CHAPTER 3

PARAMETERISED $H_{\infty}$ LOOP SHAPING DESIGN FOR A NONMINIMUM PHASE SWITCHING SYSTEM

3.1 INTRODUCTION

Modern robust techniques have not reached wider acceptance in areas of power electronics control design. This is because of the common belief that they involve complicated and intensive high-level mathematics with vague guidelines and background theory unfamiliar. It is intended to show in this chapter the straightforward way of posing a control problem in a robust framework. The procedure provides the robust parameter tuning of nonminimum phase switching converters and reduced order controllers. The procedure is applied to voltage mode controlled DC/DC converters, conventionally modeled as presented in the previous chapter.

In order to make a device insensitive to the component tolerances of power electronics and uncertainties on both the supply and consumer side, the concepts of robust control (Ojo 1995, Naim et al 1997, Doyle et al 1989, McFarlane and Glover 1992) are adopted. These references discussed widely on the nominal model of the plant, structure and bounds of uncertainty, robust (“uncertainty resistive”) performance, robust stability, their measurement, and procedures that optimize control with respect to these values. Though $H_{\infty}$ control has been used in the literature as an optimal solution to achieve robustness issues in DC/DC converters, the selection of weighting functions in $H_{\infty}$ control design is non-trivial and invariably incorporates an iterative
procedure where the weights are modified in the case of the resulting system failing to meet design specifications. Overcoming this difficulty the $H_\infty$ loop shaping design combines classical loop shaping ideas with $H_\infty$ robust stabilization. Suggestions on the selection of the weighting functions to shape the singular values of the nominal model is explicitly presented in McFarlane and Glover (1990). Motivated by the literature, the application of $H_\infty$ loop shaping control approach for the voltage regulation of the nonminimum phase power converter is presented in this chapter to propose single parameter based tuning of $H_\infty$ controllers. The performance and stability specifications of the DC/DC converter control system are clearly stated, $H_\infty$-optimal synthesis is carried out and robust controllers are obtained. The proposed single parameter based tuning of $H_\infty$ controller is simple, systematic and assumes the design specifications such as tracking performance, bandwidth and robustness in the modelling of uncertainty as constraints on the gain responses of the closed-loop transfer functions. The advantage of the proposed single parameter based loop shaping controller not only retains the advantages of the existing $H_\infty$ control strategy but also affords an additional tuning parameter which reflects a trade-off between stability robustness and time domain performance. It can also be used to modify the output response. Transient behaviour and robustness to uncertainties of closed loop systems are tested for the robust controllers by means of simulation using a nonlinear model for the DC/DC converter that includes switching effects.

3.2 PROBLEM STATEMENT

The nominal converter model for $H_\infty$ controller in Figure 3.1 is given in Equation (3.1) as transfer function from the control variable to the output voltage where $\hat{v}_{in}$ is the plant input disturbance signal, $\hat{v}_0$ is the output voltage disturbance signal, $\hat{v}_{ref}$ is the reference disturbance signal, $\hat{r}_i$ is the
perturbation in the load and \( e = v_o - v_{0\text{ref}} \) is the error in reference tracking. \( V_{in}, V_0, V_{0\text{ref}} \) and \( R_L \) are nominal values of input, output, reference voltage and load resistance respectively. \( v_o \) and \( v_{0\text{ref}} \) are the instantaneous values of output voltage and reference voltage.

![Schematic diagram of \( H_\infty \) control of boost converter](image)

**Figure 3.1 Schematic diagram of \( H_\infty \) control of boost converter**

Here \( G(s) \) represents the plant, \( K_\infty(s) \) is the \( H_\infty \) controller.

\[
G(s) = \frac{v_{0\text{ref}}(s)}{d(s)} = \frac{k_{dc}(s + z_1)(s - z_2)}{s^2 + 2\delta\omega_n s + \omega_n^2}
\]  

(3.1)

\( \delta \) and \( \omega_n \) are damping ratio and natural frequency of the converter. \( z_1 \) and \( z_2 \) are the zeros. \( k_{dc} \) the DC gain of the converter.

The first step towards design of \( H_\infty \) loop shaping controller is to choose appropriate pre and post compensator.

Assuming that \( x \in \mathbb{R}^n \) is the state vector of the shaped plant, \( u(t) \in \mathbb{R}^m \) is the control input and \( y(t) \in \mathbb{R}^p \) is the controller input, the augmented
plant $\hat{G}$ as suggested by McFarlane and Glover (1992) displayed in Figure 2.3 in the previous chapter can be described by equations of the form

$$\begin{align*}
\dot{x} &= A_s x + B_s u \\
y &= C_s x + D_s u \\
u &= K_\infty e
\end{align*}$$  \tag{3.2}

where $A_s$ is the state matrix, $B_s$ is the input matrix, $C_s$ is the output matrix and $D_s$ is the transmission matrix of the shaped plant.

The objective is to find a controller $K_\infty$ such that

- the spectral radius of their product is less than $\varepsilon_{\text{max}}^{-2}$.
- the performance and the stability robustness trade-off are ensured through the choice of a single design parameter say $\lambda$.

### 3.2.1 Choice of Pre-and/or Post Compensators

Using pre-and/or post compensators $W_1$ and $W_2$, the singular values of the original plant $G$ are shaped to give the compensated plant $\hat{G} = W_2GW_1$ a desired open-loop shape. This step contains all of the ingredients of the classical techniques. The shaping functions $W_1$ and $W_2$ are controlled by the designer and the properties of the resulting controller depend upon these functions. The choice of pre and post compensators are made in such a way that a single design parameter $\lambda$ is left for the designer to achieve the required performance. The shaped plant contains no hidden unstable modes, and by using Equation (3.3) a compromise between performance at low frequencies and robust stability at high frequencies are obtained.
\[ \sigma (\tilde{G}(\omega)) \gg 1 \text{ for low frequency and } \]
\[ \bar{\sigma} (\tilde{G}(\omega)) \ll 1 \text{ for high frequency} \]  

(3.3)

where \( \sigma \) is the minimum singular value, \( \bar{\sigma} \) is the maximum singular value and \( \tilde{G}(\omega) \) is the shaped plant model.

All the frequencies are of interest at the regulated output of the converter. So the post-plant weighting function \( W_2 \) is chosen as a constant. The pre-plant weighting function exhibits dynamic shaping. Attenuation at low frequencies, reduction in bandwidth and phase advance to reduce the roll-off rates at crossover are all dependent on \( W_1 \). For a good shape the choice of \( W_1 \) could be a model of a lead compensator. The converter transfer function should not contain any unstable modes. In general if \( G \) contains any unstable modes the loop shaping weights should be chosen not to hide those modes in \( \tilde{G} \). For the chosen converter problem the pre compensator \( W_1 \) and \( W_2 \) the post compensator structure are as follows:

\[
W_1 = \frac{\lambda(s + \delta W_n^n)}{(s + z_1^1)} \text{ and } W_2 = 1 
\]  

(3.4)

In the proposed design, the pre compensator zero location is placed sufficiently close to the real part of the converter poles, so that it cancels the left half nominal plant pole and introduces the single design parameter \( \lambda \) to reflect the trade-off between stability robustness and time domain performance.
3.2.2 Robust Stabilisation

For the shaped plant \( \tilde{G} \), the minimal state space realization is obtained as follows:

\[
A_s = \begin{bmatrix} 0 & 1 \\ -w_n^2 & 0 \end{bmatrix}, \quad B_s = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\]

\[
C_s = \begin{bmatrix} -\lambda k_{dc} \delta w_n z_2 \\ \lambda k_{dc} (\delta w_n - z_2) \end{bmatrix}, \quad D_s = [\lambda k_{dc}]
\]

(3.5)

The design indicator \( \varepsilon_{\text{max}} \) is evaluated using the stabilizing solutions of the two algebraic Riccati equations, \( X \) and \( Y \) as given by McFarlane and Glover (1992).

\[
\varepsilon_{\text{max}} = \left(1 + \rho_{\text{max}} (YX)^{1/2}\right)^{-1/2}
\]

(3.6)

\[
(A_s - B_s N^{-1} D_s^T C_s)^T X + X (A_s - B_s N^{-1} D_s^T C_s) - XB_s N^{-1} B_s^T X + C_s^T R^{-1} C_s = 0
\]

(3.7)

\[
(A_s - B_s D_s^T R^{-1} C_s)^T Y + Y (A_s - B_s D_s^T R^{-1} C_s)^T - Y C_s^T R^{-1} C_s Y + B_s N^{-1} B_s^T = 0
\]

(3.8)

where \( R = I + D_s D_s^T \) and \( N = I + D_s^T D_s \).

An optimal \( H_\infty \) controller is constructed using the following generalized state space description restricting the robustness index to lie between 0.3 and 1.
\[
\begin{align*}
Q\dot{x} &= (Q(A_s - B_s N^{-1} B_s^T X - B_s N^{-1} D_s^T C_s) - \varepsilon_{\max}^{-2} Y C_s^T C_s) x + \varepsilon_{\max}^{-2} Y C_s^T y \\
u &= -B_s^T X x
\end{align*}
\]

(3.9)

where \( Q = \left( I - \varepsilon_{\max}^{-2} Y X \right) \)

The structure of \( Q \) matrix is such that some of its eigenvalues are non zero. A reduced order realization is obtained based on Singular Value Decomposition (SVD).

### 3.2.3 SVD Based Compact Control Structure

Let the matrix \( Q \) which is close to singular, be applied with SVD as given by Equation (3.10).

\[
Q = USV^T = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^T
\]

(3.10)

\( U \) and \( V \) are unitary matrices and \( S \) is a diagonal matrix with positive diagonal elements arranged in the descending order. \( \Sigma \) is a diagonal matrix having dimension equal to rank of \( Q \). The \( H_\infty \) controller defined in Equation (3.9) is now modified as follows.

\[
\begin{align*}
USV^T \dot{x} &= (USV^T (A_s - B_s N^{-1} B_s^T X - B_s N^{-1} D_s^T C_s) - \varepsilon_{\max}^{-2} Y C_s^T C_s) x + \varepsilon_{\max}^{-2} Y C_s^T y \\
u &= -B_s^T X x
\end{align*}
\]

(3.11)
Upon replacing $V^T x$ by $z$, Equation (3.11) reduces to

$$\begin{align*}
US\dot{z} &= (USV^T (A_s - B_s N^{-1} B_s^T X - B_s N^{-1} D_s^T C_s) - \epsilon_{max}^{-2} YC_s^T C_s)Wz + \epsilon_{max}^{-2} YC_s^T y \\
u &= -B_s^T XV z
\end{align*}$$

(3.12)

Premultiplying both sides of Equation (3.12) by $U^T$ and partitioning $z$ into $z_1$ and $z_2$, Equations (3.13) and (3.14) are obtained.

$$\begin{align*}
S\dot{z} &= [(SV^T (A_s - B_s N^{-1} B_s^T X - B_s N^{-1} D_s^T C_s) - U^T \epsilon_{max}^{-2} YC_s^T C_s)W]z + [U^T \epsilon_{max}^{-2} YC_s^T ]y \\
u &= -B_s^T XV z
\end{align*}$$

(3.13)

$$\begin{align*}
\dot{z}_1 &= A_k z_1 + B_k y \\
u &= C_k z_1 - D_k y
\end{align*}$$

(3.14)

where

$$A_k = \begin{bmatrix}
SM_{11} - SM_{12} SM_{22}^{-1} SM_{21} \\
\Sigma
\end{bmatrix}, \quad B_k = \begin{bmatrix}
IM_{11} - SM_{12} SM_{22}^{-1} IM_{21} \\
\Sigma
\end{bmatrix},$$

$$C_k = \begin{bmatrix}
OM_{11} - OM_{12} SM_{22}^{-1} SM_{21} \\
\Sigma
\end{bmatrix}, \quad D_k = -\begin{bmatrix}
OM_{12} SM_{22}^{-1} IM_{21}
\end{bmatrix}$$

and

$$\begin{bmatrix}
SM_{11} & SM_{12} \\
SM_{21} & SM_{22}
\end{bmatrix} = [(SV^T (A_s - B_s N^{-1} B_s^T X - B_s N^{-1} D_s^T C_s) - U^T \epsilon_{max}^{-2} YC_s^T C_s)W],$$

$$\begin{bmatrix}
IM_{11} \\
IM_{21}
\end{bmatrix} = [U^T \epsilon_{max}^{-2} YC_s^T ], \quad [OM_{11} \quad OM_{12}] = [-B_s^T XV]$$
The $H_\infty$ Controller can now be obtained as

$$K(s) = \frac{U(s)}{Y(s)} = \begin{bmatrix} C_k (sI - A_k)^{-1} B_k + D_k \end{bmatrix}$$  \hspace{1cm} (3.15)

### 3.2.4 The Modified Final $H_\infty$ Controller Structure

Following the discussion in the previous section, the precompensator, $W_1$ is chosen with an initial value of $\lambda$. The minimal state space realization for the shaped plant is computed and the unique stabilizing solution to the two Riccati equation is obtained and the $\varepsilon_{\max}$ condition is checked. Using SVD the compact controller structure is obtained as given by Equation (3.15).

For the given Boost converter problem, the $H_\infty$ controller given by Equation (3.15) is found to be having the following structure:

$$K(s) = \frac{K_c (s + z)}{s + p}$$  \hspace{1cm} (3.16)

where $K_c$ is the DC gain, $z$ and $p$ are the zero and pole of the $H_\infty$ controller.

For different values of $\lambda$, the design procedure is repeated to obtain $K_c$, $z$ and $p$ satisfying the robust margin $\varepsilon_{\max}$. Using proper curve fitting technique, the parameter of the $H_\infty$ controller are identified as follows:
This derivation is useful to fix \( \lambda \) satisfying the design requirement.

The modified final feedback controller \( K_\infty \) is constructed by combining the \( H_\infty \) controller \( K \) with the shaping functions \( W_1 \) and \( W_2 \) such that \( K_\infty = W_1 K W_2 \)

\[
K_\infty(s) = \frac{\lambda(s + \delta w_n)}{(s + z_1)} \frac{K_c(\lambda)(s + z(\lambda))}{s + p(\lambda)}
\]  

The validity of the proposed \( H_\infty \) design technique is demonstrated through a numerical example in the following section.

### 3.3 DESIGN EXAMPLE

In the design example, the parameters for boost converter are chosen as follows: \( V_{in} = 12 \) V, \( V_0 = 24 \) V, \( L = 200 \) \( \mu \)H, \( C = 220 \) \( \mu \)F, \( R_L = 44 \) \( \Omega \), \( r_L = 0.7 \) \( \Omega \), \( r_c = 0.1 \) \( \Omega \), \( r_d = 0.1 \) \( \Omega \), \( r_s = 0.1 \) \( \Omega \), and \( D = 0.54 \). The switching frequency is chosen as 240 kHz. For the above set of parameters, the boost converter depicted in Figure 3.1 has the following control input to the output voltage nominal transfer function. The presence of right-half plane zero shows the nonminimum phase behaviour of the converter.

\[
G(s) = \frac{\hat{v}_0}{d}(s) = \frac{-0.118(s + 45460)(s - 42420)}{s^2 + 4311s + 5.2 \times 10^6}
\]
Inspection of the open-loop gain plot of the nominal plant implies that considerable improvement is required in the performance characteristics, especially amplification at low frequencies, attenuation at high frequencies and improvement in the closed-loop bandwidth as shown in Figure 3.2. For the averaged and linearized plant model Equation (3.19), the following pre and post compensator are chosen.

$$W_1 = \frac{\lambda(s + 4120)}{(s + 45460)}, \quad W_2 = I \quad (3.20)$$

where $\lambda$ is a design parameter. The weights are chosen such that no unstable hidden modes are created in shaped plant $\tilde{G}(s)$. Figure 3.2 shows the plant model is well shaped. The optimum $\epsilon_{max}$ in Equation (3.6) depends only on $\lambda$.

![Figure 3.2](image)

**Figure 3.2** The gain values of the nominal plant and the shaped plant model
Now the transfer function of the shaped plant is:

\[
\tilde{G}(s) = \frac{-0.118\lambda(s + 4120)(s - 42420)}{s^2 + 4311s + 5.2 \cdot 10^6}
\]  

(3.21)

3.3.1 Algorithm of the \( H_\infty \) Tuning

The algorithm of the tuning procedure may be summarized as follows:

1. Choose an initial value of \( \lambda \) as 1 and obtain the minimal state space realization for the shaped plant Equation (3.21).

2. The unique stabilizing solution to Equations (3.7) and (3.8) are then obtained and are used in Equation (3.6) to compute \( \varepsilon_{\text{max}} \).

3. Verify if \( \varepsilon_{\text{max}} \) computed in the previous step, fulfills the prescribed conditions, i.e. between 0.3 and 1 which ensures the closed-loop system to have good, robust stability. If this is true go to next step, otherwise increment or decrement \( \lambda \) suitably and repeat steps (1), (2) and (3) till the condition on \( \varepsilon_{\text{max}} \) is satisfying.

4. Solve Equation (3.15) to obtain the compact control structure as given in Equation (3.16).

5. Repeat the above steps by incrementing \( \lambda \) and recalculate the controller parameters \( K_c, z \) and \( p \). Using proper curve fitting technique, the controller parameters \( K_c, z \) and \( p \) are tabulated as \( \lambda \) dependent functions in Table 3.1.
Finally the $H_\infty$ controller $K_\infty$ is cascaded with the shaping functions $W_1$ and $W_2$ to obtain the modified controller as follows:

$$K_\infty(s) = \frac{K_c(s + z) \lambda(s + 4120)}{s + p(s + 45460)}$$

(3.22)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Function of $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{\text{max}}(\lambda)$</td>
<td>$0.0022\lambda^2 - 0.055\lambda + 0.8357$</td>
</tr>
<tr>
<td>$K_c(\lambda)$</td>
<td>$0.0038\lambda^2 - 0.0415\lambda + 0.8728$</td>
</tr>
<tr>
<td>$z(\lambda)$</td>
<td>$14.416\lambda^2 - 188.66\lambda + 3331$</td>
</tr>
<tr>
<td>$p(\lambda)$</td>
<td>$11.645\lambda^2 - 158.56\lambda + 3420$</td>
</tr>
</tbody>
</table>

3.4 PERFORMANCE ANALYSIS

The block diagram of $H_\infty$ control of a boost converter is depicted in Figure 3.1. The performance of the closed-loop system with the proposed controller described above has been examined by MATLAB 7.0.1 and SIMULINK tool set. The simulation step is set as $1\mu$s for the system. The process output responses for four typical $\lambda$ settings are shown in Figure 3.3. It is found that as $\lambda$ becomes larger, the response time for the closed-loop system becomes shorter. However, the overshoot becomes larger, and the robust margin $\varepsilon_{\text{max}}$ becomes smaller, which means that the robust stability of the closed loop system becomes worse. To obtain a fast setpoint response $\lambda$ can be chosen larger ($\lambda = 3.5$ with $\varepsilon_{\text{max}} = 0.668$), while to be robust against a greater variation of plant parameters $\lambda$ can be chosen smaller ($\lambda = 0.5$ with
Thus the trade-off between time domain performance and stability robustness can be directly reflected through the choice of design parameter $\lambda$. Here for further analysis, $\lambda$ is chosen as 0.5 with $\epsilon_{\text{max}} = 0.81$.

Figure 3.3 Output voltage for different $\lambda$ settings

The performance of the controller is compared with a traditional voltage mode Equation (3.23) and Naim’s controller (Naim et al 1997) both in time and frequency domain. Applying standard design techniques (Pressman 1991), the transfer function of a simple VM controller is obtained as follows:

$$C(s) = \frac{-K \left( s + \frac{1 - D}{3\sqrt{LC}} \right)^2}{s \left( s + \frac{1}{rC} \right) \left( s + \frac{1}{rc} \right)}$$

This control strategy is derived following two basic guidelines:

(1) to make the low-frequency open-loop gain as large as possible and
(2) to obtain a loop gain of less than one at a frequency, which is about half a decade lower than the frequency of the RHPZ.

For the above comparison, the same converter configuration of Figure 3.1 is considered with the $H_{\infty}$ controller, $K_{\infty}(s)$ being replaced by simple voltage mode Equation (3.23) and Naim’s controller (Naim et al 1997). The output voltage response for the VM (Pressman 1991), Naim’s controller (Naim et al 1997) and the proposed controller ($\lambda = 0.5$ and $\varepsilon_{\text{max}} = 0.81$) are shown in Figure 3.4. It is concluded that the proposed controller gives better performance compared to VM controller (Pressman 1991) in providing fast transient performance and Naim’s controller (Naim et al 1997) in terms of overall performance.

![Figure 3.4](image.png)

**Figure 3.4** Output voltage response for VM (Pressman 1991), Naim’s controller (Naim et al 1997) and Proposed ($\lambda = 0.5$) controllers
3.4.1 Steady-state Performance

A conventional non-regulated power supply using a full bridge diode rectifier with a capacitor filter is used as a voltage source. The voltage provided by this source is polluted by a second (100 Hz), third (150 Hz), fourth (200 Hz), fifth (250 Hz), sixth (300 Hz) and seventh (350 Hz) harmonics of amplitude 1V. The peak-peak amplitude of the ripples in the output voltage of the closed-loop system is tabulated for the controllers mentioned above in Table 3.2. It is observed that the distortion in the output voltage is considerably less compared to voltage mode controller (Pressman 1991). That is, the $H_\infty$ controller is capable of attenuating the harmonic voltage disturbances present in the power supply. Also Table 3.2 shows that, the proposed controller with $\lambda$ values chosen as 0.5 and 1 outperforms the other controllers especially when the voltage provided by the source is polluted mainly by a second harmonic, i.e. at $6.28.10^2$ rad/sec. Here $\omega$ is in rad/sec.

Table 3.2 Harmonic attenuation property: Peak-peak amplitude of the ripple in the output voltage when a sinusoidal perturbation is introduced on nominal operating voltage

<table>
<thead>
<tr>
<th>Type</th>
<th>Peak-Peak Amplitude of the Ripple Input Disturbance Attenuation ($v_{in} = 12 + 1\sin\omega t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\omega=6.28.10^2$</td>
</tr>
<tr>
<td>Proposed</td>
<td>$\lambda$</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
</tr>
<tr>
<td>VM (Pressman 1991)</td>
<td>0.4</td>
</tr>
<tr>
<td>Naim (Naim et al 1997)</td>
<td>0.2</td>
</tr>
</tbody>
</table>
A tabulation of the data in terms of the load and line regulation properties is also given in Tables 3.3 and 3.4, respectively. Table 3.3 shows percentage changes in the steady state value of output voltage for three different input voltage conditions (11 V, 12 V and 10 V) in case of load variations. Minimum load is taken as 440 Ω which is ten times the full load. In Table 3.4, percentage changes in the steady state value of output voltage is tabulated for minimum (440 Ω), half (88 Ω) and full load (44 Ω) conditions in case of input voltage variations from 11 V to 13 V. According to Table 3.3, the maximum load-regulation error is restricted to 1% of $v_o$ (nominal condition). Similarly it can be found from Table 3.4 that the maximum line regulation error occurs at full load $R_L = 44 \, \Omega$, with a deviation of 0.54 % of $v_o$ (nominal condition) which is much lower compared to Naim’s controller.

Table 3.3 Load regulation property: Output voltage deviation at steadystate for different input voltage conditions in case of load variations

<table>
<thead>
<tr>
<th>Input Voltage</th>
<th>Voltage Deviation: $\Delta v_o = v_{o(440 , \Omega)} - v_{o(44 , \Omega)}$</th>
<th>Percentage change: $\frac{\Delta v_o}{v_{o(nominal , condition)}} \times 100%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VM (Pressman 1991)</td>
<td>Naim (Naim et al 1997)</td>
</tr>
<tr>
<td>$v_i = 11 , V$</td>
<td>0.02 V</td>
<td>0.15 V</td>
</tr>
<tr>
<td></td>
<td>0.08 %</td>
<td>0.62 %</td>
</tr>
<tr>
<td>$v_i = 12 , V$</td>
<td>0.003 V</td>
<td>0.064 V</td>
</tr>
<tr>
<td></td>
<td>0.012 %</td>
<td>0.265 %</td>
</tr>
<tr>
<td>$v_i = 13 , V$</td>
<td>0.01 V</td>
<td>0.09 V</td>
</tr>
<tr>
<td></td>
<td>0.004 %</td>
<td>0.37 %</td>
</tr>
</tbody>
</table>
Table 3.4  Line regulation property: Output voltage deviation at
steady-state for different loading conditions in case of input
voltage variations

<table>
<thead>
<tr>
<th>Loading Condition</th>
<th>Voltage Deviation: ( \Delta V_0 = V_{0(v_i=11 \text{ V})} - V_{0(v_i=13 \text{ V})} )</th>
<th>Percentage change: ( \frac{\Delta V_0}{V_{0(nominal \ condition)}} \times 100% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VM (Pressman 1991)</td>
<td>Naim (Naim et al 1997)</td>
<td>Proposed ((\lambda=0.5))</td>
</tr>
<tr>
<td>Minimum load (440 (\Omega))</td>
<td>0.001 V</td>
<td>0.15 V</td>
</tr>
<tr>
<td>Half load (88 (\Omega))</td>
<td>0.005 V</td>
<td>0.19 V</td>
</tr>
<tr>
<td>Full load (44 (\Omega))</td>
<td>0.02 V</td>
<td>0.5 V</td>
</tr>
</tbody>
</table>

3.4.2  Transient Performance

Figures 3.5, 3.6 and 3.7 show the behaviours of output voltage for
the three controllers in the case of reference voltage step changes from 24 to
25 V as start up performance at 10 ms and from 25 to 24 V at 15 ms.

1. Figure 3.5 shows the system response with voltage mode
controller (Pressman 1991). During start up, the output
voltage overshoot is close to 0.2 V with about 5 ms settling
time.

2. Figure 3.6 shows the system response with Naim’s controller
(Naim et al 1997). During start up, the output voltage
overshoot is close to 0.4 V with 2 ms settling time. When
reference voltage changes down to 24 V, the response is
slower because of the discharge time for the capacitor C and
the overshoot of output voltage is large.
3. Figure 3.7 shows the system response with the proposed loop shaping controller with $\lambda = 0.5$. When reference voltage increases to 25 V, output voltage increases fast and with smaller overshoot to the desired value. When the reference voltage changes down to 24 V, the response is better than the other controllers.

The undershoot is a typical characteristics of the converter which is dominant under fast transient performance. The VM controller (Pressman 1991) response chooses a very limited bandwidth and is able to control the undershoot limiting the fast transient performance.

Table 3.5 lists the transient specifications in terms of percentage overshoot ($M_p$), Peak time ($t_p$) and settling time ($t_s$) at nominal operating condition $v_i = 12\,\text{V}$ and $R_L = 44\,\Omega$ for the three controllers.

![Figure 3.5 Voltage mode controllers (Pressman 1991) performance under step changes of reference voltage](image)
Figure 3.6 Output voltage waveform for step change of reference voltage with Naim’s controller (Naim et al 1997)

Figure 3.7 Output voltage waveform for step change of reference voltage with the proposed controller ($\lambda = 0.5$)
3.4.3 Robust Performance

Table 3.6 shows robustness property of the three controllers against parameter variations. The parameters of the converter, used for studies on robust performance given in Table 3.6 correspond to Figure 3.1. Without changing the controller parameters, the transient specifications in terms of percentage overshoot ($M_p$), Peak time ($t_p$) and settling time ($t_s$) of output voltage are investigated for the three designs and presented in Table 3.6. From the comparisons, it is concluded that the proposed controller maintain a good performance with reduced overshoot and oscillations against parameter variations and is superior to Naim’s controller (Naim et al 1997), whereas the performance of the traditional VM controller (Pressman 1991) exhibits a strong degradation in terms of its sluggish nature. It is important to notice the limitation that a larger settling time of the response is seen for VM controller (Pressman 1991) from Figure 3.4 and Table 3.6.

Table 3.5 Transient specifications of output voltage at nominal operating condition $V_i = 12V$ and $R_L = 44 \Omega$

<table>
<thead>
<tr>
<th>Nominal Input Voltage</th>
<th>Naim (Naim et al 1997)</th>
<th>VM (Pressman 1991)</th>
<th>Proposed ($\lambda = 0.5$ and $\epsilon_{\text{max}} = 0.81$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_p$ (%) $t_p$ (msec) $t_s$ (msec)</td>
<td>$M_p$ (%) $t_p$ (msec) $t_s$ (msec)</td>
<td>$M_p$ (%) $t_p$ (msec) $t_s$ (msec)</td>
</tr>
<tr>
<td>12 V</td>
<td>6.03 1 6</td>
<td>4.17 1 7</td>
<td>2.3 1 1</td>
</tr>
</tbody>
</table>
Table 3.6  Robustness property: Transient specifications of output voltage for parameter variations at nominal operating condition $V_i = 12V$ and $R_L = 44 \ \Omega$, $L = 200 \ \mu \text{H}$, $C = 220 \ \mu \text{F}$, $r_1 = 0.2 \Omega$, $r_c = 0.1 \Omega$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Naim (Naim et al 1997)</th>
<th>VM (Pressman 1991)</th>
<th>Proposed ($\lambda = 0.5$ and $\epsilon_{max} = 0.81$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mp (%)</td>
<td>$t_p$ (msec)</td>
<td>$t_s$ (msec)</td>
</tr>
<tr>
<td>$L$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>250\mu H</td>
<td>6.23</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>300\mu H</td>
<td>7.06</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>150\mu H</td>
<td>4.57</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>250\mu F</td>
<td>5.61</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>300\mu F</td>
<td>4.99</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>150\mu F</td>
<td>14.5</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>$r_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7 \Omega</td>
<td>1.87</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0.5 \Omega</td>
<td>2.91</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0.1 \Omega</td>
<td>11.6</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$r_c$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05 \Omega</td>
<td>8.31</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>0.2 \Omega</td>
<td>12.5</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>0.01 \Omega</td>
<td>12.5</td>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>

In summary, the proposed controller has the best overall performance. It is evident that the proposed implementation method exhibits a faster recovery of the steady state with lower overshoot.
3.5 CONCLUSION

A compact single parameter based $H_\infty$ loop shaping control strategy is proposed in this chapter. Its application to the regulation of DC/DC boost converter is investigated. The salient features of the proposed scheme are listed as follows:

- The additional tuning parameter proposed in the loop shaping controller reflects the trade-off between stability robustness and time domain performance of the closed-loop system and it can be used to modify the output response.
- The adopted model for perturbations is also described and is found to be attenuating a selected group of harmonic components (in the audible range) contained in the output voltage.
- Simulations are carried out to test the static and dynamic performances of the system. From the analysis of the results, it is concluded that the proposed controller outperforms the existing ones in providing robust tracking performance and provides a simple design and compact structure.

However, the proposed controller exhibits poor performance for line and load disturbances. The reason for poor performance is due to non-representation of the disturbance effect in the small signal model considered for controller design. Hence, an enhanced loop shaping design procedure is proposed in the next chapter.