma L} = \cosh \gamma L - \sinh \gamma L \) and simplifying

\[
Z_{\text{in-tap}} = \left( \frac{\cosh (\gamma L_{\text{tap}}) + \sinh (\gamma L_{\text{tap}})}{\sinh (\gamma L_{\text{tap}}) + \cosh (\gamma L_{\text{tap}})} \right) \left( 1 - \frac{Z_R - Z_0}{Z_R + Z_0} \right) \left( 1 - \frac{Z_R - Z_0}{Z_R + Z_0} \right) \tag{3.34}
\]

Substituting \( Z_{\text{in-tap}} \) from the equation (3.34) for inductive load in the equation (3.28) to obtain the distribution matrix \( T_m \), and similarly substituting \( Z_{\text{in-tap}} \) from the equation (3.31) for open tap. Hence a wide variety of loops can be analysed by finding the ABCD parameters of each section accordingly by either transmission matrix ‘\( T_m \)’ of main line or transmission matrix ‘\( T_d \)’ of the distribution line and performing matrix multiplication.

### 3.2.5 The Channel Model

Having obtained the transmission matrices for main and distribution lines, the ABCD parameters for the entire network may be found by multiplying the ABCD parameters for the individual sections along the loop as shown in the figure 3.5 and given by the equation (3.13). With the ABCD matrix for the entire network the transfer function of the total loop can be obtained. The transfer function of a loop can be found from the equation (3.26), as follows

\[
V_1 = AV_2 + BI_2 = AV_2 + B \frac{V_2}{Z_R}
\]

\[
H(f) = \frac{V_2}{V_1} = \frac{Z_R}{AZ_R + B} \tag{3.35}
\]

\[
H(f) = 20\log_{10}|H(f)|dB
\]
Hence the transfer function $H(f)$ of the entire network is obtained by evaluating the ABCD parameters of the individual section. In bottom-up approach the prior knowledge of network ie loop topology is essential. The indoor power line structure varies randomly, hence the loop or network topology estimation for indoor power lines is discussed in the next section.

### 3.3 Loop Topology Estimation

The indoor power line network varies according to the customer requirement in the customer premises having many distribution branches which can be connected to the electrical appliances. Hence a method is required to know the indoor power line network topology.

Two popular schemes of loop topology estimation for telephone lines are available in the literature [63], namely

- **Double Ended Loop Testing (DELT)**
- **Single Ended Loop Testing (SELT)**

DELT is performed with equipment at both the ends of the power line network viz customer end (indoor power socket) and the area transformer. DELT is not economically feasible for pre service deployment estimation as it involves sending a person with instruments to the customer end. Also DELT cannot be used when there is no physical connection between the network end and area transformer.

In SELT the measurement is done from one end only, preferably from the area transformer. SELT is less expensive and faster than DELT and used during initial deployment and post deployment trouble shooting [64]. A SELT measurement technique relies on an analysis of reflections that is produced by discontinuities in the medium, to fully characterise the link. It is more economical and attractive to probe the line with the same DSL modem employed for regular DSL services.

There are two methods of loop topology estimation under the SELT, one is **Time Domain Reflectometry (TDR)** and the second is **Frequency Domain Reflectometry (FDR)**. TDR is the older method that employs pulse as an excitation
signal. Once the transmitted pulse encounters a discontinuity in the cable, a portion of the energy is reflected back to the receiver. The elapsed time of arrival of the echo signal determines the location of the discontinuity. The shape and the polarity of the echo provide a signature for identifying the type of discontinuity that produced the echo [65]. The time for the reflection to reach the transmitter and its shape are used to calculate the distance and type of discontinuity.

Use of pulse in TDR measurements requires an external pulse generator and receiver that are not feasible with a DSL modem. In addition to the need of external equipment, tradeoff between the SNR and resolution are the major limitations of the conventional TDR. To overcome the this limitations, correlation time domain reflectometry (CTDR) method for the estimation of loop topology is introduced in this chapter. Complementary codes are used as a probe signal instead of conventional pulse in the measurement phase. Complementary codes are sent through existing DSL modem and hence no additional equipment is required.

FDR transmits signals with different frequencies, and measures the echo at these frequencies. The incident and the reflected signal together form a standing wave on the line that is analysed and used to estimate the distance discontinuity [67, 68].

Application of SELT for a power line environment is investigated. The SELT estimation process basically contains two phases, a measurement phase and an Interpretation phase. The proposed method aims at using the blocks of existing DSL modem; thus eliminating need of any external equipment for measurement. The measurement phase is completed in two steps;

i) when CTDR (Correlation Time Domain Reflectometry) measurements are carried out at different sockets (due to large number of BTs) and

ii) When a FDR (Frequency Domain Reflectometry) measurement is performed for the whole Power line loop.

Interpretation of measured data is again a two-step procedure that involves computation and can be executed either in the modem itself or offline where more
resources are available. The two steps in interpretation are

i) Extraction of an approximate loop topology from CTDR measurements.
ii) FDR signature of the CTDR estimated loop is simulated and is compared with the FDR measurement in a mean squared sense to arrive at a most probable estimate of the power line network.

Results have been presented for test loops of plain line, single bridge tap and line with two BTs.

As the length of the power line is limited, CTDR method where complementary codes are used as a probe signal instead of conventional pulse in the measurement phase is selected for topology estimation [66]. Complementary codes are sent through existing DSL modem and needs no additional equipment. Tone spacing and ADSL PSD masks specified by ITU G.992.3 [51] are followed for transmitting the signal. In this method distance to the discontinuity is estimated by correlating the input signal with the echo. The position of the peaks in the correlated signal indicates the discontinuity. To find the type of discontinuity step by step maximum likelihood algorithm is used with data de-embedding[65]. The discontinuities are identified one at a time using step by step method and data de-embedding helps in identifying far end discontinuities of lines. Major advantage of this method is that it does not require any prior knowledge of the loop. Use of complementary codes as probe signal improves the range and resolution of CTDR.

However when the number of bridge taps (BTs) is high and close to each other, CTDR from one end does not help to unravel the complete topology. To overcome this limitation, CTDR measurements from either side of the sockets are carried out to estimate the taps close to each of the sockets. The topology learning from multipoint CTDR application is then overlaid to generate the complete topology. The predicted topology is verified with multipoint FDR measurements [68]. This procedure is explained to greater detail in the next section.
3.3.1 Multi-Point CTDR method

PLC network typically consists of a large number (more than 5) of bridge taps. It is observed that the received echo is more influenced by the first few bridge taps and the effect of remaining bridge taps is minimal. At each bridge tap discontinuity, the transmitted signal strength is reduced to $\sim 30\%$ (transmission coefficient of bridge tap $\sim -0.3$) of incident value. This reduction when cumulated, results in too weak echo (reflection from 2nd discontinuity is $\sim 2.7\%$ ($0.33*0.33*0.33$) of the input signal, considering the signal loss only due to the discontinuity) and hence becomes insensitive beyond the 3rd tap. Due to this reason, CTDR SELT technique using a single point measurement is not capable of complete topology estimation. De-embedding technique applicable for telephone line is also found to be limited in handling high number of bridge taps. A multipoint measurement is used to overcome this limitation. Figure 3.9 shows a typical in home PLC network with the measurement points. In the multipoint method, TDR measurement from each socket is analyzed to predict the finite segments close to the measuring socket.

![Figure 3.9: Schematic of PLC network](image-url)
The predicted line segments accuracy is improved by Nelder-mead optimization algorithm by comparing the simulated echo of the predicted loop with the received echo in the localized time period. Applicability of the iterative method for the complete network is tedious for PLC, as it often converges to local minima due to high number of variables (number of variables = 2*no of taps +1). To overcome this limitation, fine tuning of the line length is performed at the line segment level and not at complete topology level.

Identified line segments close to each socket are used to construct the complete topology. Single point CTDR and FDR method is capable of predicting the topology for test loops that have one or two bridge taps. However single point measurement based method is not enough for cases with multiple BTs, hence multipoint CTDR and FDR is employed. FDR verification step for the complete topology from multipoint CTDR is presented in the next section.

### 3.3.2 FDR Verification

Predicted loop topology is validated by means of FDR method at multiple sockets. \( R(\Phi, f_n) \) is the simulated FDR data for predicted topology. \( \hat{R}(f_n) \) measured FDR data. Where ‘n’ is the transmitted tone. Predicted loop \( \Phi \) is used to generate the FDR data \( R(\Phi, f_n) \). This is compared with the measured FDR data \( \hat{R}(f_n) \) in terms of mean square error (MSE) value as shown in Equation 3.36.

\[
MSE = \left( \sum_{n=1}^{N} \left| R(\Phi, f_n) - \hat{R}(f_n) \right|^2 \right)^{\frac{1}{2}}
\]

(3.36)
This method is validated for a wide range of power line topologies [68]. From this estimated topology, the transfer characteristic of the power line is derived and the channel capacity is calculated. Figure 3.10 gives the flow chart for the combined multipoint CTDR and FDR method.

![Flow chart for loop topology estimation](image)

**Figure 3.10:** Flow chart for loop topology estimation
3.4 Simulation Results and Analysis

Methodology to obtain the transfer function and the loop topology estimation are discussed in the previous section. In this section we obtain the transfer function (TF) for defined class of residential power lines. In telephone lines standard topologies are listed in the literature available for standards [49] that facilitates easy comparison. However there are no standard test cases available for power lines. Here we have projected a set of test cases which represent a typical indoor residential power line network, where there are straight line sections, distribution lines with open end and with residential loads connected which are inductive in nature.

Transfer function of the channel is obtained for the different test cases shown in figure 3.10, plain line with increasing length, line with one, two, five and ten bridge taps with open and loaded conditions. Most of the residential loads are inductive; hence the inductive load of 600mH is considered with constant -140dbm/Hz AWGN noise and observed that the attenuation increases with increasing length, increasing number of taps and with load.

3.4.1 Simulation conditions

Power lines specifications of AWG12 and AWG14 are used for simulation. However because of a significant number of the bridge taps inside the house, there is an additional frequency dependent attenuation in power cables. The topologies of indoor power lines considered for analysis are shown in Figure 3.11. The frequency dependant transfer function is computed using the method described in section 3.2.1 to 3.2.4 above. Specifically we compute the ABCD parameters $T_m$ and $T_d$ using equations (3.25) & (3.28) respectively using 14AWG Power cable for both cases with open bridge taps and also with an inductive load of 600mH that is typical in Fans and machines inside a house.
3.4.2 Test Loop Topologies

Simulation results are presented below for the test loops shown in the figure 3.11. Residential power line network is such that there number of distribution branches or the bridge taps at several points to connect various loads. Hence line with one and two bridge taps for the analysis and more practical conditions of the residential power line network with five and ten taps for open BT and with AWGN of -140dBm/Hz and with the inductive loads are considered.

Test loops considered are

- Plain line as in the figure 3.11A, the transfer characteristics are observed for varying lengths of 600mts, 1200mts and 3000mts to represent short, medium and long lines.

- To study the effect of a tap, line with one distribution branch of length 10mts at the rear end is shown in the figure 3.11B, similarly a distribution branch of the same length at the front end as shown in figure 3.11C is considered.

- To study the effect of increasing taps, a line with two taps and varying bridge tap length as shown in the figure 3.11D is considered.

- As mentioned above the condition near to practical condition of the indoor power line, line with 5taps and 10taps as shown in figure 3.11E & 3.11F has been considered for analysis.

The frequency dependant normalized transfer functions 20log H(f) are shown in Figures 3.12 to 3.22 for the test loops shown in Figure 3.11.
Figure 3.11: Indoor power line network topologies
3.4.3 Transfer Function of Test Loops

Test loop1: plain line with the length of 600mts, 1200mts & 3000mts

The frequency response of the plain length of line shown in figure 3.11A with the power line lengths 600mts, 1200mts and 3000mts is shown in the figure 3.12. As observed the attenuation increases by 20dB as the line length doubles.

![Figure 3.12: Frequency response of loop 1(600,1200 & 3000mts)](attachment)

Test loop2: Line with BT at the rear end without & with load

The frequency response for the test loops 3.11B and 3.11C (with distribution branches) are shown in the figures 3.13 to 3.16. Note the presence of a dip in figures 3.13 -3.16 is due to the presence of a bridge tap that acts as tuned LC circuit absorbing power at the resonance frequency. At resonant frequency which depends on L & C the impedance is maximum, hence there is a dip at that frequency. Attenuation is extremely high (-20dB without load & -100dB with load) when the bridge taps (BT) are connected to the load compared to open BT as shown in figure 3.14 & 3.16. Not much difference in the attenuation characteristics is observed for the
two cases wherein the bridge taps at the front and rear end due to the constant LC values of the tap irrespective of the location of the tap.

**Figure 3.13:** Frequency response of loop 2 with BT open

**Figure 3.14:** Frequency response of loop 2 with load
Test loop 3: Line with BT at the front end without & with load

Figure 3.15: Frequency response of loop 3 with BT open

Figure 3.16: Frequency response of loop 3 with load
Test loop 4: Line with two BT’s without & with load

Frequency response of the test loop 3.11D, viz with two taps of different lengths are shown in the figure 3.17 & 3.18. As observed and compared with the single tap, the attenuation is the same but with the more number of dips due to more mismatch. Attenuation increases with the load in the bridge tap due to impedance mismatch.

Figure 3.17: Frequency response of loop 4 with BT open

Figure 3.18: Frequency response of loop 4 with load
Test loop 5: Line with 5BT’s without & with load

Frequency response of the test loops 3.11E & 3.11F, viz with five and ten taps considering the practical cases inside the building are show in the figure 3.19 to 3.22. The notches are deep due to more power loss in each junction as shown in figure 3.19 & 3.21. With these bridge taps connected to the load the characteristics are shown in the figure 3.20 & 3.21, the observation is that, the notches are also deeper along with increase in attenuation.

Figure 3.19: Frequency response of loop 5 with BT open

Figure 3.20: Frequency response of loop 5 with load
Test loop 6: Line with 10 BT’s without & with load

Figure 3.21: Frequency response of loop 6 without load

Figure 3.22: Frequency response of loop 6 with load
3.4.4 Simulation Results: Loop Topology Estimation

Loop topology estimation of power lines is discussed in the section 3.3. As the length of the power line is limited, CTDR method is selected for topology estimation. However when the number of bridge taps (BTs) is high and is close to each other, a multipoint CTDR-FDR method is used to estimate the power line network.

The simulation conditions for the test loops are:

- For the better resolution, 1 to 1024 tones (4.4 MHz) are used with 2 bits in each tone in VDSL2 technology.
- Complementary codes are used as probe signal.
- Transmit signal PSD as per ITU standards G992.3 for VDSL2 increased by -10dB is considered.
- For FDR validation tone numbers 1 to 512 are considered as the length of power line segments are small.

The test loops considered for the verification of the developed method are:
Test loop1 (figure 3.11A) is a plain line, loop 3 (figure 3.11C), loop 4 (figure 3.11D) lines with one and two BTs respectively.

Test loop1: Plain Line

The desired plot of correlation amplitude versus distance in meters for PLC- test loop 1 is shown in Figure 3.11A. Single peak indicates the line is a plain line without any bridge tap. It may be noted that this type of lines are rare in power line network but is considered as a reference test loop for verification of this algorithm. The ‘X’ value corresponding to the peak is 602 m which indicates the distance of the discontinuity.
Using the iterative process with a single point time domain echo signal (Section 3.3.1), the line length is fine tuned as 599.99 m. This predicted line topology is validated with FDR method as shown in Figure 3.24. Mean square error between the actual and estimated reflected signal is 0.0077 which confirms the accuracy of prediction. MSE in the order of $1e^{-3}$ in PLC validation is found sufficient for good accuracy.
Test loop 3: Line with BT at the front end

For PLC-test loop 3, the correlated amplitude variation with distance is shown in Figure 3.25. Table 3.2 shows the initial estimated loop from the location of the peaks and the final topology using CTDR based optimization. The FDR validation of this estimation is shown in Figure 3.26 with an MSE of 0.0035 which confirms the accuracy of prediction.

![Figure 3.25: Correlation amplitude Vs distance for PLC-test loop3](image)

**Table 3.2: Estimated topology for test loop 3**

<table>
<thead>
<tr>
<th>Initial estimated topology</th>
<th>Final estimated topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>(in meters)</td>
<td>(in meters)</td>
</tr>
<tr>
<td><img src="image" alt="Initial topology" /></td>
<td><img src="image" alt="Final topology" /></td>
</tr>
<tr>
<td>X: 10.6</td>
<td>X: 10</td>
</tr>
<tr>
<td>Y: 50.87</td>
<td>Y: 49.9</td>
</tr>
<tr>
<td>551.13</td>
<td>550.1</td>
</tr>
</tbody>
</table>
Test loop 4: Line with two BT’s

For test loop 4 the correlation amplitude versus distance plot is given in Figure 3.27. Two set of peaks in the figure indicates presence of two bridge taps in the loop. Table 3.3 shows the comparison of initial estimated loop and the optimized loop topology. The comparison between the received FDR echo and the simulated echo of the estimated loop topology is shown in Figure 3.28 and the mean square error between them is 0.0015.
Table 3.3: Estimated topology for test loop 4

<table>
<thead>
<tr>
<th>Initial estimated topology</th>
<th>Final estimated topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>(in meters)</td>
<td>(in meters)</td>
</tr>
<tr>
<td>- 8.4 - 10.6 - 201.4 - 199.2 - 201</td>
<td>- 9.9 - 9.9 - 200 - 199.9</td>
</tr>
</tbody>
</table>

- indicates the measurement point

Figure 3.28: FDR validation for PLC-test loop 4

3.5 Conclusion

In this chapter the indoor power line (AWG14) channel model based on the two-port network theory (bottom-up approach) has been proposed. ABCD parameters are useful in characterizing two-port networks. Each network is independently characterised by its own ABCD parameters. The effective ABCD parameters for the entire system can be given by simple matrix multiplication of the individual
parameters. Power line modeling has been done by deriving the transmission matrix for the main path and the distribution path for open BT and BT with the inductive load, typically considering the equivalent inductive load for the loads like fan, motor in the residential power line network. Simulation has been done for the test cases which are comparable to the practical indoor power line network. Frequency response has been obtained for test loops and observed that the attenuation doubles as the line length doubles. Attenuation increases as the number of taps increases and is worse if the loads are connected to the bridge taps.

Realizing that, to obtain the transfer function in bottom-up approach loop topology is essential hence an innovative multipoint CTDR based SELT methodology is developed for power line topology estimation. Test loops of varying complexities by changing the number of bridge taps and tap lengths are used to validate the methodology.