1 SENSITIVITY ANALYSIS

1.1 Single Period Solution

The sensitivity of the solution obtained for the single period context with respect to the input parameters is checked. The values of the input parameters for the base case are obtained either from literature, or from general norms observed in real life. The values of the input parameters for the base case are presented in Table 4.

Table 1: Values of the input parameters of the base case

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Source/Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Distribution</td>
<td>Uniform with [7000, 8000]</td>
<td>Burke et al.</td>
</tr>
<tr>
<td>Unit Retail Price of the product</td>
<td>700</td>
<td>(2009)</td>
</tr>
<tr>
<td>Costs quoted by the Respective Suppliers</td>
<td>[621, 625, 627]</td>
<td></td>
</tr>
<tr>
<td>Mean yields of the Respective Suppliers</td>
<td>[0.9, 0.9, 0.9]</td>
<td></td>
</tr>
<tr>
<td>S.D. of yields of the Respective Suppliers</td>
<td>[0.029, 0.029, 0.029]</td>
<td></td>
</tr>
<tr>
<td>Cost quoted by the Back-up Supplier</td>
<td>650 [300 as advance for</td>
<td>See Footnote¹</td>
</tr>
<tr>
<td></td>
<td>booking, 350 after delivery]</td>
<td></td>
</tr>
<tr>
<td>Unit Salvage Value for unsold products</td>
<td>30</td>
<td>Burke et al.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2009)</td>
</tr>
<tr>
<td>Penalty for per unit unmet demand</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Avg. Inventory Holding cost per unit</td>
<td>30</td>
<td>See Footnote²</td>
</tr>
<tr>
<td>Inventory on hand at the beginning</td>
<td>40</td>
<td>See Footnote³</td>
</tr>
</tbody>
</table>

¹ Cost quoted by the back-up supplier is chosen higher than the maximum of the costs quoted by the regular suppliers. The division of the cost is done as about 50% of the total cost is to be paid as advance. However this division does not affect the nature of the sensitivity curves with respect to other parameters.

² As a standard practice, retail firms often calculate monthly inventory holding cost as 3%-5% of the material cost. However, this value can vary from product to product.
For measuring the sensitivity of the solution with respect to one parameter, the value of the parameter is varied within a feasible range keeping the values of the other parameters constant and optimal solution is obtained for each of the cases. Finally the solutions are plotted against the values of the parameter to study the sensitivity.

1.1.1 Sensitivity with respect to the Price of the Product

**Figure 1:** Plot of optimal order quantities against different values of price

To study the sensitivity of the optimal solution with respect to price, the optimal solutions are obtained for 100 different values of price (ranging from 700 to 1690 with a gap of 10) keeping the values of the other parameters same as in the base case (as given in Table 4). In Figure 1, the optimal order quantities are plotted against different values of the price quoted by the firm to its customers. Here $q_i$ denotes the optimal quantity to be ordered to the $i^{th}$ Supplier ($i = 1, 2, 3$) and $q_0$ denotes the optimal quantity to be booked to the back-up supplier for emergency. It can be observed that as price increases, $q_1$ decreases, and simultaneously $q_2$ increases. Also, as the price

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3 The value of inventory on hand at the beginning of the period is taken as a random integer as it does not affect the nature of the sensitivity curves with respect to other parameters.
crosses 1070, Supplier 3 starts getting an order in the optimal solution; and as price further increases, $q_2$ increases at a comparatively slower rate and $q_3$ shows an increasing trend. Also, it can be observed that the optimal order limit to the back-up supplier $q_0$ increases as the price increases. The logical explanation to this behavior can be given as following: as the price increases, it allows the firm a higher margin, which can counter the expected stock-out cost by a higher amount of diversification. Mathematically, the standard deviation of the total supply can be given as (for $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$):

$$
\sigma_{TOTAL} = \sqrt{\sigma_1^2 q_1^2 + \sigma_2^2 q_2^2 + \sigma_3^2 q_3^2} = \sigma \sqrt{q_1^2 + q_2^2 + q_3^2}
$$

It is apparent from the expression that as the diversification increases the standard deviation of the total supply decreases. Also, the expected number of stock-outs can be expressed as:

$$
\prod_{i=1}^{n} \int_{0}^{1} dt_i g_i(t_i) \left[ \int_{Q+q_0}^{\infty} [z - (Q + q_0)] f(z) dz \right]
$$

$$
= \prod_{i=1}^{n} \int_{0}^{1} dt_i g_i(t_i) \left[ \frac{z^b}{2} \left[ \frac{z}{Q+q_0} - (Q + q_0) z^{b}_{Q+q_0} \right] \right]
$$

$$
= \frac{b^2}{2} - b(\bar{r}_1 q_1 + \bar{r}_2 q_2 + \bar{r}_3 q_3 + q_0) + \frac{1}{2}(\bar{r}_1^2 + \sigma_1^2)q_1^2
$$

$$
+ \frac{1}{2}(\bar{r}_2^2 + \sigma_2^2)q_2^2 + \frac{1}{2}(\bar{r}_3^2 + \sigma_3^2)q_3^2 + \bar{r}_1 \bar{r}_2 q_1 q_2 + \bar{r}_1 \bar{r}_3 q_1 q_3
$$

$$
+ \bar{r}_2 \bar{r}_3 q_2 q_3 + \frac{q_0^2}{2} + \bar{r}_1 q_1 q_0 + \bar{r}_2 q_2 q_0 + \bar{r}_3 q_3 q_0
$$

Now, it is apparent from the expression of the expected number of stock-outs that higher amount of diversification leads to reduction in expected stock-out cost. Hence, if the price increases,
optimally the firm should go for higher amount of diversification justifying the behavior observed in the price sensitivity graph.

1.1.2 Sensitivity with respect to the Salvage Value of the Product

To study the sensitivity of the optimal solution with respect to the salvage value of the product, the optimal solutions are obtained for 55 different salvage values (ranging from 30 to 300 with a gap of 5) keeping the values of the other parameters same as in the base case (as given in Table 4). In Figure 2, the optimal order quantities are plotted against different salvage values. Again $q_i$ denotes the optimal quantity to be ordered to the $i^{th}$ Supplier ($i = 1, 2$) and $q_0$ denotes the optimal quantity to be booked to the back-up supplier for emergency. It may be noted that Supplier 3 does not get any order in any of the cases within the given range of salvage value. It can be observed from Figure 2 that as the salvage value increases, $q_1$ steadily increases, $q_2$ steadily decreases till it reaches zero and $q_0$ remains unaffected. This behavior can be explained in the following way: as the salvage value increases, the unsold products at the end of the period

**Figure 2:** Plot of optimal order quantities against different salvage values
fetch higher revenue, so it is logical for the firm to try fulfilling the demand for the period from the lowest cost supplier and whatever amount is left at the end of the period generates a sizable revenue for its high salvage value. Since there is no relationship between the salvage value and the quantity to be booked to the back-up supplier for emergency, \( q_0 \) remains insensitive to change in salvage value.

1.1.3 Sensitivity with respect to the Penalty Cost

**Figure 3:** Plot of optimal order quantities against different values of penalty costs

For studying the sensitivity of the optimal solution with respect to the unit penalty cost faced by the firm for each unmet demand, the optimal solutions are obtained for 55 different values of unit penalty costs (ranging from 30 to 570 with a gap of 10) keeping the values of the other parameters same as in the base case (as given in Table 4). In Figure 3, the optimal order quantities are plotted against different values of the penalty cost faced by the firm in case of stock-out. The graph is very similar to the price sensitivity graph as discussed earlier. In this case, as the penalty cost increases, the effect of stock-out becomes more and more magnified.
Hence, the firm optimally should try to reduce the expected number of stock-outs. So, by using the same logic as discussed in the price sensitivity graph, the firm can reduce the expected number of stock-outs by diversifying more and more to different suppliers. Hence in the graph, it can be observed that if the penalty cost increases, the optimal order quantity to the lowest cost supplier decreases, whereas the optimal order quantities to the higher cost suppliers and the optimal order limit to the back-up supplier increase.

1.1.4 Sensitivity with respect to the Inventory Holding Cost

**Figure 4:** Plot of optimal order quantities against different values of inventory holding costs

The sensitivity of the optimal solution with respect to inventory holding cost per unit is studied by observing the behavior of the solutions for 31 different values of inventory holding costs per unit (ranging from 30 to 45 with a gap of 0.5) keeping the values of the other parameters same as in the base case (as given in Table 4). In Figure 4, the optimal order quantities are plotted against different values of inventory holding costs per unit. It can be observed from Figure 4 that increment in inventory holding cost leads to slight decrease in $q_1$ and slight increase in $q_2$ and $q_0$ in the optimal solution. It is logical because, if the inventory holding cost per unit increases, the
firm should try to reduce the excess amount of inventory held by the firm, which can be done by reducing the uncertainty in supply by diversifying the order to more suppliers. Also, as the amount to be booked to the back-up supplier does not add up to the amount of inventory held by the firm, it is natural for the firm to rely more on the back-up supplier in case inventory holding cost is high.

1.1.5 Sensitivity with respect to the Mean Yield of the Minimum Cost Supplier

Figure 5: Plot of optimal order quantities against different values of mean yield of the minimum cost supplier

To study the sensitivity of the optimal solution with respect to the mean of the yield distribution of the minimum cost supplier ($\bar{r}_1$), the behavior of the solutions for 55 different values of mean of the yield distribution of the minimum cost supplier (ranging from 0.9 to 0.36 with a gap of 0.01) keeping the values of the other parameters same as in the base case (as given in Table 4). In Figure 5, the optimal order quantities are plotted against different values of mean of the yield distribution of the minimum cost supplier. It can be observed from Figure 5 that, as $\bar{r}_1$ decreases from 0.9 to 0.67, $q_1$ remains fairly stable but $q_2$ increases steadily. This happens because a
decrease in $\bar{r}_1$ means lower amount of quantity received from that supplier, which is to be substituted by other comparatively more reliable supplier. As $\bar{r}_1$ falls below 0.67, then supplier 3 also starts getting the order, and $q_1$ starts decreasing steadily. This happens because if the yield of the supplier becomes very low, it is beneficial for the firm to diversify the order to different suppliers rather than maintaining the same level of order with the low yield supplier.

1.1.6 Sensitivity with respect to the Standard Deviation of Yield of the Minimum Cost Supplier

**Figure 6**: Plot of optimal order quantities against different values of standard deviation of the yield distribution of the minimum cost supplier

For studying the sensitivity of the optimal solution with respect to the standard deviation of the yield distribution of the minimum cost supplier ($\sigma_1$), the behavior of the solutions for 55 different values of standard deviation of the yield distribution of the minimum cost supplier (ranging from 0.029 to 0.083 with a gap of 0.001) keeping the values of the other parameters same as in the base case (as given in Table 4). In Figure 6, the optimal order quantities are plotted against different values of standard deviation of the yield distribution of the minimum cost supplier. It
can be observed that the nature of the sensitivity curve with respect to the standard deviation of
the yield of the minimum cost supplier is similar to that for the mean yield of the minimum cost
supplier, though the effect is more pronounced for the case of standard deviation. Unlike the
previous scenario, here, as $\sigma_1$ increases, the minimum cost supplier immediately encounters a
rapid fall in order quantity placed to them in the optimal solution. This is logical because in the
previous case, though the mean yield of the supplier was reduced, the amount of order to be
received by the firm from that supplier was relatively guaranteed due to small value of $\sigma_1$. In
comparison, in this case, the amount to be received from the supplier becomes more and more
wayward because of the increment in $\sigma_1$.

1.2 Multi-period Solution

The sensitivity of the analytical solution obtained for the multi-period context for infinite time
horizon with respect to the input parameters is checked. Similar to the case of the single period
solution, for measuring the sensitivity of the solution with respect to one parameter, the value of
the parameter is varied within a feasible range keeping the values of the other parameters
constant and optimal solution is obtained for each of the cases. Again, the solutions are plotted
against the values of the parameter to study the sensitivity.

For the base case, the cost quoted by the back-up supplier is set as 640 similar to the base case
for the single period solution. However, the apportionment of that cost for the multi-period
scenario is done as follows: 50 as advance for booking the amount to the back-up supplier and
600 during the time of receipt of the order. The logic behind this notion is that, when the firm
plans for a longer time horizon, it builds a long-time relationship with the back-up supplier,
which generates a trust between the two concerned parties. Also, from the back-up supplier’s
point of view, if some units remain unsold for one particular period, he can sell those units in the
subsequent periods. Therefore, this leads to a lower amount of advance payment to the back-up supplier. The discount factor per period is taken to be 0.9. The values of the other input parameters for the base case are kept the same as it was for the single period solution (as shown in Table 4).

1.2.1 Sensitivity with respect to the Price of the Product

![Order Quantities vs Price](image)

**Figure 7**: Plot of optimal order quantities against different values of price

To study the sensitivity of the optimal solution with respect to price, the optimal solutions are obtained for 100 different values of price (ranging from 700 to 1690 with a gap of 10) keeping the values of the other parameters same as in the base case (as given in Table 4). In Figure 7, the optimal order quantities are plotted against different values of the price quoted by the firm to its customers. Unlike for the single period scenario, in the optimal solution, $q_1$ remains stable over different values of price, the other suppliers do not get any order and $q_0$ shows an increasing trend with increase in price. This behavior is due to the ability of the firm to carry the unsold amount to the next period, so it is natural for the firm to order the entire amount from single
supplier with lowest cost and carry the excess amount to the next period for generating revenue. Also, if the price increases, it becomes more important for the firm for not conceding any loss of sales, therefore prompting an increase in \( q_0 \) for back-up in case of emergency.

### 1.2.2 Sensitivity with respect to the Penalty Cost

![Order Quantities vs Penalty Cost](image)

**Figure 8:** Plot of optimal order quantities against different values of penalty costs

For studying the sensitivity of the optimal solution with respect to the unit penalty cost faced by the firm for each unmet demand, the optimal solutions are obtained for 55 different values of unit penalty costs (ranging from 30 to 570 with a gap of 10) keeping the values of the other parameters same as in the base case. In Figure 8, the optimal order quantities are plotted against different values of the penalty cost faced by the firm in case of stock-out. Similar to the single period scenario, this graph behaves almost similar to the sensitivity curve with respect to price for most of the range. However, when the penalty faced by the firm for unit loss of sales becomes more than 470, the effect of stock-out becomes so magnified that it prompts the firm to go for dual sourcing for reducing supply uncertainty.
1.2.3 Sensitivity with respect to the Inventory Holding Cost

Figure 9: Plot of optimal order quantities against different values of inventory holding costs

The sensitivity of the optimal solution with respect to inventory holding cost per unit is studied by observing the behavior of the solutions for 31 different values of inventory holding costs per unit (ranging from 30 to 45 with a gap of 0.5) keeping the values of the other parameters same as in the base case. In Figure 9, the optimal order quantities are plotted against different values of inventory holding costs per unit per period. It can be observed from Figure 9 that increment in inventory holding cost leads to decrease in $q_1$ and increase in $q_0$ in the optimal solution. The other suppliers do not receive any order in the optimal solution. This again stems due to the ability of the firm to carry the excess inventory to the next period, therefore, it is optimal for the firm to place order to single supplier with lowest cost. As inventory holding cost per unit per period increases, it becomes beneficial for the firm to book additional order to the back-up supplier for which the firm does need to pay any inventory holding cost, rather than placing order to the minimum cost supplier and incurring a hefty inventory holding cost.
1.2.4 Sensitivity with respect to the Mean Yield of the Minimum Cost Supplier

![Order Quantities vs Mean Yield of Supplier1](image)

**Figure 10:** Plot of optimal order quantities against different values of mean yield of the minimum cost supplier

To study the sensitivity of the optimal solution with respect to the mean of the yield distribution of the minimum cost supplier ($\bar{r}_1$), the behavior of the solutions for 55 different values of mean of the yield distribution of the minimum cost supplier (ranging from 0.9 to 0.36 with a gap of -0.01) keeping the values of the other parameters same as in the base case. In Figure 10, the optimal order quantities are plotted against different values of mean of the yield distribution of the minimum cost supplier. It can be observed from Figure 10 that, as $\bar{r}_1$ decreases from 0.9 to 0.46, $q_1$ increases and the other suppliers do not get any order. Once $\bar{r}_1$ falls below 0.46, then supplier 2 starts getting the order and increases steadily with further decrease in $\bar{r}_1$, and $q_1$ shows a steady decreasing trend. This happens because a decrease in the mean yield of Supplier 1 means a lower proportion of quantity received from that supplier; hence it is sensible to place a larger order to that supplier. Even if the supplier supplies a high amount, it can be carried forward to the next period as inventory to generate revenue in the following periods. However, if
the mean yield falls below a certain level, it is no longer beneficial for the firm to place order to single supplier, but the firm should diversify its sourcing to two suppliers to reduce uncertainty in supply.

1.2.5 Sensitivity with respect to the Standard Deviation of Yield of the Minimum Cost Supplier
**Figure 11**: Plot of optimal order quantities against different values of standard deviation of the yield distribution of the minimum cost supplier

For studying the sensitivity of the optimal solution with respect to the standard deviation of the yield distribution of the minimum cost supplier ($\sigma_1$), the behavior of the solutions for 55 different values of standard deviation of the yield distribution of the minimum cost supplier (ranging from 0.029 to 0.083 with a gap of 0.001) keeping the values of the other parameters same as in the base case. In Figure 11, the optimal order quantities are plotted against different values of standard deviation of the yield distribution of the minimum cost supplier. It can be observed from the graph that single sourcing to the lowest cost supplier with a stable order quantity is more beneficial for the firm till $\sigma_1$ reaches a value of 0.057. Further increase in $\sigma_1$ leads to decrease in $q_1$ and increase in $q_2$ in the optimal solution. For lower value of $\sigma_1$ the optimal order quantities remain unaffected with change in $\sigma_1$ because, as the firm has the opportunity to carry the excess inventory forward and back-order the unmet demand to the next period, the net effect on the expected profit due to decrease in stock-out/inventory holding by diversifying the sourcing to dual suppliers does not compensate for the increase in materials cost due to diversification. However, as $\sigma_1$ becomes higher than 0.057, the net effect on the expected profit due to stock-out/inventory holding magnifies so much that it becomes beneficial for the firm to diversify its sourcing to dual suppliers.