CHAPTER 4

REDEFINED a priori KNOWN FUNCTION

4.1 INTRODUCTION

In this chapter the drawbacks of the existing reaching law methods are discussed. To improve the performance a redefined a priori known function is suggested in this chapter. The advantages achieved with this redefined function are demonstrated by applying it to different types of systems. Further, the limitations of the redefined law are also investigated.

4.2 NEED FOR REDEFINITION

With controllers designed according to GR strategy, relatively higher unsymmetrical oscillations present initially in the system response when the system is subjected to parameter uncertainties and time-varying external disturbances. This requires a larger control effort and deteriorates the quality of the output. Further, chattering is unavoidable in GR strategy and the amount of chattering is to be compromised with the settling time.

Though the controllers based on BBR strategy are effective for systems subjected to bounded perturbations and eliminates the unwanted chattering in system dynamics, it encounters the following drawbacks. First, when the system is subjected to time invariant external disturbances and with no parameter variation the transition from reaching mode to sliding mode is abrupt and such sudden changes in the system dynamics are generally not
preferred. When the system is subjected to time-varying disturbances, the system response overshoots the QSM band - an undesirable quasi-sliding mode characteristic. This is minimized with the controllers designed using MBR strategy, but the trajectory is set to cross the switching hyperplane. Further, the control effort required by the BBR/MBR controllers increase monotonically with increase in $k^*$, user defined constant in the Equation (3.34) and hence, may not be suitable for systems with larger time-delay. It is also noticed that the systems become unstable when subjected to parameter variations, irrespective of the types of the controller used. Hence, it is necessary to design a controller, which will eliminate the above said problems.

4.3 DEFINITION

Considering the abrupt change in the system dynamics when subjected to time-invariant external disturbance, it is felt that this sudden change may be minimized by bringing down the differences in switching function values between the successive steps. It requires the improvement in the a priori known function, since the present value of it determines the value of the switching function in the subsequent step. This is accomplished by modifying the a priori known function $s_d(k)$ given in Equation (3.34). To avoid ambiguity the redefined function is designated as $s_p(k)$ and given as

$$s_p(k) = \frac{k^* - k}{k^*} s(k - 1)$$  \hspace{1cm} (4.1)

It is already well established that the value of the switching function $s(k)$ decreases in each step in reaching mode and higher the rate at which it falls is faster the convergence to the sliding hyperplane or better settling-time of the system. Hence, according to the redefined a priori known
function given in Equation (4.1), $s_p(k)$ also decreases at a faster rate than in BBR strategy, driving the system in to the sliding mode quickly. Unlike BBR strategy, where the step size is determined by $k$ alone, in the redefined strategy the step size additionally depends on the magnitude of the previous value of $s(k)$ and hence, faster and smoother convergence is achieved. Due to this the settling time is reduced and transition from reaching mode to sliding mode is also found to be smoother. Figure 4.1 illustrates the evolution of $s_d(k)$ and $s_p(k)$ for a typical system. With $k$ set to 30, the improvement in the settling time for the same system, when controlled by three different types of controllers, may be observed from Figure 4.2.

![Evolution of a priori known function for k = 30](image.png)

**Figure 4.1 Evolution of a priori known function**
Further, with the redefined \textit{a priori} known function \( s_p(k) \) the system given in Equation (3.21) satisfies the reaching condition of the quasi-sliding mode condition on the sliding surface \( s(k) = 0 \) for any \( k > 0 \), if the following conditions are satisfied.

\[
\begin{align*}
\text{s}(k) > 0 & \quad \Rightarrow \quad 0 \leq \text{s}(k+1) < \text{s}(k) \\
\text{s}(k) < 0 & \quad \Rightarrow \quad \text{s}(k) < \text{s}(k+1) \leq 0 \\
|\text{s}(k)| = 0 & \quad \Rightarrow \quad |\text{s}(k+1)| = 0
\end{align*}
\quad (4.2)
\]

In fact, with the redefined strategy the quasi-sliding mode approaches the ideal quasi-sliding mode condition.

The control effort is measured as (Bartoszewicz 1998)

\[
J = \sum_{k=0}^{n} |u(k)|
\quad (4.3)
\]
required in all the three strategies and it is observed that lesser control effort is required in the redefined technique than in the previous techniques viz. GR, BMR and MMR strategies. The control quality criterion, defined as (Bartoszewicz 1998),

\[ Q = \sum_{k=0}^{n} |y(k)| \]  (4.4)

which estimates the amount of ripple in the output, has also improved in the proposed strategy. To illustrate the improvements with the redefined \textit{a priori} known function the response of systems under different environmental conditions are simulated with existing strategies and proposed strategy and presented in the following section.

4.4 \hspace{1cm} \textbf{NUMERICAL EXAMPLES}

Consider a discrete system with following parameters.

\[
\begin{align*}
    A &= \begin{bmatrix} 1 & 1 \\ 0 & 0.5 \end{bmatrix}, & b &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
    C &= \begin{bmatrix} 1 & 1 \end{bmatrix}, & x_0 &= \begin{bmatrix} 1000 \\ 0 \end{bmatrix}
\end{align*}
\]  (4.5)

This system, subjected to different environmental conditions, is simulated with different types of controllers discussed above and the response in each case is plotted.

4.4.1 \hspace{1cm} \textbf{System Subjected to Time-Invarying External Disturbance}

In this case the exogenous disturbance vector assumed is

\[
f = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]  (4.6)
with no parameter variation i. e. \( \Delta A = 0 \). The typical responses are simulated and shown in Figures 4.3, 4.4 and 4.5. It is observed from Figure 4.4 and Figure 4.5 that the settling time of the system improves appreciably when controlled by the reaching law with redefined \textit{a priori} known function. With \( k \) set to 20, the system settles exactly at \( kT = 20 \) with BBR strategy and approximately around \( kT = 14 \) with the redefined strategy. Further it is found from all the three figures that the dynamics and also the phase trajectory are smoother with redefined strategy than in Bartoszewicz strategy, an appreciable factor.

Figure 4.3  Evolution of switching function \( s(x) \) with time-invariant disturbance
Figure 4.4 System output with time-invarying disturbance

Figure 4.5 Phase trajectory of system with time-invarying disturbance
For BBR strategy and redefined strategy, the control quality criterion $Q$, given by Equation (4.4), found to be 11,500.000 and 6293.585 respectively, which shows the improvement with redefined strategy, in terms of efficiency. The improvement in control quality effort $J$, given by Equation (4.3) may also be observed with the calculated values for BBR and redefined strategies at 649 and 601 respectively.

For different values of $k$, the controllers according to GR, BBR and redefined strategies are simulated and the required control effort $J$, control quality $Q$ and the settling time $t_s$ are estimated and tabulated in Table 4.1. It is observed that the GR strategy is independent of $k^*$ for obvious reasons. For the controllers designed according to BBR strategy the control quality rapidly deteriorates with the increase in $k^*$ and there is marginal improvement in the required control effort with the settling time around the selected $k^*$. With the redefined controller also the control quality diminishes, but still better than the BBR strategy. The required control effort remains almost same and better than that of BBR strategy. The settling time with the redefined controller is observed to be the best among the three strategies and improves for higher values of $k^*$.

4.4.2 System Subjected to Time-Varying External Disturbance

The same system given by Equation (3.21) is considered but exogenous disturbance taken is time-varying and given as

$$f = \begin{bmatrix} 0 \\ 0.04 \times \lvert 50-k \rvert - 1 \end{bmatrix}$$  (4.7)
Table 4.1 Performance analysis of different DSM strategies for systems with time-invariant external disturbances and without parameter variation

| $k^*$ | Strategy          | Control Quality $Q = \sum_{k=0}^{n} |y(k)|$ | Control Effort $J = \sum_{k=0}^{n} |u(k)|$ | Settling Time ($t_s$) |
|-------|-------------------|------------------------------------------|------------------------------------------|----------------------|
| 10    | Gao               | 16460.810                               | 910.940                                 | 60                   |
|       | Bartoszewicz      | 6500.00                                 | 699.00                                  | 11                   |
|       | **Redefined Priori** | **4660.000**                           | **622.87**                              | **10**               |
| 20    | Gao               | 16460.810                               | 910.940                                 | 60                   |
|       | Bartoszewicz      | 11500.00                                | 649.00                                  | 21                   |
|       | **Redefined Priori** | **6293.585**                           | **601.00**                              | **17**               |
| 30    | Gao               | 16460.810                               | 910.940                                 | 60                   |
|       | Bartoszewicz      | 16500.00                                | 632.33                                  | 31                   |
|       | **Redefined Priori** | **7549.461**                           | **601.00**                              | **20**               |
| 50    | Gao               | 16460.810                               | 910.940                                 | 60                   |
|       | Bartoszewicz      | 26500.00                                | 619.00                                  | 51                   |
|       | **Redefined Priori** | **9543.127**                           | **601.00**                              | **26**               |

The parameter variations ($\Delta A$) are neglected in this case also. This system is simulated for $k = 0$ to 100 and the dynamics of the system obtained under this condition are plotted in Figures 4.6 and 4.7. From Figure 4.6 it is observed that the system dynamics remain on the switching line $s(x) = 0$, once it reaches it and is also void of chattering apart from smoother transition from reaching phase to sliding phase. A closer look of the response when controlled by BBR strategy is also presented along with that of redefined strategy in Figure 4.8, for better understanding. For $k = 100$, the width of the QSM band is calculated as 2, ranging from -1 to +1, i.e. $\delta_d = 1$, and from
Figure 4.6 it is clear that the dynamics of the BBR strategy overshoots the QSM band. Hence, the controller designed using modified BBR strategy, called as MBR strategy, is used and the system dynamics is shown in Figure 4.8. Though the dynamics are restricted within the QSM band, it is set to cross the switching plane with MBR based controllers as observed from Figure 4.8. The advantage of the controllers based on the redefined a priori known function is evident from Figure 4.8 as the dynamics lie on the switching plane once it is reached. With $k^*$ set to 30, the calculated values of the control quality criterion $Q$ are respectively 7549.461 and 16567.520 for redefined and Bartoszewicz strategies. Hence, a better control quality criterion $Q$ is observed with the redefined strategy. The control effort $J$ measured are 601 and 583.783 for redefined and Bartoszewicz strategies respectively, which shows that the control effort is better in case of Bartoszewicz strategy, but at the cost of steady-state accuracy. However, the settling time is found to be better with redefined strategy.

![Figure 4.6 Evolution of $s(x)$ with time-varying external disturbance](image-url)
Figure 4.7 Comparison of $s(x)$ with time-varying external disturbance

Figure 4.8 Evolution of $s(x)$ with MBR and redefined strategies
The system performance is analyzed for different values of $k^*$ under the above mentioned environmental condition and the estimated parameters are tabulated in Table 4.2. It is clear from the Table 4.2 that the redefined strategy settles early with lesser control quality than Bartoszewicz strategy and better steady-state accuracy is achieved with the redefined strategy at the cost of increased control effort.

Table 4.2 Performance analysis of different DSM strategies for systems with time-invariant external disturbances and without parameter variation

<table>
<thead>
<tr>
<th>$k^*$</th>
<th>Strategy</th>
<th>Control Quality</th>
<th>Control Effort</th>
<th>Settling Time (ts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Bartoszewicz</td>
<td>6597.120</td>
<td>649.82</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Redefined</td>
<td><strong>4690.216</strong></td>
<td><strong>622.877</strong></td>
<td><strong>11</strong></td>
</tr>
<tr>
<td>20</td>
<td>Bartoszewicz</td>
<td>11586.320</td>
<td>600.620</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Redefined</td>
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<td><strong>601.000</strong></td>
<td><strong>18</strong></td>
</tr>
<tr>
<td>30</td>
<td>Bartoszewicz</td>
<td>16567.520</td>
<td>583.753</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Redefined</td>
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<td><strong>601.00</strong></td>
<td><strong>22</strong></td>
</tr>
<tr>
<td>50</td>
<td>Bartoszewicz</td>
<td>26505.920</td>
<td>548.020</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Redefined</td>
<td><strong>9543.127</strong></td>
<td><strong>601</strong></td>
<td><strong>30</strong></td>
</tr>
</tbody>
</table>

4.4.3 System Subjected to Parameter Variation

The system is now subjected to parameter variation and analyzed using both MBR and redefined prior strategies. The parameter perturbation vector assumed is

$$\Delta A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad (4.8)$$
The simulated results are given in Figure 4.9 and Figure 4.10. From Figure 4.9 it is obvious that the system is unstable when controlled by MBR strategy. Figure 4.10 shows that the redefined strategy discussed in this section too fails to stabilize the system under the influence of parameter uncertainty. Hence, these strategies require improvement to meet this shortfall.

Figure 4.9 Evolution of s(x) when subjected to parameter variation – Bartoszewicz strategy
Figure 4.10  Evolution of $s(x)$ when subjected to parameter variation  
redefined a priori strategy

4.5  CONCLUSION

The drawbacks of the existing controllers and the need for redefined controllers are discussed in this chapter. The a priori known function is redefined and the controllers designed according to this redefined strategy are applied to stabilize the system under various environmental conditions. The advantages of redefined controllers over the controllers based on Bartoszewicz strategies are also presented. It is also observed that the controllers designed according to both Bartoszewicz strategy and redefined strategy is unable to stabilize the system when subjected to parameter perturbation and hence required improvements.