CHAPTER 1

INTRODUCTION

1.1 GENERAL

The Linear Programming Problem (LPP) of the mathematical programming model consists of a linear objective function and a set of linear constraints to be fulfilled. It has become an important model in modern theoretical and applied mathematics. It is also proved to be one of the most effective tools in operations research for decision making. But one of the common problems in the practical application of LPP is the difficulty in determining the proper values of model parameters. i.e., some or all of the model parameters may be random variables. What is needed is a way of formulating the problem so that the optimization will directly take the uncertainty into account. One such approach for mathematical programming under uncertainty is Stochastic Programming (SP). Therefore SP is an optimization technique in which the constraints and/or the objective function of an optimization problem contains certain random variables. The source of random variables may be several depending on the nature and type of the problem. SP models were first formulated by Dantzig (1955).

In the real world, problems are modelled as SP in various spheres. For example, modelling of an investment portfolio so as to meet random liabilities and modelling of strategic capacity investments, power systems, financial planning (Carino et al 1994), telecommunication network planning (Sen et al 1994), supply chain management (Fisher et al 1997) etc. Different
types of SP models have been developed to suit the different purposes of management. The first type of SP is the Expected Value (EV) model, which optimizes the expected objective function subject to some expected constraints. The second, Chance Constrained Programming (CCP) was pioneered by Charnes and Cooper (1959) as a means of handling uncertainty by specifying a confidence level at which it is desired that the stochastic constraint holds. After that, Liu (2002) generalised CCP to the case with not only stochastic constraints but also stochastic objectives. In the recent past, SP has been applied to the problems having multiple, conflicting and non-commensurable objectives, where generally there does not exist a single solution which can maximize (minimize) all the objectives. This gives rise to another mathematical programming model called Multi-Objective Stochastic Programming (MOSP) and if the objective of MOSP is non-linear, then Multi-Objective Non-Linear Stochastic Programming (MONLSP) is obtained.

There is another real world situation, where a decision maker often faces a problem of optimizing ratio(s). For example, in a manufacturing company, one may have to optimize output/cost, profit/cost etc. Therefore, a linear Fractional Programming (FP) model is the one used to optimize the ratio of two linear functions subject to a set of constraints along with non-negative conditions on the underlying variables. Similarly, one can define Non-Linear Fractional Programming (NLFP) as optimizing the ratio of two functions in which at least one function is non-linear, subject to a set of constraints along with non-negative conditions on the underlying variables. If the objective coefficients and/or constraints are random, then one may have Non-Linear Stochastic Fractional Programming (NLSFP). If there are more objectives, then Multi-Objective Non-Linear Stochastic Fractional Programming (MONLSFP) is obtained. Moreover, in the recent past there have been varieties of CCP models introduced for many engineering and management applications. Among them, the CCP model for reliability
optimization is a prominent one. In this, the CCP model is developed for Redundancy Allocation Problem (RAP), wherein redundancy is added to system components to improve system reliability. Another important CCP model is in the field of Data Envelopment Analysis (DEA), wherein the stochastic efficiency of a chosen Decision Making Unit (DMU) is measured using other DMUs.

There are a number of solution techniques available in literature to obtain optimal solutions for all the decision making problems described above. The logical analysis and conclusions of all decision making problems are based on the concept of models and model building. The formulation of any mathematical programming model generally involves a detailed study of the system, data collection and identification of decision variables and construction of the objective function and system constraints. In the course of formulating a model, model builders often tend to include inadvertently, variables/constraints and/or objective functions that may not play a role in defining the feasible set. The presence of such redundant constraints/variables and/or objective functions is hardly disputed and can play havoc with solution procedures and bring down the computational efficiency. It is well known that every additional constraint/variable and/or objective function in a problem demands more computational effort and computer memory. The identification of such embedded non-role play constraints/variables and/or objective functions in the model without affecting the character of the system is as difficult as solving any mathematical programming problem itself.

1.2 REDUNDANCY AND NEED FOR STUDYING REDUNDANCY

Anything omitted without affecting the concerned system is redundant. In the light of this definition, redundancy is something that permits reduction of a system to a smaller size having the same properties as the
original system. A redundant constraint/variable in a mathematical programming problem plays no role when it comes to solving for a final optimal solution.

1.2.1 Need for Redundancy Removal

The existence of redundancies in a mathematical programming model can not only play havoc in solution procedures but also mislead good decisions, which can indirectly affect the organization. This paved the way for several researchers to develop algorithms for identification and removal of redundancies in order to obtain a reduced model. There always had been a need for developing simple algorithms to detect \textit{apriori} redundant constraints at minimal computational effort. The main objective of these algorithms had been to remove all redundancies in a model, which is not always possible in practice. This has led to many improvements in developing algorithms to identify as many redundancies as possible in a given problem. In order to find optimal solutions efficiently, in the first part of the research work a few redundancy algorithms are proposed for LPP and mainly for SP problems. The idea used for finding redundancy in these SP problems is to convert them into equivalent deterministic models.

The proposed approach of developing redundancy algorithms for the purpose of finding optimal solutions efficiently, suffers from the following limitations:

(i) Finding the deterministic equivalent of SP model is not easy in many cases;

(ii) It is very difficult to determine how much redundancy is present in a given model until it is solved.
There is always a question whether it is worth spending time to identify redundancy when there are sophisticated computers to solve large scale problems in less time. These limitations were motivating factors to think of artificial intelligence technologies such as a Genetic Algorithm (GA) for solving a variety of SP problems. Therefore in yet another attempt to finding optimal solutions efficiently, in the second part of the research work different GAs are developed for solving some special classes of SP problems. Here one need not reduce the problem size and hence the problem can be taken for solving directly. Another advantage is that the GA does not require any complex mathematical calculations and is simple to apply. A GA is a stochastic search method for optimization problems based on the mechanics of natural selection and natural genetics (i.e., survival of the fittest). The GA has been well documented in the literature, such as in Holland (1975), Goldberg (1989), Michalwich (1994) and Fogel (1984).

1.3 NEED FOR SOLVING SP PROBLEMS THROUGH GENETIC ALGORITHM

The following are the reasons as to why SP problems may be solved using GA:

- Most of the times data involved in the problem are not normal. Stochastic parameters involved in SP problems may follow any of the continuous distributions such as Weibull, Gamma etc., and these are difficult to handle.

- Deriving deterministic equivalents of the SP problem is not easy for all distributions. The basic solution technique of an SP problem has been to convert the stochastic constraints to their respective deterministic equivalents and it is proved to be difficult due to multivariate integration and is possible if the
stochastic parameters involved in the SP problem follow some specific distributions such as Uniform, Normal, Lognormal and Exponential. Therefore, traditional procedures cannot be used

- When the stochastic parameters involved in the problem follow mixed distributions, conversion of the stochastic constraint into its corresponding deterministic equivalent is tedious and there are no compact methods available to do so

- Even after a great struggle some of the stochastic constraints with known distribution may be converted into deterministic form. But to solve such a reduced non-linear programming model, the constraints need to obey properties of differential calculus, which happen to fail most of the time

From the above, it is clear that deriving deterministic equivalent for other stochastic parameters is really difficult. Even using the standard software packages such as Mathematica, Maple, Matlab, the multivariate integration cannot be computed. Therefore with the rapid development of computer technology and software, artificial intelligence technologies such as GA has been found to offer an edge over conventional methods to deal with complicated problems.

1.4 OBJECTIVES OF THE STUDY

In this section, the problems considered in the proposed research work and its objectives are discussed briefly.

1.4.1 Finding Redundancy in LPP and MONLSFP Problems

For a speedy way of obtaining the optimal solution, redundancies are identified and removed from the following problems.
1.4.1.1 To find redundant constraints and redundant variables in LPP

The objective of the proposed Direct method is to provide an apparent reduced LP model after removing irrelevant constraints and unattractive variables. The reduced LP model is finalized only after the apparent LP model is validated. For some of the LPPs, the Direct method yields optimal solution without using Simplex method. In case the Direct method fails to yield the optimal solution, the Simplex procedure is invoked with the Advanced basis generated by the Direct method.

1.4.1.2 To find redundant objective function(s) in MONLSFP problems

Redundancy in constraints and variables are usually studied in linear, integer and non-linear programming problems. Therefore the objective is to provide an algorithm that identifies redundant objective function(s), if any, in MONLSFP problems. The methodology used in this algorithm is the derivation of deterministic equivalence of stochastic objective functions. The derived non-linear constraint forms of objective functions are then linearised for finding redundant stochastic objective function(s).

1.4.1.3 To find redundant objective function(s) and redundant constraint(s) in MONLSFP problems

As an extension of the previous work, an algorithm is developed with an objective of finding both redundant objective function(s) and redundant constraint(s) in MONLSFP problems. The deterministic equivalents of stochastic constraints and stochastic objective functions are derived and then linearised. The redundancy algorithm is applied to the linearised model for finding redundant constraints and redundant objective functions.
1.4.2 Simulation based Genetic algorithm techniques for solving some special classes of SP problems

The success of GAs in processing SP problems such as Chance Constrained Fractional Programming (CCFP), Chance Constrained Reliability Stochastic Optimization (CCRSO) and Chance Constrained Data Envelopment Analysis (CCDEA), have been exploited by combing GA with Simulation.

1.4.2.1 To develop stochastic simulation based genetic algorithm for CCFP problems

Generally deriving the deterministic equivalence of a Stochastic Fractional Programming (SFP) problem with random coefficients in the objective function and/or in the constraints is difficult and is possible only if the random coefficients follow some specific distributions with known parameters. Therefore the objective here is to develop a stochastic simulation based GA for solving CCFP problems. Monte Carlo simulation is employed to check the feasibility of chance constraints.

1.4.2.2 To develop stochastic simulation based genetic algorithm for CCRSO problem

The objective of this research work is to address the CCRSO problem, where the objective is to maximize system reliability for the given chance constraints. For this purpose, an n-stage series system with m-chance constraints of the RAP is considered. A complete GA is developed for the CCRSO problem with various cases of randomness (with known distributions) considered for resource variables.
1.4.2.3 To develop stochastic simulation based genetic algorithm for CCDEA problems

In this work, the P-model CCDEA problems have been introduced, which include the concept of “Satisficing”. The basic approach to solve the CCDEA has been to convert chance constraints and the P-model objective function into their deterministic equivalents. This approach is difficult in some cases and in many cases it fails to produce the desired optimal solution when the data involved follow some specific continuous distributions. Therefore the objective is to provide a stochastic simulation based genetic algorithm which does not require any conversion technique. A case study problem involving data pertaining to a total of 31 banks (both public and private banks) is solved to verify the effectiveness of the proposed GA.

1.5 AN OVERVIEW OF THE THESIS

The problems stated and contributions made for solving the above mentioned problems have been organised in this thesis under seven chapters.

The first half of Chapter 2 provides review of literature, background and related work with regard to various redundancy algorithms developed so far in the field of LPP and SP problems. Subsequently detailed literature survey, contributions on how researchers have used GA as an optimization tool for their various SP problems are explained.

Chapter 3 describes a few redundancy algorithms for identifying redundant variable(s)/constraint(s) and/or objective function(s). In the first part, a Direct method of solving the LPP problem is proposed. The second part proposes a new approach for finding redundant objective function(s) if any for MONLSFP problems. Finally, as an extension a more versatile
redundancy algorithm is introduced for the purpose of identifying both redundant objective function(s) and redundant constraint(s) in MONLSFP problems. Numerical examples are introduced to illustrate the effectiveness of all the algorithms.

In chapter 4, initially a CCFP model is defined with parametric form. Then a stochastic simulation based GA is developed as a solution technique for the parametric form of CCFP problems. In this, feasibility of chance constraints is checked by Monte Carlo simulation which is explained in a lucid manner. Finally, the solution procedure is illustrated with a few numerical examples. The results obtained through the GA are compared with analytical approach results, wherever possible.

In chapter 5, initially the stochastic integer programming problem is discussed for an $n$ - stage series system with $m$ chance constraints. Simple algorithms for random number generation for various cases of random resource variables of the CCRSO problem are also given. In the next section, handling of stochastic constraints using Monte Carlo simulation has been explained. Finally, GA and Monte Carlo simulation are effectively combined leading to a well structured GA for solving CCRSO problems. Effectiveness of the proposed GA is illustrated by a numerical example.

The GA is developed in chapter 6 for solving the P-model of CCDEA problem, which includes the concept of “Satisficing”. In this approach, how the stochastic objective functions and chance constraints are directly used within the genetic process is well explained. To demonstrate managerial applications of the above approach, a case study problem is considered and solved to check whether the DMU under consideration is stochastically efficient or not.
Chapter 7 concludes the thesis with new contributions made in the field of redundancy algorithms. The versatility of the GA as an optimization tool is also well explained in solving various SP problems such as CCFP, CCRSO and CCDEA problems. Finally, directions for future research are given for interested scholars.

1.6 A COMPLETE RESEARCH FRAMEWORK

In the field of Operations Research, researchers often encounter real problems in which the Decision Maker wishes to optimize objective function(s) subject to several constraints involving random coefficients. Therefore with the requirement of considering randomness, different types of SP have been developed to suit the different purposes of Engineering and Management problems. Though there are methods available for solving these problems, they suffer from limitations (i) in terms of speed in obtaining the optimal solution (ii) in handling constraints where random coefficients follow continuous distributions such as Weibull, Gamma distributions etc. These issues have been taken up as a research problem and to address the problem new algorithms have been proposed and presented as a research framework sown in Figure 1.1.
Figure 1.1 Research Framework