CHAPTER 8

SIMULTANEOUS OPTIMIZATION

8.1 GENERAL

The multi objective simultaneous optimization technique incorporating MRA/RSM is quite useful for optimizing multi responses. The optimization phase of multi response problem can result in a trade off between different responses. Eventually, the goal is to find a possible solution that leads to the best combination of responses. There are several techniques that have been developed to allow the simultaneous optimization of several responses. There are major types of simultaneous optimization designs namely, overlaying contour plots, priority based approach, desirability function approach and loss function approach. When there are more than three design variables, overlaying contour plots becomes awkward, because the contour plots are two dimensional. The desirability function approach is one of the most widely used methods in industry for the optimization of multiple response processes. Desirability functions serve to turn multiple response optimization problems into single response problems. The desirability function is sensitive to a change in the value of property. This section briefly reviews desirability function approach developed for simultaneous optimization.

Setting of process variables to meet a required specification of quality characteristics (response variable) in a process is one of the common problems in the process quality control. But the response variable generally
has more than one quality characteristics in the process and the experimenter attempts to optimize all of them simultaneously. It is hard to satisfy all the characteristics (models) simultaneously because the characteristics are different.

The aim of this chapter includes the springback and bend force optimization of bending process using the concept of simultaneous optimization. The simultaneous optimization was carried out with five variables with an objective of minimizing both the bend force and springback. This was carried out in two steps: first the bend force and springback were individually optimized using linear programming technique. Second, the combination of both responses was optimized by Harrington’s desirability function, a multi criteria decision making method. The simultaneous optimization was carried out for both MRA and RSM models.

### 8.2 Harrington’s Desirability Approach

In the optimization, it would be possible to arrive at different combinations of the levels of the controllable factors for different responses. The functional approach based on desirability function developed by Harrington has been adopted for multi-response optimization. The response variable ‘\( Y_i \)’ can be transformed to an individual desirability value ‘\( d_i \)’ with the help of desirability function. The transformation can be represented as

\[
d_i = \exp(-\exp(-Y_i))
\]  

(8.1)

for one sided transformation. It varies over the range

\[
0 \leq d_i \leq 1
\]  

(8.2)
where if the response $Y_i$ is at its goal or target, then $d_i = 1$, and if the response is outside an acceptable region, $d_i = 0$. Individual desirability of all responses can be combined to get a single value of desirability by the expression,

$$D_c = (\Pi d_i)^{1/m} \quad (8.3)$$

where ‘m’ - Number of responses

The composite or overall desirability ‘$D_c$’ for the multi responses has been computed for the conditions and the larger value of these has been identified as the optimum operating combination of the levels. The composite desirability is a measure of how well it has satisfied the combined goals for all the responses. It has a range of zero to one.

### 8.3 METHODOLOGY

The desirability approach consists of the following steps

- Conduct experiments and fit response models for all responses. Simultaneous consideration of multi responses involves first building an appropriate model for each response and then trying to find a set of operating conditions that in some sense optimizes all responses or at least keeps them in desired ranges.

- Define individual desirability functions for each response

- Maximize the overall desirability ‘$D_c$’ with respect to the controllable factors.
8.4 SIMULTANEOUS OPTIMIZATION OF MRA MODELS

The purpose of the work is to extend the MRA models into the multiple response optimizations using the desirability function approach. The obtained multiple regression equations for validation are shown in equations (8.4) and (8.5) (refer to equations (7.4) and (6.4)). The MRA between the parameters and response was done with a sample size of 25. The equations obtained were as follows:

\[
\text{Bend Force} = -5985.63 - 27.4835d + 34958n + 48.7697r_p \\
+ 1595.92v_p + 5.55356w + 0.215322d^2 \\
- 49505.5n^2 + 0.280332r_p^2 + 558.963v_p^2 \\
+ 0.0528968w^2 + 54.0721dn + 0.496631dr_p \\
+ 11.3823dv_p - 0.169912dw - 189.479nr_p \\
- 5536.93nv_p + 2.39213nw - 4.75857r_pv_p \\
+ 0.0282183r-fw - 10.2686vw \tag{8.4}
\]

\[
\text{Springback} = 134.868 - 1.76939d - 902.346n - 3.56834r_p \\
+ 40.5456v_p + 0.988406w + 0.0630572d^2 \\
+ 1691.7n^2 + 0.205291r_p^2 + 113.487v_p^2 \\
+ 0.0106328w^2 - 1.771690dn + 0.125020dr_p \\
+ 1.14855dv_p - 0.0360303dw + 0.155119nr_p \\
- 79.3695nv_p - 1.62916nw - 6.89611r_pv_p \\
+ 0.0165057r_fw + 1.92601v_pw \tag{8.5}
\]

where \(d\) is the punch travel in mm, \(n\) is the strain hardening exponent, \(r_p\) is the punch radius in mm, \(v_p\) is the punch velocity in mm/s and \(w\) is the width of the sheet in mm.
8.4.1 Objective Function

The objective function is minimization of bend force/springback effect by optimizing the parameters simultaneously.

Bend Force: \[ f(d,n,r_p,v_p,w) \] \hspace{1cm} (8.6)

Springback: \[ f(d,n,r_p,v_p,w) \] \hspace{1cm} (8.7)

Subject to,

Punch travel (d) : \[ d = 25 \]

Strain hardening exponent (n) : \[ 0.307 < n < 0.325 \]

Punch radius (r_p) : \[ 6 < r_p < 14 \]

Punch velocity (v_p) : \[ 0.3577 < v_p < 0.802 \]

Width of the sheet (w) : \[ 30 < w < 70 \]

The constraints are identical for bend force and springback optimization.

8.4.2 Determination of Optimum Parameter Combination

The optimum values 25 mm, 0.325, 6 mm, 0.3577 mm/s and 30 mm for a minimum bend force and 25 mm, 0.307, 14 mm, 0.3577 mm/s and 70 mm for minimum springback are obtained. Hence, the optimum combinations are not the same for the two response parameters. The Harrington’s desirability function is applied for simultaneous optimization of both the responses and the results are reported in Table 8.1.
Table 8.1 Concurrent Optimization and Desirability Function of Multiple Responses for MRA

<table>
<thead>
<tr>
<th>d</th>
<th>n</th>
<th>r_p</th>
<th>v_p</th>
<th>w</th>
<th>Bend force (N)</th>
<th>Springback (Degrees)</th>
<th>Composite Desirability</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.325</td>
<td>6</td>
<td>0.3578</td>
<td>30</td>
<td>146.93</td>
<td>6.72</td>
<td>0.999397</td>
</tr>
<tr>
<td>25</td>
<td>0.307</td>
<td>14</td>
<td>0.3578</td>
<td>70</td>
<td>291.68</td>
<td>1.09</td>
<td>0.845262</td>
</tr>
</tbody>
</table>

The composite desirability ‘D_c’ of the two responses have been computed for both the conditions and the larger value of these has been identified as the optimum operating combination of the levels. From the Table 8.1, the first set of values (bend force), which yields higher desirability, has been found to be the optimal setting for minimizing both the bend force and the springback.

8.5 SIMULTANEOUS OPTIMIZATION OF RSM MODELS

The purpose of the work is to extend response surface model based on central composite design into the multiresponse optimization using the desirability function approach. The obtained RSM equations for validation are shown in equations (8.8) and (8.9) (refer to equations (7.8) and (6.9)). The RSM between the parameters and response was done with a sample size of 32. The equations obtained were as follows

\[
\text{Bend Force} = 18593 + 36 \, d - 129582 \, n \, - 52 \, r_p + 2670 \, v_p + 77 \, w \\
+ 228383 \, n^2 + 4 \, r_p^2 + 192 \, v_p^2 - 74 \, d \, n - 76 \, n \, r_p \\
- 9296 \, n v_p - 241 n \, w + 10 \, r_p \, v_p + 2 \, v_p w \quad (8.8)
\]

\[
\text{Springback} = -154.636 - 0.0472818 \, d + 1244.31 \, n - 3.01444 \, r_p \\
+ 2.39891 \, v_p - 1.00288 \, w + 0.00272544 \, d^2 \\
- 2284.54 \, n^2 - 0.00796597 \, r_p^2 - 0.754979 \, v_p^2
\]
+ 0.0000688837w^2 + 0.792544d r_p + 0.013809 dv_p  
- 0.27028 dw + 7.26978 nr_p + 10.3627 nv_p + 2.68639 nw 
- 0.113738 r_p v_p + 0.00690452 r_p w + 0.0761883 v_p w  (8.9)

where d is the punch travel in mm, n is the strain hardening exponent, r_p is the punch radius in mm, v_p is the punch velocity in mm/s and w is the width of the sheet in mm.

### 8.5.1 Objective Function

The objective function is the minimization of bend force/springback effect by optimizing the parameters simultaneously.

\[
\text{Bend Force : } f (d, n, r_p, v_p, w) \quad (8.10)
\]

\[
\text{Springback : } f (d, n, r_p, v_p, w) \quad (8.11)
\]

Subject to,

- Punch travel (d) : \( d = 25 \)
- Strain hardening exponent (n) : \( 0.307 < n < 0.325 \)
- Punch radius (r_p) : \( 6 < r_p < 14 \)
- Punch velocity (v_p) : \( 0.3577 < v_p < 0.802 \)
- Width of the sheet (w) : \( 30 < w < 70 \)

The constraints are identical for bend force and springback optimization

### 8.5.2 Determination of Optimum Parameter Combination

The optimum values 25 mm, 0.325, 14 mm, 0.3577 mm/s and 30 mm for a minimum bend force and 25 mm, 0.307, 14 mm, 0.3577 mm/s and
70 mm for minimum springback are obtained. Hence, the optimum combinations are not the same for the two response parameters. The Harrington’s desirability function is applied for simultaneous optimization of both the responses and the results are reported in Table 8.2.

Table 8.2 Concurrent Optimization and Desirability Function of Multiple Responses for RSM

<table>
<thead>
<tr>
<th>d</th>
<th>n</th>
<th>r_p</th>
<th>v_p</th>
<th>w</th>
<th>Bend force (N)</th>
<th>Springback (Degrees)</th>
<th>Composite Desirability</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.325</td>
<td>14</td>
<td>0.3577</td>
<td>30</td>
<td>446</td>
<td>9.7</td>
<td>0.999969</td>
</tr>
<tr>
<td>25</td>
<td>0.307</td>
<td>14</td>
<td>0.3577</td>
<td>70</td>
<td>628</td>
<td>3.8</td>
<td>0.988877</td>
</tr>
</tbody>
</table>

The composite desirability ‘D_c’ of the two responses has been computed for both the conditions and the larger value of these has been identified as the optimum operating combination of the levels. From Table 8.2, the first set of values (bend force), which yields higher desirability, has been found to be the optimal setting for minimizing both the bend force and the springback.

8.6 CONCLUDING REMARKS

Based on the experiments, a simultaneous optimization study was conducted and the following conclusions were drawn.

- Punch travel of 25 mm, n of 0.325, r_p of 14 mm, v_p of 0.3577 mm/s, w of 30 mm give the optimum results for both responses, springback and bend force using MRA/RSM models.
- The optimal is valid only within the specified range of the parameters.