Chapter – 5

Modified Multivariate Exponentially Weighted

Moving Average Control Chart
5.1 Introduction

The rapid growth of modern statistics, which is the science of winding and twisting network connecting Mathematics, Scientific Philosophy, computer software and other Intellectual sources of the Millennium, has led to an increased interest in detecting assignable causes in several simultaneous related quality characteristics or process variables. When the characteristics are mutually correlated, the univariate control charts are not as sensitive as multivariate control charts.

Hotelling (1947) has proposed the multivariate control chart for mutually correlated characteristics. This control chart is based only on the most recent observation. This control chart is insensitive to detect assignable causes of production processes in small and moderate changes in mean vector. This chart has given an out – of – control signal as soon as,

\[ Q_i = (X_i - \bar{X})' S^{-1} (X_i - \bar{X}) > h, \]
where h (>0) is a specified control limit. \(X_i\) is an observable \(p \times 1\) random vector representing the \(i^{th}\) individual observation of the \(p\) characteristics,

\[
\begin{bmatrix}
X_{i1} \\
X_{i2} \\
\vdots \\
X_{ip}
\end{bmatrix}
\quad i=1,2,3,...,t,
\]

where \(i\) is the age of the sample. The estimated mean vector, whose components are the means of each characteristic, is

\[
\bar{X} =
\begin{bmatrix}
\bar{X}_1 \\
\bar{X}_2 \\
\vdots \\
\bar{X}_p
\end{bmatrix}
\]

where \(\bar{X}_j = \frac{1}{t} \sum_{i=1}^{t} X_{ij}, j=1,2,...,p\) and the estimated covariance matrix is

\[
S = \frac{1}{t-1} \sum_{i=1}^{t} (X_i - \bar{X})(X_i - \bar{X})'.
\]

To construct the multivariate control chart the charting statistics for observation \(X_i\) is

\[
Q_i = (X_i - \bar{X})' S^{-1} (X_i - \bar{X}).
\]

If one assumes that the estimates of \(\bar{X}\) and \(S\) are the true population mean vector \(\mu\) and \(\Sigma\) respectively, then \(Q_i \sim \chi^2\) variate with \(p\) degrees of freedom which is used to specify the control limits. In this
case, the lower control limit is \( LCL = \chi^2_{1-\alpha/2}(p) \), and the upper control limit is \( UCL = \chi^2_{\alpha/2}(p) \) where \( \chi^2_{\alpha}(p) \) is \( 1-\alpha \) percentile of the \( \chi^2 \) distribution with \( p \) degrees of freedom.

If one assumes that \( i^{th} \) observation \( X_i \) is independent of both \( \bar{X} \) and \( S \), then the charting statistics, \( Q_i \sim F \) distribution with \( p \) and \( t-p \) degrees of freedom which is used to specify the control limits. In this case, the lower control limit is \( LCL = \frac{p(t-1)}{t(t-p)} F_{\alpha/2}(p,t) \), and the upper control limit is

\[
UCL = \frac{p(t-1)}{t(t-p)} F_{1-\alpha/2}(p,t),
\]

\( F_{\alpha}(p,t-p) \) is the \( 1-\alpha \) percentile of the \( F \) distribution with \( p \) and \( t-p \) degrees of freedom.

Gnanadesikan and Kettering (1972) have showed that if \( p \) is small, then the exact distribution \( Q_i \sim \frac{(t-1)^2}{t} B(p/2,(t-p-1)/2) \) is apt to specify the control limits. In this case the LCL and UCL are respectively

\[
LCL = \frac{(t-1)^2}{t} \times \frac{p}{[p/(t-p-1)]} F\left(1-\alpha/2; p, t-p-1\right) F\left(1-\alpha/2; p, t-p-1\right)
\]

\[
UCL = \frac{(t-1)^2}{t} \times \frac{p}{[p/(t-p-1)]} F\left(\alpha/2; p, t-p-1\right) F\left(\alpha/2; p, t-p-1\right)
\]

in terms of percentiles from the \( F \) distribution. Tracy, Young and Malson (1972) have used the equations (5.1) and (5.2) to study the multivariate control charts for individual observations.
Woodall and Ncube (1985) have introduced multivariate CUSUM control charts. It involves simultaneous univariate charts which are frequently applied to correlated observations. In univariate CUSUM control chart, the sequence of points is correlated so the diagnosis of pattern of recognition on the CUSUM chart is very difficult. Also the successive values of the cumulative sums differ by only one observation, so the CUSUM chart will often exhibit runs or other behaviors due to these correlations.

Healy (1987) has showed that an univariate CUSUM chart based on a linear combination of the variables can be used in the multivariate situation. This chart detects a shift in the mean vector in only one specified direction.

Hawkins (1991) has developed this approach into several directions. This method is more effective than that of Woodall and Ncube (1985), Lowry and Woodall (1992) have proposed a multivariate EWMA chart which is a multivariate extension of the univariate EWMA chart. In the multivariate case, vectors of EWMA are defined

\[ Z_i = RX_i + (I-R) Z_{i-1} \]

where \( i = 1, 2, \ldots, t \) and is the age of the sample and \( Z_0 = \mu \) and \( R = \text{diag} (r_1, \ldots, r_p) \), \( 0 < r_j \leq 1 \), \( j = 1, 2, \ldots, p \). The MEWMA chart gives out of control signals as soon as
\[ Q_i = (Z_i - \bar{Z})' \sum_{z_i}^{-1} (Z_i - \bar{Z}) > h_i, i=1,2,...,t, \]

where \( h_1 > 0 \) is chosen to achieve a specified control limit and \( \sum_{z_i} \) is the covariance matrix of \( Z_i \).

If \( r_1 = r_2 = ... = r_p = r \), then the MEWMA vectors can be written as

\[ Z_i = r X_i + (1-r) Z_{i-1}, \ i = 1,2,...,t. \]

In this case

\[ \sum_{z_i} = \frac{r}{2 - r} \sum \]

The asymptotic covariance matrix,

\[ \sum_{z_i} = \frac{r}{2 - r} \sum_j^{p} \] as \( i \to \infty \)

Here the weights \( 0 < r_j \leq 1, j = 1,2,.... \), \( p \) are the only choice. For smaller process shifts in the mean vector, the reasonable weight is 0.1 which is recommended by Lowry and Woodall (1992).

This chapter focuses on the study of a modified multivariate exponentially weighted moving average control chart. The MEWMA control chart can be modified in the following ways:

- How many past and current observations can be viewed as the weighted moving averages of a production process?
- The weights of the moving averages are not a choice
• The moving averages and their weights depend on the nature of changing direction of a production process

An example is given under to illustrate the potential of the control chart.

5.2 Markov Dependence Production Process

If the production process remains in control at the target value $\mu$, then the sample cumulative sum should fluctuate stochastically around zero. If the mean shifts upward to some value $\mu_1 > \mu_0$, then an upward or positive drift will develop in the cumulative sum. If the mean shifts downward to some value $\mu_1 < \mu_0$, then a downward or negative drift will develop in the cumulative sum. Therefore we may construct a Markov chain (MC) as in Karlin (1966) which consists of three states as a result of the directions of the changes of the observed values, namely 0 indicates that there is a constant change in the observed values, $+1$ indicates a positive direction or upward movement of the consecutive observed values and $-1$ represents a downward movement or negative direction exhibits in the successive observed values. Keeping this in mind, we define $X_n$ as the type of changes of directions of the observations at the $n^{th}$ time, then the sequence of changes $(X_n; \ n = 0, 1, 2, 3, \ldots)$ forms a MC for the state space $\{-1, 0, +1\}$ with a unit step transition probability matrix $P = (p_{ij})$, where

$$p_{ij} = \Pr \{X_{n+1} = j \mid X_n = i\}, \ i, j = -1, 0, +1.$$
From the MC, we find the probability distribution of the changing direction of the process. If the MC is an irreducible a periodic positive recurrent, then the limiting distribution and the stationary distribution are the same. Otherwise the stationary distribution of the finite state MC is numerically obtained. The stationary probabilities $\pi_{-}, \pi_{0}$ and $\pi_{+}$ of the MC are given by $\Pi P = \Pi$ so that $\pi_{-} + \pi_{0} + \pi_{+} = 1$.

The limiting probability

$$\lim_{n \to \infty} p_{y}^{(n)} = \pi_{j}, \quad j = -1, 0, +1,$$

where $p_{y}^{(n)} = \Pr\{X_{m+n} = j | X_{m} = i\}$.

is the $n^{th}$ step transition probability which gives the probability that from the state is at the $m^{th}$ time, the state $j$ is reached at $(m + n)^{th}$ time $n$ steps, i.e., the probability of transition from the state $i$ to the state $j$ in exactly $n$ steps. On the basis of the probabilities, we can find out whether the process has moved upward or downward or whether there has been no change in the long run.

We construct a MC consisting of the three states as a result of the direction of the changes of the observed values. In this method of constructing is valid and easier to use, since it takes into account only the direction of changes of the observed values.
The procedure to detect the nature of a process involves the following steps:

Step (1): Determine the negative change, constant change or Positive change of the given observations as $-$, 0, + respectively.

Step (2): Count the number of transition from negative changes to itself and it is denoted by $n_{00}$. Similarly negative changes to no change by $n_{01}$, negative changes to positive direction as $n_{02}$, no change it itself as $n_{11}$, no change to positive direction as $n_{12}$, positive changes to negative direction as $n_{20}$, positive changes to constant change as $n_{21}$, positive changes to itself as $n_{22}$.

Step (3): Represent all the transitions in a table given below:

<table>
<thead>
<tr>
<th>Details</th>
<th>$-$</th>
<th>0</th>
<th>+</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-$</td>
<td>$n_{00}$</td>
<td>$n_{00}$</td>
<td>$n_{00}$</td>
<td>$n_0.$</td>
</tr>
<tr>
<td>0</td>
<td>$n_{10}$</td>
<td>$n_{11}$</td>
<td>$n_{12}$</td>
<td>$n_1.$</td>
</tr>
<tr>
<td>+</td>
<td>$n_{20}$</td>
<td>$n_{21}$</td>
<td>$n_{22}$</td>
<td>$n_2.$</td>
</tr>
<tr>
<td>Total</td>
<td>$n_0$</td>
<td>$n_1$</td>
<td>$n_2$</td>
<td>$n-2$</td>
</tr>
</tbody>
</table>
Step (4): Determine the three states of MC with transition probabilities as

\[ P = \begin{pmatrix} \frac{n_{00}}{n_{0}} & \frac{n_{01}}{n_{1}} & \frac{n_{02}}{n_{2}} \\ \frac{n_{10}}{n_{0}} & \frac{n_{11}}{n_{1}} & \frac{n_{12}}{n_{2}} \\ \frac{n_{20}}{n_{0}} & \frac{n_{21}}{n_{1}} & \frac{n_{22}}{n_{2}} \end{pmatrix} \]

Step (5): From the MC, we may find out the probability distribution of the nature of the changing directions.

5.3 Construction of a Modified MEWMA Control Chart

We propose a Modified Multivariate Exponentially Weighted Moving Averages (MMEWMA) based on the nature of the process. Suppose a production process has three changes, then a MMEWMA is defined as

\[ Z_i = R_+ X_i + R_- X_{i-1} + R_0 Z_{i-1}, i = 1, 2, \ldots, t, \]

And the starting vector required with the first sample at i = 1 is \( Z_0 = \overline{X} \), and \( R_+ + R_- + R_0 = I_{p \times p} \) where
The MMEWMA $Z_i$ is the weighted average of all the previous sample vector values. To demonstrate this $Z_{i-1}$ may be used to obtain $Z_i$.

That is

$$Z_i = R_+ X_i + R_0 \{ R_+ X_{i-1} + R_0 \{ R_+ X_{i-2} + R_0 Z_{i-2} \} \}.$$ 

Using recursively for $Z_{i:j}$, $j = 1, 2, \ldots, i$, we have

$$Z_i = R_+ \sum_{j=0}^{i} R_0^j X_{i-j} + R_- \sum_{j=0}^{i} R_0^j X_{i-j-1} + (R_0^2) Z_0.$$ 

Thus

$$\sum_{Z_i} = \sum_{j=0}^{i} R_+ R_0^{i-j} + \sum_{j=0}^{i} R_- R_0^{i-j}.$$ 

The weights $R_+ R_0^j$ and $R_- R_0^j$ decrease geometrically with the age of the sample mean vector, and the assumption that the sample vectors $X_1, X_2, \ldots, X_i, \ldots$ are independent with covariance $\Sigma$. The $(k, l)^{th}$ element of $\sum_{Z_i}$ is

$$\pi^+_k \pi^+_l + \frac{1 - \left( \pi^+_0 \right)^l}{\left[ 1 - \pi^+_0 \right]} \sigma_{kl} + \pi^-_k \pi^-_l - \frac{1 - \left( \pi^-_0 \right)^l}{\left[ 1 - \pi^-_0 \right]} \sigma_{kl},$$

where $\sigma_{kl}$ is the $(k,l)^{th}$ element of $\sum_{Z_i}$. If

$$\pi^+_0 = \pi^+_1 = \pi^+_2 = \ldots = \pi^+_p = \pi_+$$

$$\pi^-_0 = \pi^-_1 = \pi^-_2 = \ldots = \pi^-_p = \pi_-$$

$$\pi^+_0 = \pi^+_1 = \pi^+_2 = \ldots = \pi^+_p = \pi_0$$
Then

\[ \sum z_i = \frac{\left[ \pi^2_+ + \pi^2_- \right] (1 - \pi^2_0)}{1 - \pi^2_0} \sum \]  

(5.5)

Thus in an actual situation, it is seen that MMEWMA control chart performs better than MEWMA and equation (5.3) can be used to detect assignable causes production process. Since the MC of the changing directions of the process is not irreducible, a periodic positive recurrent, then

\[ \text{as } i \to \infty \sum z_i = \frac{\pi^2_+ + \pi^2_-}{1 - \pi^2_0} \sum \]  

(5.4)

It is independent of the age of the sample. In the steady state situation, the asymptotic covariance matrix \( \sum z \) of the equation (5.4) can be used to detect the assignable causes in a production process because the limiting distribution is equal to the stationary distribution of the changing direction of the process.

If a production process has only two changing direction upward and downward, we define

\[ Z_i = R_+ X_i + R_- Z_{i-1} \quad i = 1, 2, \ldots, t, \]

So that \( R_+ + R_- = I_{p \times p} \), then the MMEWMA chart is same as Lowry and Woodall (6) MEWMA chart but their weights are the probability distribution of the changing direction of the process.

If there is only one shift in a production process, then
Now the MMEWMA chart becomes the same as Hotelling (1947) or Tracy, Young and Malson (1992) multivariate control chart.

5.4 Numerical Example

Tracy, Young and Malson (1992) have illustrated how to construct the multivariate control chart for individual observation in the Table 5.1, which involves the simultaneous measurement of three variables, percentages of impurities ($X_1$), temperature ($X_2$) and concentration strength ($X_3$) of a particular substance in a production process. For the same example, we construct a MMEWMA and MEWMA charts. The directions of changes -, 0 and + are shown to the respective observations.
### Table 5.1

Measurements of Impurities, Temperature and Concentration

<table>
<thead>
<tr>
<th>i</th>
<th>% Impurities</th>
<th>Temperature</th>
<th>Concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.92</td>
<td>85.77</td>
<td>42.26</td>
</tr>
<tr>
<td>2</td>
<td>16.90⁺</td>
<td>83.77⁻</td>
<td>43.44⁺</td>
</tr>
<tr>
<td>3</td>
<td>17.38⁺</td>
<td>84.46⁺</td>
<td>42.74⁻</td>
</tr>
<tr>
<td>4</td>
<td>16.90⁻</td>
<td>86.27⁺</td>
<td>43.60⁺</td>
</tr>
<tr>
<td>5</td>
<td>16.92⁺</td>
<td>85.23⁻</td>
<td>43.18⁻</td>
</tr>
<tr>
<td>6</td>
<td>16.71⁻</td>
<td>83.81⁻</td>
<td>43.33⁻</td>
</tr>
<tr>
<td>7</td>
<td>17.07⁺</td>
<td>86.08⁺</td>
<td>43.33⁻</td>
</tr>
<tr>
<td>8</td>
<td>16.93⁻</td>
<td>85.95⁻</td>
<td>43.41⁺</td>
</tr>
<tr>
<td>9</td>
<td>16.71⁻</td>
<td>85.73⁻</td>
<td>43.28⁻</td>
</tr>
<tr>
<td>10</td>
<td>16.88⁺</td>
<td>86.27⁺</td>
<td>42.59⁻</td>
</tr>
<tr>
<td>11</td>
<td>16.73⁻</td>
<td>83.46⁻</td>
<td>44.007⁺</td>
</tr>
<tr>
<td>12</td>
<td>17.07⁺</td>
<td>85.81⁺</td>
<td>42.78⁻</td>
</tr>
<tr>
<td>13</td>
<td>17.60⁺</td>
<td>85.92⁺</td>
<td>43.11⁺</td>
</tr>
<tr>
<td>14</td>
<td>16.90⁻</td>
<td>84.23⁻</td>
<td>43.48⁺</td>
</tr>
</tbody>
</table>
The following are the measures of the MEWMA control charts based on Lowry and Woodall (1992) with the choice weight $r = .1$. The charting statistics

$$Q_t = (Z_t - \bar{Z})' \sum_{z_i}^{-1} (Z_t - \bar{Z})$$

where $\sum_{z_i}^{-1} = \frac{1}{19} S$.

Here $Z_0 = \bar{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$

$$= \begin{bmatrix} 16.83 \\ 85.19 \\ 43.21 \end{bmatrix}$$

$$S = \begin{pmatrix} .365 & -.022 & .100 \\ -.022 & 1.036 & -.245 \\ .100 & -.245 & .224 \end{pmatrix}$$

$$\bar{Z} = \begin{bmatrix} \bar{Z}_1 \\ \bar{Z}_2 \\ \bar{Z}_3 \end{bmatrix} = \begin{bmatrix} 16.71 \\ 85.17 \\ 43.19 \end{bmatrix}$$
The MEWMA’s charting statistics $Q_i$ are given in Table 5.2

**Table 5.2**

Computation of MEWMA Charting Statistics

<table>
<thead>
<tr>
<th>$i$</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
<th>MEWMA $Q_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.6390</td>
<td>85.2480</td>
<td>43.1150</td>
<td>.5616</td>
</tr>
<tr>
<td>2.5</td>
<td>16.6651</td>
<td>85.1002</td>
<td>43.1475</td>
<td>.4849</td>
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<tr>
<td>3</td>
<td>16.7366</td>
<td>85.0362</td>
<td>43.1068</td>
<td>1.2246</td>
</tr>
<tr>
<td>4</td>
<td>16.7529</td>
<td>85.1596</td>
<td>43.1561</td>
<td>1.3549</td>
</tr>
<tr>
<td>5</td>
<td>16.7696</td>
<td>85.1666</td>
<td>43.1585</td>
<td>.4917</td>
</tr>
<tr>
<td>6</td>
<td>16.7636</td>
<td>85.0309</td>
<td>43.2147</td>
<td>.5499</td>
</tr>
<tr>
<td>7</td>
<td>16.7942</td>
<td>85.2072</td>
<td>43.2446</td>
<td>.7607</td>
</tr>
<tr>
<td>8</td>
<td>16.8078</td>
<td>85.2072</td>
<td>43.2446</td>
<td>.7607</td>
</tr>
<tr>
<td>9</td>
<td>16.7980</td>
<td>85.2595</td>
<td>43.2481</td>
<td>.9878</td>
</tr>
<tr>
<td>10</td>
<td>16.8062</td>
<td>85.3606</td>
<td>43.1823</td>
<td>1.20722</td>
</tr>
<tr>
<td>11</td>
<td>16.7986</td>
<td>85.1705</td>
<td>43.2641</td>
<td>1.20722</td>
</tr>
<tr>
<td>12</td>
<td>16.8257</td>
<td>85.2345</td>
<td>43.2157</td>
<td>.80550</td>
</tr>
<tr>
<td>13</td>
<td>16.3913</td>
<td>85.3031</td>
<td>43.2051</td>
<td>7.7516</td>
</tr>
<tr>
<td>14</td>
<td>16.4222</td>
<td>85.1958</td>
<td>43.2326</td>
<td>5.61215</td>
</tr>
</tbody>
</table>
There is no constant change in the three random vectors $X_1$, $X_2$ and $X_3$. So the two states MC's of the three characteristics $X_i$, $i = 1, 2, 3$ are respectively

$$
\begin{pmatrix}
1 & 4 \\
5 & 5 \\
5 & 2 \\
7 & 7
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
2 & 4 \\
6 & 6 \\
4 & 2 \\
6 & 6
\end{pmatrix}
$$

All the three MC's are irreducible, a periodic and positive recurrent. Hence the limiting and stationary distributions are equal. The stationary probabilities of the change of directions the process $X_1$, $X_2$, and $X_3$ are

$$
\left\{ \pi_+^{(1)} = \frac{25}{53}, \pi_-^{(1)} = .5 \right\}, \left\{ \pi_+^{(2)} = .5, \pi_-^{(2)} = .5 \right\} \text{ and } \left\{ \pi_+^{(3)} = .5, \pi_-^{(3)} = .5 \right\}
$$

respectively. In this case the matrices are

$$
R_+ = \begin{pmatrix}
.5 & 0 & 0 \\
0 & .5 & 0 \\
0 & 0 & .5
\end{pmatrix}
\quad \text{and} \quad
R_- = \begin{pmatrix}
.5 & 0 & 0 \\
0 & .5 & 0 \\
0 & 0 & .5
\end{pmatrix}
$$

The two matrices $R_+$ and $R_-$ are equal, so the weights of the MMEWMA chart is $r = .5$. 
Table 5.3

Computation of MMEWMA Charting Statistics

<table>
<thead>
<tr>
<th>I</th>
<th>Z₁</th>
<th>Z₂</th>
<th>Z₃</th>
<th>MMEWMA Qᵢ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.8177</td>
<td>85.48</td>
<td>42.735</td>
<td>8.73633</td>
</tr>
<tr>
<td>2</td>
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<td>43.0875</td>
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</tr>
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<td>85.4063</td>
<td>43.2569</td>
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<td>16.8059</td>
<td>84.5641</td>
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</tr>
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<td>43.3424</td>
<td>1.83587</td>
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<td>85.9641</td>
<td>42.9662</td>
<td>1.7767</td>
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<td>43.4831</td>
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<td>85.2611</td>
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<td>43.1208</td>
<td>2.62357</td>
</tr>
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<td>14</td>
<td>17.0825</td>
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<td>0.86082</td>
</tr>
</tbody>
</table>
In this case \( z_1 = 16.8140 \), \( z_2 = 85.2100 \), and \( z_3 = 43.2021 \) and 
\[
\sum z_i = \frac{1}{13} S, \quad \text{where} \quad S = \begin{pmatrix} .365 & -.022 & .100 \\ -.022 & 1.036 & -.245 \\ .100 & -.245 & .224 \end{pmatrix}
\]

when \( p = 3 \), \( t = 14 \) and \( \alpha = .01 \), equations (5.1) and (5.2) result in \( \text{LCL} = .082 \) and \( \text{UCL} = 8.55 \).

**Figure 5.1**

Figure 5.1 shows the corresponding MEWMA and MMEWMA control charts. We observe from Figure 5.1 that none of the observations
are outside the control limits in the MEWMA method, i.e., the process is under statistical control based on the MEWMA technique. However, in the MMEWMA chart, observation 1 lies outside the upper control limit. Hence we may conclude that there are some assignable causes in the production process and this may be due to the low level percentage impurities for the first observation. Thus the production process is out of control.

*For this study of the MMEWMA charts, whenever a production process involves several simultaneous related quality characteristics designs in a manner that has been available for the analysis of univariate control charts. Further Markov dependence has been used to tract effectors of the simultaneous related quality characteristics.*