CHAPTER 6

STOCHASTIC SIMULATION BASED GENETIC ALGORITHM FOR CHANCE CONSTRAINED DATA ENVELOPMENT ANALYSIS PROBLEMS

6.1 INTRODUCTION

Data Envelopment Analysis (DEA) as developed by Charnes et al (1978) and extended by Banker et al (1984) (BCC) and Charnes et al (1985) (Additive model) is a non-parametric linear programming method for estimating production frontiers and evaluating the relative efficiency of Decision Making Units (DMUs) with multiple outputs and multiple inputs. One of the advantages of the DEA method is that it does not require either *apriori* weights or explicit specification of functional relations between the multiple outputs and inputs. For surveys of DEA, readers are referred to Charnes and Cooper (1985), Banker et al (1989), and Seiford and Thrall (1990). In these treatments, the input and output data are always assumed to be deterministic.

However the extreme point nature of the deterministic DEA method makes efficiency measurement highly sensitive to changes in data causing significant problems when data are not known with certainty. With the enrichment of the DEA literature, practitioners have acknowledged the need to incorporate data variability and uncertainty within the deterministic DEA models with the aim of addressing measurement errors, as well as the inherent stochastic nature of production processes. For example, Banker (1993) incorporated statistical elements into DEA and developed a non-parametric approach with maximum likelihood methods used to effect inferences in the presence of statistical noise. Hall and Simar (2002) have proposed a technique for estimating a boundary point in the presence of statistical noise. Since this is basically a univariate technique, Simar (2007) adapted the Hall-Simar methodology to a multivariate frontier setup providing stochastic version of DEA estimators. This improves the performance of the standard DEA estimators in the presence of noise.

Proceeding in a different direction, Land et al (1993) utilized chance constrained programming theory to develop a chance constrained efficient frontier which envelops a given set of observations most of the time. In their chance constrained format, the test of efficiency for a DMU is conducted by stochastically comparing its inputs and outputs with best practice which is formed as a weighted arithmetic mean of all observations. Olesen and Petersen (1995) developed chance constrained version of the multiplier form of the CCR model using the marginal chance constraints. Cooper et al (1996) incorporated Simon's (Simon 1957) "Satisficing Concepts" into DEA and developed a satisficing DEA model with CCP theory reflecting theories of behaviour in social psychology. Bruni et al (2009) in their P-model of CCP utilized Joint chance constraints to extend the concept of "Stochastic efficiency" to a measure called "alpha stochastic efficiency".

Solution methods for solving these CCDEA problems so far have been to convert objective function and chance constraints to their respective deterministic equivalents according to the predefined confidence levels and then solve the deterministic model by the traditional solution procedure. The converting process is usually difficult and possible in some special cases. Therefore, scientists and researchers started looking at an easy-to-use tool such as "hybrid intelligent algorithm" for solving CCDEA problems.

In the present study, a stochastic simulation based Genetic algorithm for solving P-model CCDEA problem has been proposed to include the concept of "Satisficing". In this algorithm, stochastic simulation is used (i) to evaluate P-model objective function (ii) to check the feasibility of chance constraints. To illustrate the above approach, a case of the Indian banking sector has been presented.

6.1.1 Application to Indian Banking Sector

The Indian banking sector has gone through structural changes since independence keeping pace with the changes in policymakers' attitude towards planning. To overcome the shackles of the colonial past and transform the stagnant agro-based economy to a self-reliant industrial economy, policymakers turned toward comprehensive economic planning in the initial years. In the interest of building up a strong industrial base, a highly restrictive industrial policy was introduced. This was accompanied by numerous import restrictions. On the other hand, to generate income and employment among the backward and underprivileged sections of population, a whole set of egalitarian socio-economic policies was developed. Among these, probably the most important step was the nationalization of major commercial banks in 1969, which allowed complete government control over the country's financial and credit resources. Over time, however, with changes in the needs of a maturing domestic economy, a new wave of economic liberalization developed. This trend was supported by the new government that came into power in 1984. Several changes were brought about to liberalize industrial licensing policies, to promote capacity expansion in existing industries, and to lift the regulations on foreign capital. The liberalization process gathered momentum in the late 1980's, as reflected by the new industrial policies of 1988 and 1990. These policy changes, in turn, called for a relaxation of the stringent regulations on the financial and banking sectors. Several deregulatory measures were introduced in commercial banking in the mid- and late 1980's.

Some liberalization began during 1980s. But in the year 1992, a substantial liberalization of India's banking and financial sector began, as part of liberalization of the economy, stimulated by the 1991 balance of payments crisis. The intent of general liberalization was to speed up growth and thereby reduce poverty. The general approach of the economy-wide liberalization was to open up the economy, give the market a greater role in price setting, and increase the private sector's role in development for example, tariffs were dropped significantly and quantitative restrictions on trade reduced.

In the financial sector, the 1991 Narasimham Committee recommendations provided a blueprint for the reforms. A number of reforms such as entry deregulation, branch delicensing, deregulation of interest rates and allowing public sector banks to increase equity in the capital market up to 49% has resulted in heightened competitive pressure in the banking sector. These changes came in the form of automatic teller machines and internet banking, huge increase in housing and consumer credit, stronger and more transparent balance sheets and product diversification (Sahoo et al 2007). A significant intent of these policies is to have a radical transformation in the operating landscape of the Indian banks. The intense competition between both domestic and foreign banks, rapid speed of innovations and introduction of new financial instruments, changing consumer's demands and desire for product augmentation have changed the way a bank conducts business and services its customers. Larger the degree of competition, the more efficient the firms would become more efficient (Akhtar 2002). However, when the structure of an industry is the product of the government regulations, the degree of competition is impaired markedly implying that the efficiency suffers negatively (Bhaumik and Dimova 2003).

Numerous attempts have been made to study the efficiency of banks in developed countries[1]. In contrast, studies analyzing the efficiency of banks in developing countries, including India are limited. In their extensive international literature survey, Berger and Humphrey (1997) noted that the vast majority of the efficiency literature focuses on the banking markets of well-developed countries with particular emphasis on the U.S. markets. The relatively scant literature on bank efficiency in emerging markets focuses mainly on the efficiency differentials among banks with different ownership status and asset size. In one of the studies, Bhattacharya et al (1997) studied the impact of the limited liberalization initiated before the deregulation of the nineties on the performance of the different categories of banks, using the DEA. Their study covered 70 banks during the period of 1986-91. They constructed one grand frontier for the entire period and measured technical efficiency of the banks under study. They found public sector banks had the highest efficiency among the three categories, with foreign and private banks having much lower efficiencies.

Das (1997) analyzed overall efficiency- technical, allocative and scale- at PSBs. In the period 1990-96, the study found a decline in overall efficiency. This occurred because there was a decline in technical efficiency, both pure and scale, which was not offset by an improvement in allocative efficiency. Saha and Ravishankar (2000) analyzed the performance of Indian banks using DEA approach for a sample of 25 public sector banks over the 1992-1995 period. They observed the increasing trend of efficiency over a period of time. On the other hand, in a study, Sathye (2003) applied DEA under variable returns to scale technology to measure the productive efficiency of 94 banks for the year 1996-1997. The results reveal that on an average, the public

[1]The brief overview of literature on efficiency measurement of banks using both parametric and non-parametric techniques in US and European countries can be seen in Dániel Holló and Márton Nagy (2006).

sector banks are relatively more efficient than foreign banks, which in turn were more efficient than that of private banks. Das et al (2004) empirically analyzed various efficiency scores of Indian banks during 1997-2003 using data envelopment analysis (DEA). They observed that Indian banks are not much differentiated in terms of input or output oriented technical efficiency and cost efficiency. However, they differ sharply in respect of revenue and profit efficiencies. Bank size, ownership and the fact of its being listed on the stock exchange are some of the factors that are found to have positive impact on the average profit efficiency and to a certain extent on revenue efficiency scores. Sahoo et al (2007) examined the productivity performance trends of the Indian commercial banks for the period: 1997-98 to 2004-05. Their findings reveal increasing average annual trends in technical efficiency for all ownership groups indicating an affirmative gesture about the effect of the reform process on the performance of the Indian banking sector. The variations in the findings are possibly due to a number of factors such as differences in the measurement of inputs and outputs, differences in periods of study and their approaches of measurement.

However, all the above studies are based on deterministic DEA models, and thus suffer from a possible lack of statistical power especially in small samples; the data can 'speak for themselves' only if the sample includes many efficient observations for a wide range of production vectors (Post 2007). The DEA by construction fixes the frontier in the relevant space and encompasses all sample observations. Thus, a small subset of data supports the frontier, making it more prone to sampling, outlier, and statistical noise problems, which may distort the measurement of efficiency. Further, the DEA models do not account for the possibility of errors-in-variables, for example due to debatable valuation for accounting data or due to uncontrollable external factors. Further, for banks, the major sources of income stems from loan and advances and different types of investment, which are considered as risky. For a risk averse bank, the response to input/output fluctuations would be one of hedging against uncertainty and this would be quite different from the attitude of a risk taking bank. The transformation of the DEA model needed to allow for these risk averse and risk taking attitudes of banks provides an important avenue for using the stochastic DEA approach in the current study. The stochastic DEA helps to decompose technical inefficiency (input and output slacks) into environmental influences, statistical noise attributable to measurement errors in the original data, and managerial inefficiency. Without such adjustments, some banks might be penalized on their performance scores due to factors beyond managerial control, while others might be rewarded for operating in favourable environments. The current study deals with the stochastic aspect of the DEA and reports the stochastic efficiency of the banks under study.

6.2 MOTIVATION

An abundance of optimization methods have been used to solve various DEA problems. The algorithms applied are either heuristics or exact procedures based mainly on modifications of dynamic programming and linear fractional programming. Most of these methods are strongly problem oriented. This means that, since they are designed for solving certain optimization problems, they can’t be easily adapted for solving other problems. In the present case, conversion of P-model CCDEA problem into its deterministic equivalent is possible only for problems which contain random coefficients following normal distributions or exponential distributions. In recent years, many studies use a universal optimization approach based on metaheuristics. The metaheuristrics are based on artificial reasoning rather than on classical mathematical programming. All metaheuristics use the idea of randomness when performing a search. GAs are one of the most widely used metaheuristics. Therefore, a stochastic simulation based GA is developed for solving P-model CCDEA problems, which include the concept of "Satisficing". To the best of our knowledge, this is the first attempt where simulation based GA has been developed and implemented in the area of CCDEA.

6.3 DIFFERENT FORMULATIONS OF DEA MODELS

6.3.1 Original Problem Formulation

Assuming that there are n DMUs, each with m inputs and s outputs, the relative efficiency score of a test DMU$_o$ is obtained by solving the following fractional programming model (CCR model) proposed by Charnes et al (1978).

$$\underset{v,u}{Max} \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \tag{6.1}$$

subject to

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1, 2, ..., n$$

$$u_r, v_i \ge 0, \quad \forall r, i,$$

where y_{rj} - amount of output r produced by DMU$_j$,

x_{ij} - amount of input i utilized by DMU$_j$,

u_r - weight given to output r,

v_i - weight given to input i.

By taking v as row vector for input multipliers and u as row vector for output multipliers, the fractional program shown in system (6.1) can be converted to a multipliers form of linear program as given below:

$$\underset{v,u}{Max}\ uy_o \tag{6.2}$$

subject to

$$vx_o = 1$$

$$-vX + uY \leq 0$$

$$v \geq 0, u \geq 0$$

where $Y = ((y_j))$ is the matrix of outputs of all units in the sample and $X = ((x_j))$ is the corresponding matrix of inputs. The above problem is run n times in identifying the relative efficiency scores of all DMUs. A DMU is considered to be efficient, if it obtains a score of 1 and a score of less than 1 implies that it is inefficient.

For every inefficient DMU, DEA identifies a set of corresponding efficient units that can be utilized as benchmarks for improvement. The benchmarks can be obtained from the dual problem as shown below:

$$\underset{\theta,\lambda}{Min}\ \theta \tag{6.3}$$

subject to

$$\theta x_o - X\lambda \geq 0$$

$$Y\lambda \geq y_o$$

$$\lambda \geq 0,$$

where θ is the measure of efficiency and $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ is a vector of weights.

6.3.2 Chance Constrained Problem Formulation

To incorporate stochastic data variations in inputs and outputs into the DEA of efficiency measurement, chance constraints can be imposed on either the primal formulation or it's dual.

The chance constrained formulation of system (6.3) proposed by Land et al (1993) is as shown below:

$$\underset{\theta,\lambda}{Min}\ \theta$$

subject to

$$P\left[Y\lambda \le y_o\right] \le \alpha$$

$$X\lambda \le \theta x$$

$$\lambda \ge 0\,.$$

The chance constraint $P\left[Y\lambda \le y_o\right] \le \alpha$ shows that only α or less than α percent of DMUs will produce output exceeding the best practice level.

6.3.3 P- model CCDEA Problem Formulation

In this P-model approach, it is the probability of efficient input/output combinations from the evaluated DMU that is maximized under a set of chance constraints. Given below is the P-model CCDEA problem formulation that extends the CCR (ratio) model of DEA.

$$\underset{v,u}{Max}\ P\left(\frac{\sum_{r=1}^{s} u_r \tilde{y}_{ro}}{\sum_{i=1}^{m} v_i \tilde{x}_{io}} \ge 1\right) \tag{6.4}$$

subject to

$$P\left(\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}} \le 1\right) \ge 1-\alpha_{j,} \qquad j = 1, 2, ..., n$$

$$u_r, v_i \ge 0, \quad \forall r, i.$$

Here "P" means probability and "$\sim$" identifies that these outputs and inputs as random variables with a known probability distribution. While $0 \le \alpha_i \le 1$is a scalar, specified in advance, which represents the allowable chance (risk) of failing to satisfy the constraints with which it is associated.

6.4 SATISFICING IN DEA

DEA with stochastic elements has recently begun to receive significant attention. Thore (1987), Banker (1993), Land et al (1994) made efforts for joining CCP with DEA in order to include data uncertainties in DEA. As an extension of the above, Cooper et al (1996) incorporated Simon's (1957) satisficing concepts into DEA models with chance constraints in order (a) to effect contact with theories of behavior in social psychology as well as (b) to extend the potential uses of DEA models to cases where 100% efficiency can be replaced by aspired levels of performance. Given below is the P-model CCDEA with "Satisficing concepts".

$$\underset{v,u}{Max}\, P\left(\frac{\sum_{r=1}^{s} u_r \tilde{y}_{ro}}{\sum_{i=1}^{m} v_i \tilde{x}_{io}} \ge \beta_o\right) \qquad (6.5)$$

subject to

$$P\left(\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}} \le \beta_j\right) \ge 1-\alpha_j, \qquad j=1,2,\ldots,n$$

$$u_r, v_i \ge 0, \quad \forall r, i.$$

Here "P" and "$\sim$" are as defined above and define β_o as an aspiration level either imposed by an outside authority or adopted by an individual for some activity. The β_j's are also prescribed constants imposed by the individual. The idea of integrating "satisficing" into efficiency analysis offers the individual the opportunity to define a satisficing level for each output and input.

6.5 STOCHASTIC EFFICIENCY

It has been shown that efficiency concepts in conventional DEA models are actually the same as Pareto efficiencies associated with different "empirical" production sets (Charnes and Cooper 1985). In this deterministic situation, the efficient frontier is truly a surface of Pareto efficiency with maximum in outputs and minimum in inputs. But in stochastic situations, one may pursue less than 100 percent efficient frontiers using CCP ideas in Charnes and Cooper (1959). Let $1-\alpha$ be the modeler's confidence level when $0 \le \alpha < 1$. Then the efficiency definition in stochastic cases developed by Cooper et al (1996) is illustrated below:

Let β_o and β_{jo} be aspiration levels prescribed for DMU$_o$ in the objective and in its constraints. Also let $\gamma_o^* = P\left[\frac{\sum_{r=1}^{s} u_r \tilde{y}_{ro}}{\sum_{i=1}^{m} v_i \tilde{x}_{io}} \geq \beta_o\right]$, where u_r^*, v_i^* are optimal values of u_r, v_i and α_{jo} be the risk of failing to satisfy the constraints for DMU$_{jo.}$

Definitions

(i) If $\beta_{jo} = \beta_o = 1$, then DMU$_o$ will have performed in a stochastically efficient manner if and only if $\gamma_o^* = \alpha_{jo}$.

(ii) If $\beta_{jo} = \beta_o = 1$, then DMU$_o$'s performance is stochastically inefficient with probability 1-γ_o^* if and only if $\gamma_o^* < \alpha_{jo}$.

Then the above two definitions translate into: Satisficing was attained or not according to whether $\gamma_o^* = \alpha_{jo}$ or $\gamma_o^* < \alpha_{jo}$.

6.6 STOCHASTIC SIMULATION

6.6.1 Constraints Check by Stochastic Simulation

Employing the stochastic simulation technique (Rubinstein 1981), check the constraints of the problem (6.5) as given below:

Consider the chance constraints

$$P\left(\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}} \leq \beta_j\right) \geq 1 - \alpha_{j,} \qquad j = 1, 2, ..., n \tag{6.6}$$

Let $g_i(u, v, \tilde{x}_j, \tilde{y}_j) = \dfrac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}}$. Then the chance constraints (6.6) can be represented as

$$P\left[g_i(u, v, \tilde{x}_j, \tilde{y}_j) \leq \beta_j \right] \geq 1 - \alpha_j, \qquad j = 1, 2, ..., n \tag{6.7}$$

where $u = (u_1, u_2, ..., u_s)$ and $v = (v_1, v_2, ..., v_m)$ are the output and input vectors of decision variables respectively, $\tilde{x}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, ..., \tilde{x}_{mj})$ and $\tilde{y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, ..., \tilde{y}_{sj})$ are the amounts of inputs and outputs random vectors utilized by the DMU_j. Each $\tilde{x}_{ij}$ and $\tilde{y}_{ij}$ have a given continuous probability distribution.

Then the N independent random vectors for each $\tilde{x}_j$ and $\tilde{y}_j$ are generated as follows:

$$\tilde{x}_j^{(k)} = (\tilde{x}_{1j}^{(k)}, \tilde{x}_{2j}^{(k)}, ..., \tilde{x}_{mj}^{(k)}), \quad k = 1, 2, ..., N, \qquad j = 1, 2, ..., n$$

$$\tilde{y}_j^{(k)} = (\tilde{y}_{1j}^{(k)}, \tilde{y}_{2j}^{(k)}, ..., \tilde{y}_{sj}^{(k)}), \quad k = 1, 2, ..., N \qquad j = 1, 2, ..., n.$$

Let $N_j^{'} (\leq N),\ j = 1, 2, ..., n$ be the number of times the following relation satisfies:

$$g_i(u, v, \tilde{x}_j, \tilde{y}_j) \leq \beta_j, \qquad j = 1, 2, ..., n\ . \tag{6.8}$$

Then by the definition of probability, (6.7) and hence (6.6) holds if

$$(N_j^{'} / N) \geq (1 - \alpha_j), \qquad j = 1, 2, ..., n. \tag{6.9}$$

6.6.2 Handling Stochastic Objective Constraint

For the following stochastic objective constraint

$$P\left[\frac{\sum_{r=1}^{s} u_r \tilde{y}_{ro}}{\sum_{i=1}^{m} u_r \tilde{x}_{io}} \geq \beta_o\right] \tag{6.10}$$

The probability value is calculated as given below:

Let $f(u, v, \tilde{x}_j, \tilde{y}_j) = \dfrac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}}$.

Thus the objective constraint of (6.10) can be represented as

$$P\left[f(u, v, \tilde{x}_j, \tilde{y}_j) \geq \beta_o\right], \tag{6.11}$$

where u and v are defined as above, $\tilde{x}_o = (\tilde{x}_{1o}, \tilde{x}_{2o}, ..., \tilde{x}_{m0})$ and $\tilde{y}_o = (\tilde{y}_{1o}, \tilde{y}_{2o}, ..., \tilde{y}_{so})$ are the amounts of input and output random vectors utilized by the DMU$_o$. Each $\tilde{x}_{io}$ and $\tilde{y}_{io}$ have a given continuous probability distributions. Then, the N independent random vectors for each $\tilde{x}_{io}$ and $\tilde{y}_{io}$ are generated as follows:

$$\tilde{x}_o^{(k)} = (\tilde{x}_{1o}^{(k)}, \tilde{x}_{2o}^{(k)}, ..., \tilde{x}_{mo}^{(k)}), \qquad k = 1, 2, ..., N$$

$$\tilde{y}_o^{(k)} = (\tilde{y}_{1o}^{(k)}, \tilde{y}_{2o}^{(k)}, ..., \tilde{y}_{so}^{(k)}), \qquad k = 1, 2, ..., N \ .$$

Let $N_o^{'} (\leq N)$ be the number of times the following relation satisfies:

$$f(u, v, \tilde{x}_o, \tilde{y}_o) \geq \beta_o$$

Then by the definition of probability, the probability value of (6.11) and hence (6.10) is equal to $N_o^{'} / N$.

6.7 STOCHASTIC SIMULATION BASED GENETIC ALGORITHM

As a stochastic search method based on the mechanics of natural selection and natural genetics, Genetic Algorithms (GAs) have demonstrated considerable success in providing good solution to many complex optimization problems and received more attentions during the past three decades. GAs have been well discussed and documented by numerous literatures. The steps of the GA are presented next.

Let

- V denote the set of individuals in the population,
- ω given individual in the population,
- K be the population size,
- M be the total number of generations that the population evolve and
- q denote a given generation.

Initialization: pop_size is defined as the number of chromosomes and pop_size chromosomes are randomly initialized. Assuming the predefined region, which contains the optimal solution (not necessarily the whole feasible set), a random point has been generated from the region. If it is feasible, then it will be accepted as a chromosome. If not, regenerate a point randomly until a feasible point is obtained. In this way, pop_size chromosomes are generated. In the example that follows, the values of $u_{r\,(r=1,2,...,s)}$ and $v_{i\,(i=1,2,...,m)}$ are chosen uniformly between 0 and the upper limits of r^{th} or i^{th} decision variables.

Constraints checking by stochastic simulation: Here, the technique of stochastic simulation is employed, which has been discussed in the previous section.

Fitness function: Since this is problem dependent, here the fitness of each chromosome is calculated according to the objective function value.

Selection: The primary objective of the selection operation is to emphasize good solutions and eliminate bad solutions in a population, while keeping the population size constant. There exists a number of ways to achieve the above tasks. Some common methods are Roulette wheel selection by Holland (1975), Tournament selection by Goldberg et al (1989), Ranking selection and others. The tournament selection method has been adopted as it has better convergence and computational time complexity properties compared to any other reproduction operator that exists in the literature. In this selection method, two individuals in the population are selected based upon the magnitude of their fitness value with respect to the rest of the population. Then of these two individuals, the one with the high fitness value is selected.

Crossover: The crossover operator is mainly responsible for the search aspect of genetic algorithm. Define a parameter p_c as the probability of crossover. Then $(p_c . p\_size)$ is the expected number of chromosomes which will undergo the crossover operation. A random number r is generated from the interval [0,1]. A chromosome $\omega_i\ (i=1,2,...,p\_size)$ is selected as a parent if $r < p_c$. The process is repeated p_size times and $(p_c . p\_size)$ parents are selected for crossover operation.

Let ω_1 and ω_2 be a pair of chromosomes, which will undergo crossover operation. Then randomly select an integer c' from the interval $(1, len)$, where len is the number of genes in a chromosome, exchange the genes after the c'^{th} gene of chromosomes ω_1 and ω_2 and produce two children. If the children are feasible, then the parents are replaced with them, otherwise, the crossover operation is repeated.

Mutation: To ensure variety and reduce the chance of local optimal convergence mutation operation is applied on chromosomes. For this, define a parameter p_m as the probability of mutation. For each chromosome, a random number r is generated within (0,1); if $r < p_m$, then the chromosome is selected for mutation. The process is repeated p_size times and $(p_m \cdot p\_size)$ parents are selected for mutation operation. Then an integer c' is generated randomly from the interval $(1, len)$ as the mutation position and the bit is mutated. If the new chromosome is infeasible, then the process is repeated until it is feasible.

6.8 GENETIC ALGORITHM FOR SOLVING CHANCE CONSTRAINED DEA PROBLEM

It is well known that GAs are capable of finding optimal solutions to problems having non-convex feasible domain. As the feasible domain of the CCDEA problem is non-convex, the following GA was developed. The following Figure 6.1 shows the structure of the GA and the various steps involved in the GA are summarized as follows:

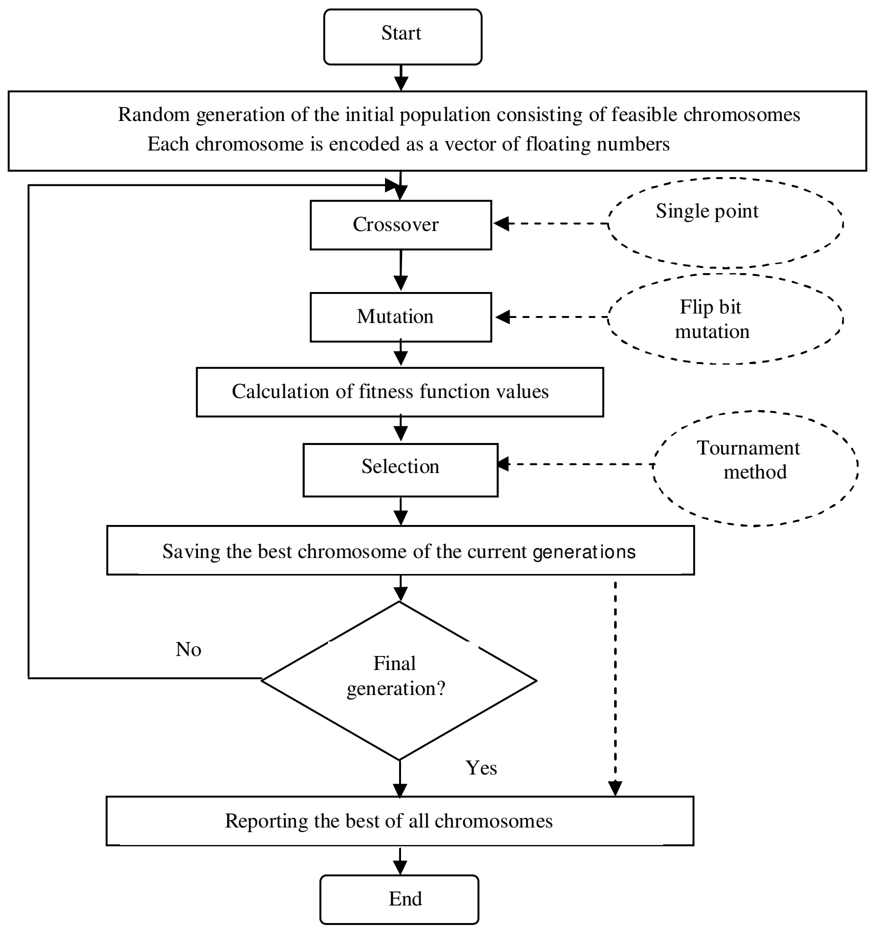

Figure 6.1 Genetic Algorithm Flowchart

Proposed Genetic algorithm

Step 1 : Randomly initialize p_size number of chromosomes according to the initialization process.

Step 2 : Check the system constraints by the technique of stochastic simulation.

Step 3 : Update the chromosomes by crossover and mutation operations after checking the feasibility of the offsprings.

Step 4 : For all chromosomes, calculate the objective values by stochastic simulation.

Step 5 : Compute the fitness of each chromosome based on the objective values.

Step 6 : Select the chromosomes according to the selection process.

Step 7 : Repeat steps 3-6 for a given number of generations.

Step 8 : Report the best chromosome as the optimal solution.

6.9 SELECTION OF INPUTS/OUTPUTS AND PREPARATION OF DATA FOR CCDEA

Selection of inputs and outputs and number of DMUs is one of the core difficulties in developing a productivity model and in preparation of the data. In the banking literature, there is a considerable disagreement among researchers about what constitute inputs and outputs of banking industry (Casu 2002; Sathye 2003). Until today, there is no all-encompassing theory of the banking firm and no agreement on the explicit definition and measurement of banks' inputs and outputs. Two different approaches appear in the literature regarding the measurement of inputs and outputs of a bank, popularly known as the "production approach" and the "intermediation approach" (Humphrey 1985). Both approaches apply the traditional microeconomic theory of the firm to banking and differ only in the specification of banking activities. The production approach, initiated by the contribution of Benston (1964) and Bell and Murphy (1968), describes banking activities as the production of services to depositors and borrowers. Under this approach, output is measured by the number and type of transactions or accounts (both deposit and loan) and inputs used are only physical units such as labor and capital, since only physical inputs are needed to provide financial services. In the absence of the availability of data on physical inputs, this approach considers operating costs as one of the inputs and excludes the interest expenses paid on deposits since deposits are viewed as outputs.

In the intermediation approach, banks are regarded as financial intermediaries. This approach is complementary to the production approach and describes banking activities as transforming the money borrowed from depositors into money lent to borrowers (Sealey and Lindley 1977, Freixas and Rochet 1997). This transformation activity stems from the different characteristics of deposits and loans. Deposits are typically divisible, liquid and riskless, while loans are indivisible, illiquid and risky. It views the banks as using deposits together with purchased inputs to produce various categories of bank assets. Outputs are measured in monetary values and total costs include all operating and interest expenses.[2]

Although there is no 'perfect approach', the intermediation approach is widely used by researchers for the reasons stated below:

(i) The use of the intermediation approach in bank productivity presents fewer data problems than with the production approach and the literature suggests that it is the most appropriate approach for evaluating entire banking industry as it is inclusive of interest expenses, which account for one-half to two-thirds of total costs of banks (Berger and Humphrey 1997).

(ii) As argued by Berger and Humphrey (1997), the intermediation approach is best suited for analyzing bank level efficiency whereas, the production approach is well suited for measuring

[2]Zhao et al. (2006) and Ramanathan (2007) for a detailed discussion on inputs and outputs used in banking study

branch level efficiency. This is because, at the bank level, the management will aim to reduce total costs and not just non-interest expenses, while at the branch level a large number of customer service processing takes place and bank funding and investment decisions are mostly not under the control of branches.

(iii) The intermediation approach is particularly important for banks, where most activities consists of turning large deposits and funds purchased from other financial institutions into loans or financing and investments (Favero and Papi 1995).

(iv) The availability of data on physical units is often a problem, whereas the monetary values of inputs and outputs may be deduced from annual account statements. Further, it is understood that the monetary values rather than physical units better capture the transformation and value addition by banks.

In the Indian context, Sahoo et al (2007) argued that besides being profit driven, banks are forced to take up economic and social responsibilities like safety of customers; financing much needed public sector expenditure in various social and economic services. This further justifies the use of the intermediation approach rather than the production approach.

Similar to many studies on banking efficiency (e.g., Aly et al 1990; Zaim 1995; DeYoung and Nolle 1998; Berger and Mester 1997; DeYoung et al 1998; Mester 1987; Berger and Humphrey 1991 and Avkiran 1999), the intermediation approach has been chosen with restricted choice of variables[3]. It

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[3]An increase in the number of outputs or inputs leads to an increase in efficiency scores. In small samples with many variables almost all units may be on the frontier.

is well known that the DEA is sensitive to variable selection. As the number of variables increase, the ability to discriminate between the DMUs decreases. The more variables are added the greater becomes the chance that some inefficient unit dominates in the added dimension and becomes efficient (Smith 1997, Coelli et al 1998). Thus to preserve the discriminatory power of the DEA, the number of inputs and outputs are kept at a reasonable level. The choice of the inputs and outputs are guided by the choices made in previous studies and also on the data availability. In the current study, two inputs – total cost (x_1) and total deposits (x_2) have been used. The input *total cost* is measured as the sum of total interest expenses and non-interest expenses including personal expenses. Non-interest expenses include service charges and commissions, expenses of general management affairs, salaries, and other expenses (including health insurance and securities portfolios). The production approach focuses on the bank's operating costs, that is, the cost of labour (employees) and physical capital (plant and equipment). The intermediary approach considers a financial firm's production process to be one of the financial intermediaries (the borrowing of funds and subsequent lending to these funds). Thus, the focus is on the total costs, including both interest and operating expenses.

Some researchers (Yeh 1996; Sathye 2003; Kao and Liu 2004) have treated interest expenses and non-interest expenses as two different inputs along with other inputs. However, following Casu and Molyneux (2003) and Sealey and Lindley (1977), both have been treated together as a single input as total cost. This kind of treatment is mainly due to the well-known dimensionality problem associated with the DEA particularly with limited sample size: a high number of variables relative to the number of observations cause more units to be wrongly identified as efficient. When too many constraints are specified, the observations tend to become incomparable against each other (Zhao et al 2006). The input, *total deposit* is taken as the sum of demand and savings deposits held by the bank and non-bank depositors.

The above two inputs are used to produce two outputs - total loans (y_1) and other earning assets (y_2). The output, *total loan* is measured as the sum of all loan accounts intermediated by banks and the output, *other earning assets* (y_2) is measured as the sum of total securities (treasury bills, government bonds and other securities), deposits with banks and equity investments.

Further, to ensure the validity of the DEA model specification, an isotonicity test (Avkiran 1999) was conducted. An isotonicity test involves the calculation of all inter-correlations between inputs and outputs for identifying whether increasing amounts of inputs lead to greater outputs. As positive inter-correlations were found, the isotonicity test was passed and the inclusion of the inputs and outputs was justified.

The basic data on inputs and outputs has been taken from the electronic database, PROWESS provided by Centre for Monitoring Indian Economy Pvt. Ltd., Mumbai for the period spanning 1994-95 to 2006-07. The sample consists of the banks for which data were available during the said period. There were 35 banks in total for which the data were available during the period of study. However, the bank of Kotak Mahindra Bank Ltd., was dropped from the sample because of excessive missing data. One of the important banks, i.e., ICICI bank[4] was dropped from the sample, as it did not fit into any of the broad cluster during the screening of the data. Due to excessive missing information during the period of study HDFC Bank Ltd., and

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[4]An increase in the number of production units yields a decrease in efficiency scores. Thus, a comparison of efficiency levels across industries requires adjustment for sample size; Zhang and Bartels (1998).

Centurion Bank of Punjab Ltd., has been removed from the list. Hence, our DMU size from 35 has dropped to 31.

Friedman and Sinuany-Stern (1998), Nunamaker (1985), and Raab and Lichty (2002) mentioned the need to have three times the number of DMUs as there are input and output variables, i.e., $3^*(m+s)<n$. In this study, with a total of two inputs and two outputs, a reasonably good sample size of 31 banks consisting of 19 public banks and 12 private banks is considered finally. The size is sufficient even when analysing the banks in terms of its ownership.

6.10 FINDINGS

This section reports the findings of the application of our proposed model in the Indian banking sector. Along with overall frontier (model 1) constructed for the entire sample of 31 banks, an ownership-based frontiers with a sample of 19 from public sector banks (model 2) and a sample of 12 from private sector banks (model 3) has been constructed. While the overall frontier examines how a particular bank is performing in the Indian banking sector as a whole, the ownership-based frontier examines how a particular bank is performing within its own group of ownership. In each model, values of α_j ($j = 1, 2, ..., n$) in all the constraints are assumed to be one of the four values 0.50, 0.10, 0.05, 0.01. The aspiration level values of β_o and β_j are assumed to be 1.

6.10.1 Choosing the Best Parameter Specifications of Stochastic Simulation based GA

In order to choose appropriate parameter values in our GA for solving P- model CCDEA problems, the following values of the population size, the crossover probability p_c and mutation probability p_m have been used.

pop_size = 40, 50, 70, 80, 100,

p_c = 0.5, 0.6, 0.7, 0.8, 0.9,

p_m = 0.04, 0.05, 0.06, 0.08.

By computer simulations on the three models using GA with various parameter specifications, it was found that the following specifications worked best.

Population size: pop_size = 50,

Crossover probability: p_c = 0.8,

Mutation probability: p_m = 0.08.

With these parameter specifications, all the three models given in section 5.10.2 are solved using the stochastic simulation based GA developed in section 5.8. Following is the guideline used to declare whether a chosen DMU (Bank) is stochastically efficient or not.

Let $\alpha_j = 0.05\,(j = 1, 2, ..., n)$, then according to definitions 1 and 2 given in section 5.5, a chosen DMU is said to be stochastically efficient if the corresponding objective function value is 0.05 and stochastically inefficient, if the objective function value is less than 0.05.

6.10.2 Stochastic Efficiency Analysis of Banks

Model 1:

Consider the following two inputs and two outputs CCDEA model in which both input and output are random variables following Normal distribution.

$$Max\ P\left(\frac{\sum_{r=1}^{2} u_r \tilde{y}_{ro}}{\sum_{i=1}^{2} v_i \tilde{x}_{io}} \geq 1\right) \text{ subject to } P\left(\frac{\sum_{r=1}^{2} u_r \tilde{y}_{rj}}{\sum_{i=1}^{2} v_i \tilde{x}_{ij}} \leq 1\right) \geq 1-\alpha_j, \quad j = 1, 2, ..., 31$$

where $u_r, v_i \geq 0, \quad \forall r, i.$

Table 6.1 Overall Frontier: Efficiency Scores of all 31 Banks

j	**Indian Banks**	α_j **= 0.50**	α_j **= 0.10**	α_j **= 0.05**	α_j **= 0.01**
1	Allahabad Bank[1]	0.9961	0.8536	0.7787	0.4628
2	Andhra Bank[1]	0.9975	0.8542	0.8091	0.6155
3	Bank of Baroda[1]	0.9936	0.8120	0.4973	0.1737
4	Bank of India[1]	*1.0000	0.8420	0.2380	0.6086
5	Bank of Maharashtra[1]	*1.0000	0.8540	0.8059	0.6882
6	Bank of Rajasthan Ltd.[2]	0.9900	0.8311	0.7843	0.5671
7	Canara Bank[1]	0.9947	0.8512	0.7989	0.1252
8	City Union Bank Ltd.[2]	*1.0000	0.8194	0.8013	0.5684
9	Corporation Bank[1]	*1.0000	0.8521	0.2145	0.6738
10	Dena Bank[1]	0.9989	0.8449	0.6122	0.4238
11	Development Credit Bank Ltd.[2]	*1.0000	0.8333	0.7885	0.7524
12	Dhanalakshmi Bank Ltd.[2]	0.9938	0.8129	0.6770	0.3599
13	Federal Bank Ltd.[2]	0.9948	0.8323	0.7920	0.6911
14	I N G Vysya Bank Ltd.[2]	0.9902	0.8346	0.6262	0.2495
15	Indian Bank[1]	0.9961	0.8403	0.8074	0.6423
16	Indian Overseas Bank[1]	0.9934	0.8492	0.8153	0.6051
17	Indusind Bank Ltd.[2]	*1.0000	0.8325	0.8090	0.7089
18	Jammu & Kashmir Bank Ltd.[2]	*1.0000	0.8421	0.6675	0.4487
19	Karnataka Bank Ltd.[2]	0.9951	0.8447	0.7803	0.6854
20	Karur Vysya Bank Ltd.[2]	0.9960	0.8331	0.7953	0.6357
21	Lakshmi Vilas Bank Ltd.[2]	0.9853	*0.8221	*0.7741	*0.7024
22	Oriental Bank of Commerce[1]	*1.0000	*0.8411	*0.7661	*0.7219
23	Punjab National Bank[1]	0.9966	0.8385	0.7740	0.7233
24	South Indian Bank Ltd.[2]	0.9883	0.8395	0.8027	0.7340
25	State Bank of Bikaner & Jaipur[1]	0.9988	0.8196	0.7531	0.1187
26	State Bank of India[1]	*1.0000	*0.8259	*0.8083	0.6853
27	State Bank of Mysore[1]	0.9970	*0.8346	*0.8041	*0.7216
28	Syndicate Bank[1]	0.9946	*0.8480	0.7978	0.7324
29	Uco Bank[1]	0.9955	*0.7738	0.7575	0.7085
30	Union Bank of India[1]	0.9930	0.8413	0.7951	0.4575
31	Vijaya Bank[1]	0.9964	0.8541	0.7726	0.7023

Table 6.1 (Continued)

j	Indian Banks	α_j = 0.50	α_j = 0.10	α_j = 0.05	α_j = 0.01
No. of Stochastically Efficient Banks		9.0000	6.0000	4.0000	3.0000
Average Efficiency Score (all banks)		0.9960	0.8356	0.6996	0.5201
Average Inefficiency Score (all banks)		0.0040	0.1968	0.4294	0.9226
Average Efficiency Score (public banks)		0.9970	0.8382	0.6664	0.4932
Average Inefficiency Score (public banks)		0.0030	0.1930	0.5006	1.0276
Average Efficiency Score (private banks)		0.9945	0.8314	0.7556	0.5658
Average Inefficiency Score (private banks)		0.0055	0.2028	0.3235	0.7674

Note: (i) * denotes the corresponding DMU is stochastically efficient (ii) superscripts 1 and 2 indicate that the corresponding bank is respectively from public sector and private sector.

Model 2:

The problem given below is the same as Model 1 except that here there are 19 DMUs.

$$Max\ P\left(\frac{\sum_{r=1}^{2} u_r \tilde{y}_{ro}}{\sum_{i=1}^{2} v_i \tilde{x}_{io}} \geq 1\right) \text{ subject to } P\left(\frac{\sum_{r=1}^{2} u_r \tilde{y}_{rj}}{\sum_{i=1}^{2} v_i \tilde{x}_{ij}} \leq 1\right) \geq 1-\alpha_j, \quad j = 1, 2, ..., 19,$$

where $u_r, v_i \geq 0, \quad \forall r, i.$

Table 6.2 Ownership Frontier: Efficiency Scores of 19 Public Sector Banks

j	**Indian Public Banks**	α_j **= 0.50**	α_j **= 0.10**	α_j **= 0.05**	α_j **= 0.01**
1	Allahabad Bank[1]	0.9961	*0.8962	0.8724	0.8234
2	Andhra Bank[1]	0.9971	*0.8754	*0.8667	*0.8008
3	Bank of Baroda[1]	0.9939	0.8793	0.8359	0.6053
4	Bank of India[1]	*1.0000	0.8840	0.4341	0.4168
5	Bank of Maharashtra[1]	*1.0000	*0.9010	*0.8700	*0.8312
6	Canara Bank[1]	0.9949	*0.8773	0.8357	0.8215
7	Corporation Bank[1]	*1.0000	0.9034	0.8720	0.8001
8	Dena Bank[1]	0.9989	0.8917	0.8504	0.8089
9	Indian Bank[1]	0.9961	0.8940	0.8699	0.6544
10	Indian Overseas Bank[1]	0.9934	0.8771	0.8517	0.8093
11	Oriental Bank of Commerce[1]	*1.0000	*0.8843	*0.8726	*0.8230
12	Punjab National Bank[1]	0.9966	0.8759	0.8576	0.7881
13	State Bank of Bikaner & Jaipur[1]	0.9988	*0.8808	*0.8755	*0.8200
14	State Bank of India[1]	*1.0000	*0.8909	0.8615	0.7828
15	State Bank of Mysore[1]	0.9900	*0.8916	*0.8689	*0.8062
16	Syndicate Bank[1]	0.9946	*0.8785	0.8549	0.8236
17	Uco Bank[1]	0.9955	*0.8499	0.8678	0.8057
18	Union Bank of India[1]	0.9939	0.8902	0.8666	0.8126
19	Vijaya Bank[1]	*1.0000	*0.8981	0.8598	0.8138
No. of Stochastically Efficient Banks		6.0000	10.0000	5.0000	5.0000
Average Efficiency Score		0.9980	0.8971	0.8661	0.8186
Average Inefficiency Score		0.0020	0.1146	0.1546	0.2216

Note: * denotes the corresponding DMU is stochastically efficient

Model 3:

Given below is yet another CCDEA model with 12 DMUs;

$$Max\ P\left(\frac{\sum_{r=1}^{2} u_r \tilde{y}_{ro}}{\sum_{i=1}^{2} v_i \tilde{x}_{io}} \geq 1\right) \text{ subject to } P\left(\frac{\sum_{r=1}^{2} u_r \tilde{y}_{rj}}{\sum_{i=1}^{2} v_i \tilde{x}_{ij}} \leq 1\right) \geq 1-\alpha_j, \quad j = 1, 2, ..., 12,$$

where $u_r, v_i \geq 0, \quad \forall r, i.$

Table 6.3 Ownership Frontier: Efficiency Scores of 12 Private Sector Banks

j	**Indian Private Banks**	α_j **= 0.50**	α_j **= 0.10**	α_j **= 0.05**	α_j **= 0.01**
1	Bank of Rajasthan Ltd.[2]	0.9900	0.8157	0.8004	0.6220
2	City Union Bank Ltd.[2]	*1.0000	*0.8310	*0.8099	*0.7391
3	Development Credit Bank Ltd.[2]	*1.0000	*0.8149	*0.7854	*0.7487
4	Dhanalakshmi Bank Ltd.[2]	*1.0000	0.8220	0.7868	0.7268
5	Federal Bank Ltd.[2]	*1.0000	0.8434	0.7988	0.7185
6	I N G Vysya Bank Ltd.[2]	0.9964	0.8351	0.6512	0.4261
7	Indusind Bank Ltd.[2]	*1.0000	*0.8450	*0.8077	0.7414
8	Jammu & Kashmir Bank Ltd.[2]	*1.0000	*0.8296	*0.8188	*0.7256
9	Karnataka Bank Ltd.[2]	0.9984	*0.8445	0.8046	0.7189
10	Karur Vysya Bank Ltd.[2]	0.9960	*0.8407	0.7797	0.7189
11	Lakshmi Vilas Bank Ltd.[2]	0.9856	0.8208	0.7871	0.7085
12	South Indian Bank Ltd.[2]	0.9915	*0.8328	0.7450	0.7215
No. of Stochastically Efficient Banks		6.0000	7.0000	4.0000	3.0000
Average Efficiency Score		0.9907	0.8242	0.7722	0.6699
Average Inefficiency Score		0.0093	0.2133	0.2950	0.4927

Note: *denotes the corresponding DMU is stochastically efficient.

The efficiency scores of the entire sample banks are reported in Table 5.1 under the overall frontier; where as, Tables 5.2 and 5.3 report the efficiency scores of public and private sector banks respectively under ownership based frontiers. The efficiency score of each bank is reported with different value of α_j. The column corresponding to α_j = 0.50 shows the efficiency score of each bank under CCR deterministic DEA model. The rest of the columns report the stochastic efficiency at three different level of α_j. In any CCP, the focus is on the system's ability to meet the constraints (α values - risk measures) with certain reliability in an uncertain environment. Tables 5.1, 5.2 and 5.3 show that (i) a decrease in α_j value starting from 0.10 results in a decrease in the number of stochastically efficient DMUs. i.e., when the constraint violation is higher, the number of stochastically efficient DMUs is also higher, meaning to say that when the specified risk level α is increased, some of the stochastically inefficient DMUs become efficient and hence they fall into stochastic efficient frontier (ii) the average efficiency score decreases gradually as α_j value decreases, this is because the model builder expects near 100% constraint satisfaction which leads to lesser number of stochastic efficient DMUs and (iii) there is also a steady increase in average inefficiency score value when α_j value decreases.

Under overall frontier, the banks that are stochastically efficient at all level of α_j are Lakshmi Vilas Bank Ltd., Oriental Bank of Commerce and State Bank of Mysore. The State Bank of India, which has a majority equity holding with RBI, is found to be stochastically efficient for α_j = 0.10 as well as α_j=0.05. However, it becomes inefficient as α_j value decreases further. None of the private banks, except Lakshmi Vilas Bank Ltd. is found to be stochastically efficient for any level of α_j, though 4 out of 12 private banks (or 33.3% of private banks) claimed to be efficient under deterministic model. However, on an average, the private sector banks are found to be stochastically more efficient than its counter sector banks for $\alpha_j < 0.10$.

The scenario in terms of the total number of efficient banks improves under both the ownerships. As it can be observed from Table 5.2 and Table 5.3, more than 50% of the banks are stochastically efficient for $\alpha_j = 0.10$. However, the number of stochastically efficient banks reduces with the fall in the value of α_j. The public sector banks that are stochastically efficient for all value of α_j are Andhra Bank, Bank of Maharashtra, Oriental Bank of Commerce, State Bank of Bikaner & Jaipur and State Bank of Mysore. The private sector banks, which are stochastically efficient for all values of α_j under ownership frontier, are City Union Bank Ltd., Development Credit Bank Ltd., and Jammu and Kashmir Bank Ltd. One can observe that under the ownership frontier, the average stochastic efficiency of the public sector bank is higher than that of private sector banks for all values of α_j. However, it will be misleading to conclude that on an average, public sector banks are dominating the private sector banks as the efficiency of these two types of banks are estimated under different frontiers. It only gives an indication that a comparatively large number of public sector banks are competing closely with each other leading to a comparatively high mean efficiency score.

6.11 MANAGERIAL IMPLICATIONS

Systematic benchmarking through efficiency measurement is one of the methods, managers can use to benchmark the efficiency of their banks. The process of benchmarking may be used to identify useful best business practices, innovative ideas, effective operating procedures, and winning strategies that can be adopted by a bank to accelerate its own progress by ensuring quality, productivity, and cost improvements (Chang and Lo 2005). However, one should be very careful while benchmarking the banks using the usual DEA models due to the influences of operating environment and measurement error on performance measurement. The measure of efficiency might be distorted because of the factors such as, error-in-variables due to the debatable valuation of accounting data, risk taking behavior of the banks under uncertainty and other uncontrollable external factors.

A rational customer will look into the consistency of performance before he/she takes a final decision. So reporting the average efficiency of the banks over a period may mislead the customers as well as the management when the data are random. Failing to capture the stochastic noise in the data might penalize some banks on their performance scores due to factors beyond managerial control, while others might be rewarded for operating in favorable environments. One can think of adopting the CCDEA that will be handy to extract the stochastic efficiency of banks.

6.12 SUMMARY

The conventional DEA method does not allow for the possibility of stochastic variations in input-output data and its ability to discriminate efficiencies among DMUs decreases as the number of input-output components increases. Stochastic variations such as measurement and specification errors, and modeller's preference structures, however, often exist in the nature of economic systems inside which the decision making units operate. Based on these considerations, this chapter provides a P-model CCDEA model which permits stochastic variability in input-output data. The major difficulty in solving the P-model CCDEA problem has been to convert it to deterministic equivalent programming. This often requires users to be more familiar with deriving the deterministic equivalent of the given problem and it may not be possible when problem parameters follow different distributions. Therefore, a stochastic simulation based GA for P-model CCDEA problems has been developed. Problems here allow for the possibility of two-sided random disturbances in input-output data. The probability distribution of random disturbances is assumed to be of any type such as Lognormal, Exponential, Weibull etc. This is in contrast to earlier research works where random disturbances are assumed to be normal in most of the cases. The proposed GA is illustrated with an example problem wherein stochastic efficiency is found for public and private banks in India. The development of the stochastic simulation based GA in this chapter is yet another application of GA in solving the stochastic optimization problems.