CHAPTER 5

MIXED INTEGER LINEAR PROGRAMMING
FOR IRRIGATION RESERVOIR OPERATION

5.1 INTRODUCTION

The model discussed in this chapter deals with the determination of the optimal irrigation release for the reservoir under study. The main objective of this study is to determine whether significant improvements might be realized from optimisation of operation of the reservoir system. Alternative management strategies are considered for selecting the best viable irrigation release policy. A strategy for decision making under deficit supply is also considered, since full crop water demands cannot be met in all season due to scarcity of water resources.

Reservoirs are the most important elements of complex water resources systems. The important objective of the reservoirs designed for water supply is the flow augmentation. Thus the controlled reservoir releases are necessary with the following two conflicting targets: (i) Promotion of effective releases and (ii) Restriction of release in preparation for droughts. Optimal reservoir operation is a valuable concept that has been shown to be practical to implement. The results can be increased efficiency with reduced conflict in the management of tradeoffs among conflicting objectives. The importance of optimising the operation of existing dams has emerged worldwide. In reservoir operation models, generally optimisation models are used to get the high performance solutions. Linear programming is probably
the most flexible and most widely used technique for optimisation models. Applications of linear programming are extensive, and water resources problems have been solved with linear programming tools for many years (Yeh 1985, Wurbs 1993).

The model proposed in the present study is the mixed integer linear programming model (MILP) for obtaining the optimal releases for irrigation. MILP is used widely in decision making in i) piecewise linearisation of non-linear problems, ii) integer solutions instead of real values, iii) choosing alternatives and vi) incorporating IF-THEN type constraints in a linear programming model. MILP deals with linear programming in which some of the variables assume integer values. The usual way to solve MILP is by relaxing integer constraints to arrive at a solution using LP. Special constraints are introduced that will iteratively force extreme points of LP model towards desired integer restrictions. Although several algorithms have been developed for MILP none of these methods are totally reliable from computational stand point, when the number of integer variables are large (Taha 1992). Branch and bound algorithms are most successful to solve these problems. In this technique, the space of all feasible solutions repeatedly partitioned and each time a LP is solved.

As mentioned earlier, MILP is used for various types of formulation, and here used to ensure that the reservoir does not spill before reaching its capacity. The MILP suggested by Shin and ReVelle (1994) to accommodate spills in a linear programming model has been used in this study. The MILP model is solved for 24 years of historical data. The variation of crop water requirements due to the stochasticity of the hydrological parameters is not considered in the model.
5.2 MODEL FORMULATION

The most challenging aspect of mathematical programming is the ability to develop a concise and accurate model for a particular problem. Here the system under study is supplying water for drinking and irrigation purposes. A fixed amount of water is allocated for drinking purpose and the balance for irrigation. The objective function considered is to maximize the irrigation releases subject to the constraints of water availability, storage capacity etc. The details of the model objective function and constraints are discussed below.

5.2.1 Objective Function

The objective function of the model is maximisation of the monthly releases for irrigation and is given by

\[ \text{Max} \sum_i \sum_j IR_{i,j} \]  \hspace{1cm} (5.1)

where i represents year with total number of years being ‘n’, j represents month, \( IR_j \) is the monthly irrigation releases.

5.2.2 Constraints

5.2.2.1 Continuity constraints

The reservoir storage changes from one period to the next are governed by the continuity equation which is given by

\[ S_{i,j+1} = S_{i,j} + I_{i,j} - E_{i,j} - IR_j - DR_j - V_{i,j} \] \hspace{1cm} (5.2)

where \( S_{ij} \) is the beginning storage in the reservoir for the month j in the year i.
\( I_{i,j} \) is the inflow to the reservoir in the month \( j \) in the year \( i \)

\( E_{i,j} \) is the evaporation loss from the reservoir during the month \( j \) in the year \( i \)

\( DR_j \) is the drinking water supply from the reservoir for the month \( j \)

\( V_{i,j} \) the spill from the reservoir during the month \( j \) in the year \( i \)

The evaporation loss from the reservoir depends on the storage volume in that period and according to Loucks et al (1981) it depends on the storage at the beginning and end of the period. Hence \( E_{i,j} \) may be approximated as

\[
E_{i,j} = A_0 e_j + A_a e_j \left( \frac{S_{i,j} + S_{i,j+1}}{2} \right)
\]

(5.3)

where \( A_0 \) is the water spread area corresponding to the dead storage volume,

\( A_a \) is the water spread area per unit of active storage above \( A_0 \).

\( e_j \) is the evaporation rate in month \( j \).

Taking \( \frac{A_a e_j}{2} \) as \( a_j \) and combining equations (5.2) and (5.3) and rearranging yields,

\[
\left( 1 + a_j \right) S_{i,j+1} = \left( 1 - a_j \right) S_{i,j} + I_{i,j} - IR_j - DR_j - V_{i,j} - A_0 e_j
\]

(5.4)

5.2.2.2 Maximum and minimum storage constraints

The following maximum and minimum constraints need to be satisfied by the reservoir
where \( S_{\text{max}} \) is the maximum storage volume of the reservoir,

\( S_{\text{min}} \) is the minimum storage volume of the reservoir.

### 5.2.2.3 Irrigation release constraint

The irrigation releases are to be either less than or equal to the irrigation demand.

\[
IR_j \leq D_j
\]

(5.6)

where \( D_j \) is the irrigation demand in the month \( j \).

In the above LP formulation, spill from the reservoir is accounted in the continuity equation (5.2), in which \( V_{i,j} \), the spill, is unbounded and \( S_{i,j} \), the beginning storage, is bounded between \( S_{\text{max}} \) and \( S_{\text{min}} \). Though correct solutions are obtained for \( IR_j \) by using equation (5.2), the resulting final storage value and spill are incorrect. Because, equation (5.2) tends to spill water, even when the reservoir is not full and it does not make any sense to spill water at such a time (Shin and ReVelle 1994). Penalties were applied for spill in the objective function so that the reservoir spills only when the storage exceeds storage capacity. Values to be used for such penalties and interpreting the final solution is always debated. Mixed integer approach was recommended by Shin and ReVelle (1994) to employ this IF-THEN condition, and the following constraint for proper accounting of spill was suggested.

\[
c_{i,j} \leq S_{i,j+1}/S_{\text{max}}
\]

(5.7)
\[ V_{i,j} \leq c_{i,j} \times B \quad \text{(5.8)} \]

\[ c_{i,j} = 0 \quad \text{or} \quad 1 \quad \text{(5.9)} \]

where \( B \) is a very large number.

The above formulated model is solved using LINDO 6.0 window version for 24 years of historical data.

### 5.3 MODEL APPLICATION

Alternative management strategies are considered for selecting the best viable irrigation release policy. In the command area three crops are generally grown. As per the original project report two crops are proposed to be raised by using the water from the reservoir, and third crop is grown using subsurface water. Reviewing the past records it can be seen that now there is no release for first crop and this causes the delay of second crop and ultimately no third crop. Over and above during high flow months more spill is experienced. Also during the drought year the release is less than 25% of the demand. Hence an effective management policy is required. An effective management policy must be able to establish a target value in reservoirs for releasing water to the priority demand even in occurrences of water scarcity. According to Wason Jompakdee (2006), during water shortage situation, for sustainable management of water it is better to go for deficit irrigation by supplying less water in non-critical growth period and maximum water during stress sensitive periods. Many researchers in the field of paddy cultivation also pointed out the above fact for getting more yield from the paddy
(Wu 1998, Li 2001, Printz and Malik 2004). By considering all the above factors, the MILP model is run for the following five different management strategies.

Management strategy 1: Supplying water for first crop and second crop with equal weightage.

Management strategy 2: Supplying water for first crop and second crop with more weightage for second crop.

Management strategy 3: Supplying water for first crop and second crop with more weightage for the critical period of the second crop.

Management strategy 4: Supplying water for first crop and second crop with more weightage for the critical period of the second crop and also ensuring a minimum of 70% of the demand during that period.

Management strategy 5: Supplying water for second crop only.

In Chapter 4 it is already mentioned that for first crop the water requirement is only for land preparation and transplantation but the second crop fully depends on irrigation water. During the second crop the farmers are practicing the method of transplantation. The growth period of rice after transplantation can be divided into 1) transplant recovery, 2) early tillering, 3) late tillering, 4) heading and flowering, 5) grain filling and 6) maturity. According to Li (2001) and Li and Baker (2004), the filled grain number will be evidently reduced, when rice suffers serious drought during the heading and flowering stage, and thus the reduction in rice yield. Thus the critical period taken in strategy 3 and strategy 4 is the period during heading and flowering stage. For the second crop the transplantation usually starts at the
third week of October. Hence the critical period for the system is during the month December and January, which is shown in the Figure 5.1.

![Figure 5.1 Critical period for the second crop](image)

5.4 COMPARISON OF DIFFERENT STRATEGIES

The results obtained from the MILP model for each strategy for the 24 years of historical data are evaluated using the following performance indicators.

As the system is mainly for the second crop, all the above performance indicators are done for the second crop. To test the model performance for the first crop another performance indicator, the number of years failed to supply more than 70% of the required demand for first crop (%Years) is calculated. To check the overall performance total spill is also determined.

All the performance indicators for each strategy are shown in Table 5.1. For a comparison the performance indicators for the historically adopted policy is also calculated using historical release data, which is also
given in Table 5.1. The optimum reservoir releases obtained for each crop from the MILP model for different management strategies are shown in Figure 5.2(a) to 5.2(f). The graph between % demand met and cumulative time in % for different strategies are shown in Figure 5.3.

From Table 5.1 it can be seen that, all the five strategies adopted newly are better than the operational procedure being practised in the system (historically adopted) as their reliabilities are more. The performance of the strategy 5 is poor because the spill is considerably high over and above the drawback of failure to supply of water for first crop.

Now among the other four strategies, for strategy 4 spill is less. Also, as observed from the comparison of Figure 5.2(a) to 5.2(f), strategy 4 can supply water for more number of years for first crop without much affecting the second crop. Comparing the four strategies, the strategy 4 gives less system reliability (Table 5.1). This supply reduction aims to minimise the possible draught impacts (i.e. a maximum supply during draught period as shown in Figure 5.3) on the system. It is also reflected from the maximum seasonal deficit of 49.8% as given in Table 5.1. Moreover, from Figure 5.3 it can be seen that the percentage demand met is comparatively uniform for strategy 4.

Thus, with acceptable system reliability strategy 4 ensures minimum spill and supplying water for first crop for maximum number of years. It also ensures a minimum of 70% of demand in the critical growth period of the second crop even in low flow years.

Hence the management strategy 4, which ensures more reliable water supply in the critical period of the plant growth during drought year too, is the best alternative management policy for the system considered. From
Table 5.1 also it is clear that the strategy 4 have resulted in better performance than the present operational procedure being practiced in the system in all aspects.

**Table 5.1 Performance indicators**

<table>
<thead>
<tr>
<th>Management Strategy</th>
<th>For second crop</th>
<th>Reliability</th>
<th>Maximum seasonal deficit (%)</th>
<th>Mean value of deficit (Mm$^3$/ha)</th>
<th>No of years failed to supply for first crop in %</th>
<th>Spill in Mm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.87</td>
<td>70.68</td>
<td>7.05×10$^{-4}$</td>
<td>58.33</td>
<td>54.16</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.92</td>
<td>70.68</td>
<td>4.96×10$^{-4}$</td>
<td>70.83</td>
<td>35.26</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.92</td>
<td>67.23</td>
<td>7.95×10$^{-4}$</td>
<td>66.66</td>
<td>35.26</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0.83</td>
<td>49.8</td>
<td>11.04×10$^{-4}$</td>
<td>33.33</td>
<td>34.29</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0.92</td>
<td>70.68</td>
<td>4.96×10$^{-4}$</td>
<td>100</td>
<td>240.17</td>
</tr>
<tr>
<td>Historically adopted</td>
<td></td>
<td>0.63</td>
<td>75</td>
<td>13.81×10$^{-4}$</td>
<td>75</td>
<td>128</td>
</tr>
</tbody>
</table>

Figure 5.2 (a) Releases obtained for Strategy 1
Figure 5.2 (b) Releases obtained for Strategy 2

Figure 5.2 (c) Releases obtained for Strategy 3
Figure 5.2 (d) Releases obtained for Strategy 4

Figure 5.2 (e) Releases obtained for Strategy 5
Figure 5.2 (f) Historical releases

Figure 5.3 Comparison of different strategy releases
5.5 CONCLUSION

Using a mixed integer linear programming based optimisation model, scope of the improvement of the performances of an existing reservoir system has been investigated. The result indicates that a management strategy with deficit irrigation by supplying less water in non-critical growth period and maximum water during stress sensitive periods is the best viable solution for better performance. By analysing the performance indicator and historical release pattern, it can be concluded that the existing reservoir operation practices can be improved through scientific allocation of the available water to yield improved performance.