APPENDIX 1

CALCULATION OF EQUIVALENT AREAS

A1.1 TRIANGULAR ENERGY DIRECTOR

Consider a standard triangular energy director. A rectangle of same area is enclosed in it as shown in the Figure A1.1. Then the width of the rectangle is given by

\[ \frac{1}{2}bh = b'h' \]

\[ \frac{1}{2} * 2 * 1.73 = b' * 1.73 \quad b' = 1\text{mm} \]

Figure A1.1 Equivalent rectangular area of triangular energy director

A1.2 SEMICIRCULAR ENERGY DIRECTOR

Consider a standard semi-circular energy director. A rectangle of same area is enclosed in it as shown in the Figure A1.2. Then the width of the rectangle is given by

\[ \frac{\pi r^2}{2} = b'h' \]

\[ \frac{\pi * 1^2}{2} = b' * 1 \quad b' = 1.57\text{mm} \]

Figure A1.2 Equivalent rectangular area of semicircular energy director
APPENDIX 2

VISCOELASTIC HEATING CALCULATION

A2.1 NEAR FIELD

The average energy dissipated \( Q = \frac{\omega \varepsilon_0^2 E''}{2} \text{J/m}^3 \text{sec} \) (A2.1)

where \( \omega = 2\pi f \), \( f \) is the applied frequency, \( \varepsilon_0 \) is the maximum strain and \( E'' \) is the loss modulus.

Aliosio et al (1972) investigated the ultrasonic welding of polycarbonate, ABS and Noryl—using rectangular energy directors. They modeled the viscoelastic heating of the energy directors assuming adiabatic heating, and used elastic analysis to estimate the strain amplitude within parts.

Maximum strain \( \varepsilon_0 = \frac{\Delta L}{L} \) (A2.2)

where \( \Delta L \) is the maximum displacement, is given by the amplitude of vibration used during welding (60 \( \mu m \)) and \( L \) is the original length—height of standard test specimen (30*10^{-3}m).

\[
\varepsilon_0 = \frac{60 \times 10^{-6}}{30 \times 10^{-3}} = \frac{2}{1000}
\]

\( E'' \) - Loss modulus is assumed as independent of temperature. For ABS value of \( E'' \) is 0.42*10^9 Pa.
Heat Flux \( \left( \frac{W}{m^2} \right) = \frac{Q \cdot V_{ED}}{A_{ED}} \) \hspace{1cm} (A2.3)

where \( V_{ED} \) is the volume of energy director and \( A_{ED} \) is the area of energy director.

\[ V_{ED} = A_{ED} \cdot \text{Height of energy Director} \] \hspace{1cm} (A2.4)

Heat flux in energy director = \( Q \) \( \times \) Height of energy director \( \frac{W}{m^2} \) \hspace{1cm} (A2.5)

Consider the standard triangular energy director with a height of 1.73mm. So, heat flux in the case of a triangular energy director:

\[ \text{Heat flux} = Q \cdot \text{Height of energy director} \cdot \frac{W}{m^2} = 0.105504 \times 10^9 \cdot 1.73 \times 10^{-3} = 182521.92 \frac{W}{m^2} \]

The energy absorption ratio is defined as the ratio of the energy absorbed by the plastic material at the energy director to that of the energy dissipated by the viscoelastic heating.

Chuah et al (2000) determined the energy absorption ratio for ABS and HDPE. The values are 48.5% for ABS and 21.1% for HDPE.
So heat flux absorbed by the ABS (amorphous) with triangular energy director = 182521.92 * 0.485 = 88523.1312 W/m²

**A2.2 FAR FIELD**

In far field welding the intensity would decrease inversely with distance from the energy source. This results from two basic causes i.e. scattering and true absorption, which are both combined with the term ‘attenuation’.

The sound pressure of a wave which decreases as a result of attenuation can be written in the form of an exponential function

\[ P = P_0 e^{-\alpha d} \quad (A2.6) \]

where \( P_0 \) and \( P \) are sound pressures at the beginning and the end of a section of length \( d \) with attenuation coefficient \( \alpha \). The attenuation coefficient \( \alpha \) when referred to intensity \( \alpha_i \), the attenuation law of intensity is

\[ I = I_0 e^{-\alpha_i d} \quad (A2.7) \]

where \( I_0 \) and \( I \) are the intensities at the beginning and the end of a section of length \( d \).

Since intensity is proportional to the square of sound pressure \( \alpha_i = 2\alpha \).
The natural logarithm of equation (A2.7) is

$$\alpha_i d = \ln \frac{I_0}{I}$$  \hspace{1cm} (A2.8)

The decibel measure is obtained by

$$\alpha_i d = 20 \log \frac{I_0}{I} \text{ db}$$  \hspace{1cm} (A2.9)

The attenuation coefficient $\alpha$ (nepers/m) for viscoelastic materials is given by the expression

$$\alpha = \frac{\omega}{2C} \tan \delta \text{ nepers/m}$$  \hspace{1cm} (A2.10)

where $C$ is the velocity sound in the viscoelastic material and $\delta$ is the phase angle. The expressions for $C$ and $\tan \delta$ are given by the following expressions

$$C = \sqrt{\frac{E^*}{\rho}}, \tan \delta = \frac{E''}{E'}$$  \hspace{1cm} (A2.11)

where $E^*$ is the complex modulus, $E'$ is the loss modulus, $E'$ is the storage modulus and $\rho$ is the density of the plastic material.

Consider the standard specimen with triangular energy director and ABS as material.

Velocity of ultrasonic waves in ABS $c = \sqrt{\frac{E^*}{\rho}} = \sqrt{\frac{2.78 \times 10^9}{1181}} = 1534.25 \text{ m/s}$
\[
\tan \delta = \frac{E''}{E'} = \frac{0.42 \times 10^9}{2.75 \times 10^9} = 0.15
\]

\[
\alpha = \frac{\omega}{2C} \tan \delta = \frac{2\pi \times 20000}{2 \times 1534.25} \times 0.15 = 6.1398
\]

neper/m

\[
\frac{6.1398}{0.115} = 53.32 \text{ db/m}
\]

But \(\alpha_1 = 2\alpha\). so \(\alpha = 106.64 \text{ db/m}\)

So decrease in the heat flux through the distance of 30mm (height of standard specimen)

\[
\alpha_i d = 20 \log \frac{I_0}{I}
\]

\[
106.64 \times \frac{30}{1000} = 20 \log \frac{I_0}{I}
\]

\[
\frac{I_0}{I} = 1.445
\]

So for triangular energy director heat flux at the joint interface

\[
\frac{88523.1312 \text{ (near field)}}{1.445} = 61261.68 \text{ W/m}^2
\]

This procedure can be repeated for HDPE also.