CHAPTER 2

MODELING OF DC/DC CONVERTERS AND REVIEW OF LOOP SHAPING $H_\infty$ TECHNIQUE

2.1 INTRODUCTION

Time averaged models are the commonly used representations for power electronic systems (Pressman 1991) for the purpose of analysis and design of controllers. These averaged models are generally non-linear in nature and have to be linearized using conventional methods. Linearized models support control design methods based on the Transfer Function (TF) approaches. The switching and the linear models of nonminimum and minimum phase converters obtained from literature are detailed in this chapter. This is followed by a detailed discussion of the loop shaping $H_\infty$ controller with the general configuration and essential features.

2.2 STATE SPACE AVERAGING TECHNIQUE

In switching systems, the functional relationships between sources, outputs, and control parameters are explored through the use of time averaging technique. Basically the DC/DC converters are two switched network converters when operated in Continuous Inductor Current Mode (CICM), in which the system is switched back and forth between two linear states under the control of duty-cycle. In each of the two positions of the switch, the system is linear. Each of the converter circuits has filter components namely the inductor (L) and capacitor (C). The parasitic elements
of the L and C components play a major role in the model of the converter influencing the controller design. The following sections describe the model of a boost and buck converter.

2.2.1 Boost Converter

Consider the DC/DC boost converter circuit shown in Figure 2.1 as referenced in Cúk and Middlebrook (1976, 1983). The internal resistances $r_1$, $r_s$, $r_d$, $r_c$ of the inductor L, switch s, diode D and capacitor C have been included in the power stage model. Inductor parasitic resistance has also been included. $v_{in}$, $v_0$, $v_c$ and $i$ denote the instantaneous input, load, capacitor drop and inductor current respectively. The output current source $i_0$ has been inserted in the circuit to model the load perturbations.

![Figure 2.1 Topology of boost converter](image)

Under the assumption that the switching frequency is much higher than the natural frequencies of the system and the switches are realized with common power semiconductor devices, the average model of the DC/DC boost converter is described as follows:

$$
\dot{q} = [A_1 d + A_2 (1-d)]q + [b_1 d + b_2 (1-d)]v_{in} + [e_1 d + e_2 (1-d)] i_0
$$

$$
v_0 = [C_1 d + C_2 (1-d)]q + [p_1 d + p_2 (1-d)] i_0
$$

(2.1)
where \( A_1 = \begin{bmatrix} (r_1 + r_s) & 0 \\ \frac{1}{L} & -\frac{a}{R_L C} \end{bmatrix}, \quad b_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \quad e_1 = \begin{bmatrix} 0 \\ -\frac{a}{C} \end{bmatrix} \)

\( C_1 = \begin{bmatrix} 0 & a \end{bmatrix}, \quad p_1 = [-ar_c] \)

\( A_2 = \begin{bmatrix} (r_1 + r_s + ar_c) & -\frac{a}{C} \\ \frac{L}{a} & -\frac{a}{R_L C} \end{bmatrix}, \quad b_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} \frac{ar_c}{L} \\ -\frac{a}{C} \end{bmatrix} \)

\( C_2 = [ar_c, a], \quad p_2 = [-ar_c] \)

\[ q = \begin{bmatrix} i \\ v_c \end{bmatrix}^T \] is the state vector, \( d \) is the duty ratio and \( a = \frac{R_L}{R_L + r_c} \).

### 2.2.2 Buck Converter

Similarly, the average model of the DC/DC buck converter shown in Figure 2.2 is described as follows. The circuit parameters are same as described in section 2.2.1.

![Figure 2.2 Topology of buck converter](image-url)
\[ \dot{q} = [A_d + A_1(1 - d)]q + [b_d + b_2(1 - d)]v_{in} + [e_d + e_2(1 - d)]i_0 \]

\[ v_0 = [C_d + C_2(1 - d)]q + [p_1d + p_2(1 - d)]i_0 \]  \hspace{1cm} (2.2)

where

\[ A_1 = \begin{bmatrix} \frac{-(r_i - ar_c - r_s)}{L} & -\frac{a}{L} \\ \frac{a}{C} & -\frac{a}{R_L C} \end{bmatrix}, \quad b_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, \quad e_1 = \begin{bmatrix} \frac{ar_c}{L} \\ -\frac{a}{C} \end{bmatrix} \]

\[ C_1 = [ar_c \ a], \quad p_1 = [-ar_c] \]

\[ A_2 = \begin{bmatrix} \frac{-(r_i - ar_c - r_s)}{L} & -\frac{a}{L} \\ \frac{a}{C} & -\frac{a}{R_L C} \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad e_2 = e_1 \]

\[ C_2 = C_1, \quad p_2 = p_1 \]

\[ q = [i \ v_c]^T \] is the state vector, \( d \) is the duty ratio and \( a = \frac{R_L}{R_L + r_c} \).

### 2.3 AC SMALL SIGNAL MODEL

A small signal model gives insight into the dynamic properties of the averaged converter. The small signal model can be derived by linearizing the given nonlinear equations around an equilibrium point (Sum 1989, Dixon 1986). Allowing a small perturbation in the variable \( v_{in} = V_{in} + \hat{v}_{in} \), \( i_0 = I_0 + \hat{i}_0 \) and \( d = D + \hat{d} \) where \( V_{in}, I_0 \) and D denote fixed values at an operating point and \( \hat{v}_{in}, \hat{i}_0 \) and \( \hat{d} \) denote deviations from this operating point. Linearization around the equilibrium point (Cúk and Middlebrook 1976, 1983) leads to the following small-signal dynamic model.
\[
\mathbf{\dot{q}} = \mathbf{A}\mathbf{q} + \mathbf{b}_w\mathbf{\dot{v}}_i + \mathbf{b}_d\mathbf{\dot{q}}_0 + \mathbf{b}\mathbf{\dot{d}}
\]

\[
\mathbf{\dot{v}}_0 = \mathbf{C}\mathbf{q} + p\mathbf{\dot{q}}_0 + [(C_1 - C_2)Q + (p_1 - p_2)I_0]\mathbf{\dot{d}}
\]

(2.3)

where

\[
A = [A_1D + A_2(1-D)], \quad \mathbf{b}_w = [b_1D + b_2(1-D)], \quad \mathbf{b}_d = [e_1D + e_2(1-D)],
\]

\[
b = [(A_1 - A_2)X + (b_1 - b_2)V_i + (e_1 - e_2)I_0], \quad Q = -A^{-1}b_wV_i - A^{-1}b_dI_0
\]

\[
C = [C_1D + C_2(1-D)], \quad p = [p_1D + p_2(1-D)]
\]

Also the converters small signal output voltage can be derived as follows:

\[
\mathbf{\dot{v}}_0 = [C(sI - A)^{-1}b_w] \mathbf{\dot{v}}_i + [C(sI - A)^{-1}b_d + p] \mathbf{\dot{q}}_0 + [C(sI - A)^{-1}b + (C_1 - C_2)Q + (p_1 - p_2)I_0]\mathbf{\dot{d}}
\]

(2.4)

The state equation (2.3) can be rewritten as in (2.5)

\[
\mathbf{\dot{q}} = \mathbf{A}_0\mathbf{q} + \mathbf{B}_0\mathbf{\dot{w}} + \mathbf{B}_d\mathbf{\dot{d}}
\]

\[
\mathbf{\dot{v}}_0 = \mathbf{C}_0\mathbf{\dot{q}} + \mathbf{D}_0\mathbf{\dot{w}} + \mathbf{D}_d\mathbf{\dot{d}}
\]

(2.5)

where

\[
\mathbf{\dot{w}} = [\mathbf{\dot{v}}_i \quad \mathbf{\dot{q}}_0], \quad \mathbf{A}_0 = \mathbf{A}, \quad \mathbf{B}_0 = [\mathbf{b}_w \quad \mathbf{b}_d], \quad \mathbf{B}_d = \mathbf{b},
\]

\[
\mathbf{C}_0 = \mathbf{C}, \quad \mathbf{D}_0 = [0 \quad p], \quad \mathbf{D}_d = [(C_1 - C_2)Q + (p_1 - p_2)I_0]
\]

In the state space formulation defined by Equation (2.5), variable \( \dot{d} \) represents the control input, variable \( \dot{w} \) represents the exogenous inputs namely the line and load variations. \( \dot{v}_0 \) is the variation in the output to be regulated.
2.4 CONTROL ISSUES

The objective of the controller design is to provide robust stability for all members of the family of DC/DC converters. Performance of the controller should also include set-point tracking, disturbance rejection, and suppression of measurement noise. The designer has to make a basic trade-off between the objectives of tracking and noise suppression as they are contradicting requirements. The problem of regulating the output voltage of the nonminimum phase switching systems has been a subject of great interest for many years and various control techniques have been proposed in the literature. A comparative study of various controllers for the boost converter is also provided by Naim et al (1997). It is worth noting that the boost converter is modeled as a bilinear second order nonminimum phase system with a highly uncertain load resistance.

Among the robust control techniques that are being currently investigated in power electronics, the $H_\infty$ approach is a good candidate in many applications due in part to its linear characteristics, and the derived controller is best suited to be used in large signal applications. The advantages of $H_\infty$ control are faster transient response and an easier-to-design control loop as well as its ability to deal with practical variations on circuit parameters and line voltage. Among the various control methods available, the $H_\infty$ control offers a choice for dealing especially with practical variations on circuit parameters and line voltage.

Nevertheless, none of the above mentioned control strategies can explicitly specify the desired degree of robustness in the design goal and hence the robustness in case of parameter variations cannot be guaranteed in advance. To ensure a prespecified degree of stability, the $H_\infty$ loop-shaping control approach (Mcfarlane and Glover 1992), which is used in this thesis, is reviewed in this chapter.
2.4.1 Review of Loop Shaping $H_\infty$ Technique

The basic configuration of the loop shaping $H_\infty$ controller introduced by McFarlane and Glover (1992) is shown in the Figure 2.3. The essential features of this approach can be summarized as follows.

- It can incorporate both performance and robust stability requirements.
- It is conceptually and computationally simple and retains many of the features encountered in the design of classical control loops using frequency domain techniques. This is important, since experienced engineers working in industry can incorporate their knowledge of the plant and classical control design into modern powerful techniques.

![Figure 2.3 The loop shaping design procedure](image-url)
Given a plant model $G$, the design approach consists of three steps.

1) Loop Shaping  
2) Robust Stabilization  
3) Construction of Final Controller

Using pre-and/or post compensators $W_1$ and $W_2$, the singular values of the original plant $G$ are shaped to give the compensated plant $\tilde{G} = W_2GW_1$ with desired open-loop shape as shown in Figure 2.3(a). This step contains all of the ingredients of the classical techniques. The shaping functions $W_1$ and $W_2$ are controlled by the designer and the properties of the resulting controller depend upon these functions.

For the shaped plant $\tilde{G}$, $H_\infty$ optimization problem is given as follows:

$\varepsilon_{\max}^{-1} = \inf_{K \text{ stabilizing}} \left\| \begin{bmatrix} (I + \tilde{G}K)^{-1} & (I + \tilde{G}K)^{-1}\tilde{G} \\ K(I + \tilde{G}K)^{-1} & K(I + \tilde{G}K)^{-1}\tilde{G} \end{bmatrix} \right\|_\infty \tag{2.6}$

Here $\varepsilon_{\max}$ is a design indicator and $K$ is the set of all stabilizing controllers. It is desired to obtain a value between 0.3 and 1 for $\varepsilon_{\max}$ (McFarlane and Glover 1992) to ensure that the loop shapes can be well approximated and the closed-loop system will have good robust stability.

Let the state space realization of the shaped plant $\tilde{G}$ be given by

\[
\begin{cases}
\dot{x} = A_S x + B_S u \\
y = C_S x
\end{cases} \tag{2.7}
\]
where $A_S, B_S$ and $C_S$ are the state, input and output matrices of the shaped plant respectively. $u$ is the control input and $y$, the measured output.

Determination of the stabilizing controller and evaluation of $\varepsilon_{\text{max}}$ in (2.6) can be solved explicitly without need for any iteration, using only two Riccati equations. As per Tan et al (1998), the robustness index $\varepsilon_{\text{max}}$ is given by

$$
\varepsilon_{\text{max}} = (1 + \rho_{\text{max}}(YX))^{-1/2}
$$

(2.8)

where $\rho_{\text{max}}$ is the greatest eigen value of the product of $Y$ and $X$. $X$ and $Y$ are the unique semi positive stabilizing solutions to the following algebraic Riccati equations:

$$
A_S^T X + XA_S - XB_S B_S^T X + C_S^T C_S = 0
$$

(2.9)

$$
A_S Y + Y A_S^T - Y C_S^T C_S Y + B_S B_S^T = 0
$$

(2.10)

The optimal $H_\infty$ controller can be constructed using the following generalized state space description:

$$
\begin{cases}
Q\dot{x} = (Q(A_S + B_S B_S^T X) + \varepsilon_{\text{max}}^{-2} YC_S^T C_S)x + \varepsilon_{\text{max}}^{-2} YC_S^T y \\
u = B_S^T Xx
\end{cases}
$$

(2.11)

where 

$$Q = (1 - \varepsilon_{\text{max}}^{-2})I + YX.$$
2.5 VALIDATION OF CONTROL DESIGN

The small-signal models discussed in section 2.3 are used to completely utilize the linear control techniques effectively to design the controller for any given non-linear system. The performance of the controller is to be tested on the actual nonlinear model of the system.

The time average model can be used for limited bandwidth inputs. Whereas the switching model or the large signal model of the boost and the buck converters can be used for input frequencies beyond switching frequencies.

The switching model of the boost converter during ON Duration is given by Equation (2.12) and during OFF Duration is given by Equation (2.13).

\[
\begin{align*}
    i &= \frac{(-r_i - r_d)}{L} i + \frac{1}{L} v_{in} \\
    \dot{v}_c &= -\frac{a}{R_L C} v_c - \frac{a}{C} i_0 \\
    v_0 &= a v_c - a r_i i_0
\end{align*}
\]

\[
\begin{align*}
    i &= \frac{(-r_i - r_d - a r_c)}{L} i - \frac{a}{L} v_c + \frac{1}{L} v_{in} + \frac{ar_c}{L} i_0 \\
    \dot{v}_c &= \frac{a}{C} i - \frac{a}{R_L C} v_c - \frac{a}{C} i_0 \\
    v_0 &= a r_c i + a v_c - a r_c i_0
\end{align*}
\]

where the parameters represent the system function as described in section 2.2.
Similarly the switching model of the buck converter during ON and OFF Duration is given by Equation (2.14) and Equation (2.15) respectively.

\[
i = \left(\frac{-r_i - r_s - ar_r}{L}\right)i - \frac{a}{L}v_c + \frac{1}{L}v_{in} + \frac{ar_c}{L}i_0
\]

\[
\dot{v}_c = \frac{a}{C}i - \frac{1}{R_c C}v_c - \frac{a}{C}i_0
\]  

(2.14)

\[
v_0 = ar_r i + av_c - ar_i i_0
\]

\[
i = \left(\frac{-r_i - r_s - ar_r}{L}\right)i - \frac{a}{L}v_c + \frac{ar_c}{L}i_0
\]

\[
\dot{v}_c = \frac{a}{C}i - \frac{1}{R_c C}v_c - \frac{a}{C}i_0
\]

\[
v_0 = ar_r i + av_c - ar_i i_0
\]  

(2.15)

Commonly used performance measure is audio susceptibility and output impedance. The audio susceptibility of the converter quantifies the amount of input variations that reaches the output as a function of frequency.

\[
F_1 = \frac{\dot{v}_0}{\dot{v}_{in}}
\]  

(2.16)

The output impedance relates to the capacity of the converter to cater to dynamic load variations.

\[
F_2 = \frac{\dot{v}_0}{i_0}
\]  

(2.17)

The task of the controller in SMPS is to minimize sensitivity to load changes, i.e., reduce the output impedance and attenuate input-output transmission, i.e., low audio susceptibility.
2.6 CONCLUSION

In this chapter, the standard configuration of the buck and boost converters are considered. The filter components are represented with their parasitic effects. The time average model is described for the purpose of derivation of small signal model. With the help of a generalized formulation, the small signal model of the buck and boost converters are presented.

The McFarlane and Glover (1992) technique based $H_\infty$ loop shaping procedure is described with the robustness check and the $H_\infty$ control structure.

The large signal model or the switching model of the converters are presented to validate the design.

Commonly used performance measures namely the audio susceptibility and output impedance are illustrated in this chapter.

This chapter provides the overall necessary backgrounds required for $H_\infty$ control of DC/DC converters and its validation.