CHAPTER 6

THERMODYNAMICAL MODELLING OF VISCOUS DISSIPATION IN MAGNETOHYDRODYNAMIC FLOW*

6.1 INTRODUCTION

The problem of magnetohydrodynamic (MHD) incompressible steady viscous flow has many important practical engineering applications in areas such as construction of heat exchangers, installation of nuclear accelerators, blood flow measurement techniques, analysis of electrically-conducting fluids such as liquid metals, plasmas and salt water and in crystal growth. Magnetohydrodynamic forced convection flow over a wedge is of considerable interest to the technical field due to its frequent role in industrial and technological applications in analysing the effects of magnetic field on the flow and heat transfer aspect over various geometries.

The conducting fluid and magnetic field interact through electric currents that flow in the fluid. The currents are induced as the conducting

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fluid move across the magnetic field lines. In turn, the currents influence both the magnetic field and the motion of the fluid. Qualitatively, the magnetohydrodynamic interactions tend to link the fluid and the field lines so as to make them move together.

According to the boundary layer theory, the velocity increases from zero at the wall surface to the free stream velocity at the edge of the boundary layer and thus velocity gradient may be appreciable even if the viscosity is very small. Analysing the shear stress and heat transfer is one of the most important objectives in the solution of the boundary layer equations. The governing equations of boundary layer flow become non-similar due to the presence of a magnetic field or variable fluid properties. The boundary layer flow of a laminar incompressible electrically conducting fluid over a wedge in the presence of transverse magnetic field has been investigated by many researchers.

Watanabe (1978, 1986) has reduced the momentum partial differential equation to ordinary differential equation by employing difference-differential method and obtained solution in a form of integral equation. The solution for the heat transfer of an electrically conducting fluid over a semi-infinite flat plate in the presence of a transverse magnetic field has been studied by Watanabe and Pop (1993) by means of difference-differential method. The problems of stagnation point and asymmetric flow have been investigated by Raptis (1991) and Chamkha (1998). Watanabe (1986) has analysed the magnetohydrodynamic boundary layer flow along a wedge and he has not considered the energy equation. Hossain (1992) has treated the
viscous and joule heating effects on MHD free convection flow with variable plate temperature. Watanabe and Pop (1993) have solved the MHD free convection flow over a wedge in the presence of a transverse magnetic field. Yih (1998) has presented an analysis of forced convection boundary layer flow over a wedge with uniform suction and blowing whereas Watanabe (1990) has investigated the behaviour of the boundary layer over a wedge with suction/injection in forced flow.

Yih (1999) has extended the work of Watanabe and Pop (1994) to investigate the heat transfer characteristic in MHD forced convection flow adjacent to a non-isothermal wedge in the presence of transverse magnetic field. An approximate numerical solution for thermal stratification on MHD steady laminar boundary layer flow over a wedge with suction or injection has been investigated by Anjalidevi and Kandasamy (2003). The MHD boundary layer flow over a flat plate for two cases, a uniform free stream velocity and a uniform hydrostatic pressure has been investigated by Sam Lawrence and Nageswara Rao (1995). Lin and Lin (1987) have proposed a similarity solution method that provides accurate solutions for laminar forced convection heat transfer for either an isothermal surface or an uniform flux boundary to fluid of any Prandtl number. Chamkha (1998) has studied steady two dimensional mixed convection flows of an electrically conducting and heat absorbing fluid near stagnation point on a semi infinite vertical permeable surface at arbitrary surface heat flux variations in the presence of a magnetic field. Motivated by the work of the above mentioned authors, the effect of viscous dissipation and stress work on the MHD boundary layer flow over a wedge are analysed here.
6.2 CONSERVATION LAWS

The conservation equations of mass, momentum and energy for steady, two dimensional, laminar flow, and constant physical properties with viscous dissipation effects are given by

\[
\begin{align*}
  u_x + v_y &= 0, \quad \text{(mass)} \quad (6.1) \\
  uu_x + vu_y &= \nu u_{yy} + U_\infty (U_\infty)_x + (\kappa B_0^2) (U_\infty - u)/\rho, \quad \text{(momentum)} \quad (6.2) \\
  uT_x + vT_y &= \alpha T_{yy} + (\nu/C_p) (u^2_y) - (u/C_p) [U_\infty (U_\infty)_x \\
  &+ (\kappa B_0^2 U_\infty / \rho)] + \kappa B_0^2 u^2 / \rho C_p, \quad \text{(energy)} \quad (6.3)
\end{align*}
\]

The initial and boundary conditions of the system are

\[
\begin{align*}
  y = 0: & \quad u = 0, \quad v = 0, \quad T = T_0 = T_\infty + Ax^{2m}, \quad \text{(uniform)} \\
  y \to \infty: & \quad u = U_\infty = C x^m, \quad T = T_\infty, \quad \text{(uniform)} \quad (6.4)
\end{align*}
\]

where \( m = \beta/(2 - \beta) \), is the Hartree pressure gradient parameter which corresponds to \( \beta = \Omega/\pi \) for a total angle of the wedge. The surface of the wedge is maintained at a variable wall temperature proportional to \( x^{2m} \). In Equations (6.4), \( A \) and \( C \) are positive numbers. In this study, the induced magnetic field and the Hall effect are neglected.

The transport equations (6.1)-(6.3) can be written in balance form as

\[
\begin{align*}
  \nabla \cdot \vec{V} &= 0, \quad (\vec{V} = iu + jv). \quad (6.5) \\
  \rho (\vec{V} \cdot \nabla) \vec{V} + \nabla \cdot \vec{P} &= 0, \quad (6.6) \\
  \rho C_p (\vec{V} \cdot \nabla) T + \nabla \cdot \vec{J}_q &= \mu (u^2_y) - u[\rho U_\infty (U_\infty)_x + \kappa B_0^2 U_\infty] + \kappa B_0^2 u^2. \quad (6.7)
\end{align*}
\]
In the energy picture the energy dissipation for the present system is given by (Gyarmati (1969))

\[ T\sigma = -J_q(\partial lnT/\partial y) - P_{12}(\partial u/\partial y) \]  \hspace{1cm} (6.8)

the heat flux \( J_q \) and \( P_{12} \) the only component of momentum flux \( \vec{D}^u \), satisfy the conservative relations connecting the independent fluxes and forces as

\[ J_q = -L_\lambda(\partial lnT/\partial y), \quad \text{and} \quad P_{12} = -L_s(\partial u/\partial y). \]  \hspace{1cm} (6.9)

The dissipation potentials \( \Psi \) and \( \Phi \) in the energy picture are given by

\[ T\Psi = (1/2)[L_\lambda(\partial lnT/\partial y)^2 + L_s(\partial u/\partial y)^2], \]  \hspace{1cm} (6.10)

\[ T\Phi = (1/2)[R_\lambda J_q^2 + R_s P_{12}^2], \]  \hspace{1cm} (6.11)

Using the Equations (6.8), (6.9), (6.10) and (6.11), Gyarmati’s variational principle in the energy picture is formulated in the following form

\[ \delta \int_{0}^{l} \int_{0}^{\infty} \left[ -J_q(\partial lnT/\partial y) - P_{12}(\partial u/\partial y) - (L_\lambda/2)(\partial lnT/\partial y)^2 \
\quad - (L_s/2)(\partial u/\partial y)^2 - (R_\lambda/2)J_q^2 - (R_s/2)P_{12}^2 \right] dydx = 0. \]  \hspace{1cm} (6.12)

**6.3 METHOD OF SOLUTION**

It is considered that the system of two dimensional MHD laminar, inviscid potential flow past an unlimited wedge placed symmetrically in a stream with apex at the origin where \( x \) and \( y \) are coordinates measured along and normal to the surface respectively. The main stream velocity and the wall
temperature are assumed to vary as power functions of distance from the start of the boundary layer respectively as

$$U_\infty = Cx^m, \quad T_0 - T_\infty = Ax^{2m},$$  \hspace{1cm} (6.13)

where $C$ and $A$ are constants and the exponent $m$ is connected with the apex angle $\pi \beta$ by the relation

$$m = \beta / (2 - \beta) \quad \text{or} \quad \beta = 2m / (m + 1).$$  \hspace{1cm} (6.14)

In equation (6.13), $T_0$ and $T_\infty$ are the temperature of the surface and the free stream temperature respectively. The present analysis is carried out for the entire range of realistic flow.

The velocity and temperature fields in their respective boundary layer regions are suitably described by the following functions

$$
\begin{align*}
\frac{u}{U_\infty} &= 3y/d_1 - 3y^2/d_1^2 + y^3/d_1^3, \quad (y < d_1) \\
u &= U_\infty, \quad \hspace{2cm} (y \geq d_1) \\
(T - T_\infty)/(T_0 - T_\infty) &= 1 - 3y/2d_2 + y^2/2d_2^2, \quad (y < d_2) \\
T &= T_\infty, \quad \hspace{2cm} (y \geq d_2)
\end{align*}
$$

\hspace{1cm} (6.15)

Where $d_1$ and $d_2$ are the momentum and thermal boundary layer thicknesses respectively. The velocity and thermal profiles (6.15) satisfy the following compatibility conditions

$$
\begin{align*}
y &= 0, \quad u = 0, \quad v = 0, \quad T = T_0, \quad T_y = 0, \\
y &= d_1, \quad u = U_\infty, \quad u_y = 0, \quad u_{yy} = 0, \\
y &= d_2, \quad T = T_\infty, \quad T_y = 0.
\end{align*}
$$

\hspace{1cm} (6.16)

The smooth fit boundary conditions $u_y = 0$ and $T_y = 0$ correspond to $P_{12} = 0$ and $J_q = 0$ at the respective edges of the boundary layers. Here
$d_1$ and $d_2$ are unknown parameters and they are to be determined by the present thermodynamic analysis.

The transverse velocity component $v$ is obtained from the mass balance Equation (6.1) as

$$
v = \left[U_{\infty}d_1\right]\left[(3y^2/2d_1^2) - 2y^3/d_1^3 + 3y^4/4d_1^3]\right]
   + \left[U_{\infty}m/x\right]\left[(-3y^2/2d_1 + y^3/d_1^2 - y^4/4d_1^3]\right]. \quad (6.17)
$$

The velocity and temperature functions (6.15) are substituted in the momentum and energy balance Equations (6.2) and (6.3), and on direct integration with respect to $y$ with the help of smooth fit boundary conditions the fluxes $P_{12}$ and $J_q$ are obtained respectively.

The expression for $P_{12}$ remains the same for any Prandtl number $Pr$. But the energy flux $J_q$ assumes different expression for $Pr \leq 1$ and $Pr \geq 1$ respectively. When $Pr \leq 1$, the expression for $J_q$ in the range $d_1 \leq y \leq d_2$ is obtained first and the expression for $J_q$ in the range $0 \leq y \leq d_1$ is determined subsequently by matching the $J_q$ expressions of the two regions at the interface. The expressions for momentum and energy fluxes $P_{12}$ and $J_q$ are as follows.

$$
-P_{12}/L_s = \left[U_{\infty}/d_1\right] + \left[mU_{\infty}^2/\nu x\right]\left[53d_1/160 - y + 3y^3/2d_1^2 - 3y^4/2d_1^3\right]
   + 3y^5/4d_1^4 - 9y^5/4d_1^4 + y^7/28d_1^5\left[53d_1/160 - y + 3y^3/2d_1^2 - 3y^4/2d_1^3\right]
   + 3y^4/d_1^4 - 9y^5/4d_1^4 + 3y^6/4d_1^5 - 3y^7/28d_1^6\left[53d_1/160 - y + 3y^3/2d_1^2 - 3y^4/2d_1^3\right]
   + \left[kB_0^2U_{\infty}/\nu \rho\right]
   \times [3y^2/2d_1 - y^3/d_1^2 + y^4/4d_1^3 - y + d_1/5]. \quad (0 \leq y \leq d_1). \quad (6.18)
$$
\[-J_q/L_\lambda = \left[ Pr U_\infty (T_0 - T_\infty) d'_2/\nu \right] \left[ 3 y^3/2 d_1 d'_2 - 9 y^5/10 d_1 d'_2 - 9 y^4/8 d_1^2 d'_2 \right] + 3 y^6/4 d_1^2 d'_2 + 3 y^5/10 d_1^2 d'_2 - 3 y^7/14 d_1^2 d'_2 + 3 d_1^2/40 d'_2 - 3 d_1^2/280 d'_2 \]
\[-3/8 + [m Pr U_\infty (T_0 - T_\infty)/\nu x] [3 y^2/d_1 - 9 y^3/4 d_1 d_2 + 3 y^5/20 d_1 d_2^3] - 2 y^3/d_1^3 + 15 y^4/8 d_1^2 d_2 - y^6/4 d_1^2 d_2 - 21 y^5/40 d_1^3 d_2 \]
\[+ 5 y^7/56 d_1^3 d_2^3 + d_1/2 - 3 d_1^2/20 d_2 - 4 d_1^4/35 d_2^3 - 9 d_2/8] \]
\[+ [Pr U_\infty (T_0 - T_\infty) d'_1/\nu] [- 3 y^3/4 d_1^2 d_2 + 9 y^5/20 d_1^2 d_2^3 + 3 y^4/4 d_1^2 d_2] \]
\[+ y^6/2 d_1^2 d_2^3 - 9 y^5/40 d_1^4 d_2^2 + 9 y^7/56 d_1^4 d_2^3 + d_1/10 d_2 - 31 d_1^2/280 d_2^3 \]
\[+ [U_\infty^3 Pr/C_p] [- 9 y^2/d_1^2 + 18 y^2/d_1^2 - 18 y^3/d_1^2 + 9 y^4/d_1^2 - 9 y^5/5 d_1^4 \]
\[+ 9/5 d_1] + [U_\infty^3 Pr/\nu x C_p] [\xi (3 y^2/2 d_1 - 4 y^3/d_1^2 + 19 y^4/4 d_1^3 \]
\[-3 y^5/d_1^3 + y^6/d_1^3 - y^7/7 d_1^3 - 3 d_1/28] + m(3 y^2/2 d_1 \]
\[-y^3/d_1^3 + y^4/4 d_1^3 + d_1/4 - d_2)]. \quad (0 \leq y \leq d_1), (Pr \leq 1) \quad (6.19) \]

\[-J_q/L_\lambda = \left[ Pr U_\infty (T_0 - T_\infty) d'_2/\nu \right] \left[ 3 y^2/4 d_2^2 - 3 y^4/8 d_2^3 - 3/8 \right] \]
\[+ [m Pr U_\infty (T_0 - T_\infty)/\nu x] [2 y - 3 y^2/4 d_2 - y^4/8 d_2^3 \]
\[-9 d_2/8 + [m U_\infty^3 Pr/\nu x C_p] [y - d_2]. \quad (d_1 \leq y \leq d_2), (Pr \leq 1). \quad (6.20) \]

\[-J_q/L_\lambda = \left[ Pr U_\infty (T_0 - T_\infty) d'_2/\nu \right] \left[ 3 y^3/2 d_1 d_2^2 - 9 y^5/10 d_1 d_2^2 - 9 y^4/8 d_1^2 d_2^2 \right] + 3 y^6/4 d_1^2 d_2^2 + 3 y^5/10 d_1^2 d_2^2 - 3 y^7/14 d_1^2 d_2^2 - 3 d_2^2/5 d_1 + 3 d_2^2/8 d_1^2 \]
\[-3 d_2^2/35 d_1^3 \] + [m Pr U_\infty (T_0 - T_\infty)/\nu x] [3 y^2/d_1 - 9 y^3/4 d_1 d_2 \]
\[+ 3 y^5/20 d_1 d_2^3 - 2 y^3/d_1^3 + 15 y^4/8 d_1^2 d_2 - y^6/4 d_1^2 d_2^2 + y^4/2 d_1^3 \]
\[-21y^5/40d_1^3d_2 + 5y^7/56d_1^3d_2^3 - 9d_2^2/10d_1 + 3d_1^3/8d_2^2 - 9d_2^5/140d_1^5] \\
+ [PrU_\infty(T_0 - T_\infty)d_1^4/\nu]\left(-3y^3/4d_1^3d_2 + 9y^5/20d_1^3d_2^3 + 3y^4/4d_1^3d_2\right) \\
- y^6/2d_1^3d_2^3 - 9y^5/40d_1^3d_2^4 + 9y^7/56d_1^3d_2^3 + 3d_2^4/10d_1^2 - d_2^2/4d_1^3 \\
+ 9d_2^5/140d_1^5] + [U_\infty^3 Pr/C_p][\left(-9y/d_1^2 + 18y^2/d_1^3 - 18y^3/d_1^4 + 9y^4/d_1^5\right) \\
- 9y^5/5d_1^6 + 9/5d_1] + [U_\infty^3 Pr/\nu x C_p]\left[\xi (3y^2/2d_1 - 4y^3/d_1^2 + 19y^4/4d_1^3 \\
- 3y^5/d_1^4 + y^6/d_1^5 - y^7/7d_1^6 - 3d_1/28) + m(3y^2/2d_1 \\
- y^3/d_1^2 + y^4/4d_1^3 - 3d_1/4)\right]. \quad (0 \leq y \leq d_1), (Pr \geq 1). \quad (6.21)

The prime indicates the differentiation with respect to x. Using the expressions of \(P_{12}\) and \(J_y\) along with the velocity and temperature functions (6.15) the variational principle (6.12) is formulated independently for \(Pr \leq 1\) and \(Pr \geq 1\) respectively. After performing the integration with respect to y the variational principles are formulated in the following forms respectively.

\[
\delta \int_0^l L_1[d_1, d_2, d_1', d_2']dx = 0, \quad (Pr \leq 1) \quad \text{and} \quad (6.22)
\]

\[
\delta \int_0^l L_2[d_1, d_2, d_1', d_2']dx = 0. \quad (Pr \geq 1) \quad (6.23)
\]

The variational principles (6.22) and (6.23) are found identical when \(d_1 = d_2\). Accordingly, the Euler-Lagrange equations are

\[
(\partial L_{1,2}/\partial d_2) - (d/dx)(\partial L_{1,2}/\partial d_2') = 0. \quad (Pr \leq 1, Pr \geq 1) \quad (6.24)
\]

Equations (6.24) are second order ordinary differential equations in terms of \(d_1\) and \(d_2\) respectively. The above equations are further simplified by introducing the non-dimensional hydrodynamical and thermal boundary layer thicknesses \(d_1^*\) and \(d_2^*\), which are given by

\[
d_1 = d_1^*\sqrt{\nu x/U_\infty} \quad \text{and} \quad d_2 = d_2^*\sqrt{\nu x/U_\infty}. \quad (6.25)
\]
The variational principles (6.22) and (6.23) subject to transformations (6.25) and the resulting Euler-Lagrange equations are obtained as simple polynomial equations

\[
(\partial L_{1,2}/\partial d_1^*) = 0, \\
(\partial L_{1,2}/\partial d_2^*) = 0. 
\]

(6.26)\hspace{1cm} (6.27)

The coefficients of these Equations (6.26) and (6.27) depend on the independent parameters \( Pr, m, \xi \) and \( Ec \), where

\[
Re = (U_\infty x/\nu), \hspace{0.5cm} \text{(Reynolds number)} \\
Pr = (\nu/\alpha), \hspace{0.5cm} \text{(Prandtl number)} \\
Ec = U_\infty^2/\{C_p[T_o(x) - T_\infty]\}, \hspace{0.5cm} \text{(Eckert number)} \\
\xi = (\kappa B_0^2x)/(\rho U_\infty). \hspace{0.5cm} \text{(Magnetic parameter)}
\]

Equation (6.26) is a simple polynomial equation in terms of boundary layer thickness whose coefficients depend on the wedge angle parameter \( m \), and the magnetic parameter \( \xi \). This equation is solved easily for any given combinations of \( m \) and \( \xi \) and corresponding hydrodynamical boundary layer thickness \( d_1^* \) is obtained as the only positive root. The polynomial Equations (6.27) are solved for the given values of \( Pr, m, \xi \) and \( Ec \) and it is found that for any value of \( Pr \) there corresponds only one real root \( d_2^* \).

6.4 ANALYSIS OF RESULTS

As usual after getting the values of \( d_1^* \) and \( d_2^* \) for given values of \( Pr, m, \xi \) and \( Ec \) the local shear stress values and local heat transfer values are
calculated with the help of the following relations respectively.

\[
\begin{align*}
\tau_\omega &= \sqrt{\nu x / U_\infty^3} \left(-P_{12}/L_s\right)_{y=0}, \\
Nu_x &= \sqrt{\nu x / U_\infty (T_0 - T_\infty)^2} \left(-J_q/L_\lambda\right)_{y=0}.
\end{align*}
\]  \ (6.28)  \ (6.29)

The main results of engineering interest are skin friction (shear stress) and heat transfer (Nusselt number) and hence these two important characteristics are analysed here.

Tables 6.1 and 6.2 represent the values of skin friction for various values of \(m\) when \(\xi=0\) and for various values of \(\xi\) when \(m=1\) respectively. In order to verify the accuracy of the obtained results by using the present technique, the obtained results are compared with Cebeci and Bradshaw (1984), Aria (1994), Lin and Lin (1987) and Yih (1999). As given in Table 6.1, it is evidently observed that when the values of \(m\) increase, the values of skin friction also increase.

When the wedge becomes a flat plate \((m=0)\) the surface skin friction value is very small and low. While the wedge angle becomes large the values of skin friction also increase rapidly. This circumstance remains same for any given value of \(m\). When \(m=1\) and for any given values of the magnetic parameter \(\xi\), the shear stress values are tabulated in Table 6.2. Since \(m=1\), this type is a particular case known as stagnation flow. From this table, it is revealed that the skin friction values increase with the increasing values of \(\xi\) with uniform interval. The increasing of the magnetic parameter \(\xi\), also increases the skin friction values.
Table 6.1  Comparison of Skin friction values for various values of \( m \) when \( \xi = 0 \)

<table>
<thead>
<tr>
<th>( m )</th>
<th>Present results</th>
<th>Cebeci and Bradshaw (1984)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.33206</td>
<td>0.337164446</td>
</tr>
<tr>
<td>0.3333</td>
<td>0.75745</td>
<td>0.759123053</td>
</tr>
<tr>
<td>1.0</td>
<td>1.23259</td>
<td>1.23334938</td>
</tr>
</tbody>
</table>

Table 6.2  Comparison of Skin friction values for various values of \( \xi \) when \( m = 1 \)

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>Present results</th>
<th>Ariel (1994)</th>
<th>Yih (1999)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.23334938</td>
<td>1.232588</td>
<td>1.232588</td>
</tr>
<tr>
<td>1</td>
<td>1.58635749</td>
<td>1.585331</td>
<td>1.585331</td>
</tr>
<tr>
<td>4</td>
<td>2.34901650</td>
<td>2.346663</td>
<td>2.346663</td>
</tr>
<tr>
<td>9</td>
<td>3.24428992</td>
<td>3.240950</td>
<td>3.240950</td>
</tr>
<tr>
<td>25</td>
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<tr>
<td>100</td>
<td>10.0833987</td>
<td>10.074741</td>
<td>10.074741</td>
</tr>
</tbody>
</table>

Table 6.3 exhibits the heat transfer values for various values of Prandtl number when \( m= 0, 1, Ec=0 \) and \( \xi = 0 \). From this table, the heat transfer values are increasing with the Prandtl numbers and the increase is rapid for higher Prandtl numbers. Numerical results for heat transfer are presented in Table 6.4 for various values of Prandtl number, Eckert number and magnetic parameter \( \xi \), for given \( m=0 \). From these tables, the obtained results are well comparable with known results and the comparison shows very good agreement.
Table 6.3  Comparison of Heat transfer values for various values of $Pr$ when $m = 0, 1$, $Ec = 0$ and $\xi = 0$

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>Present results ($Nu_x$)</th>
<th>Lin and Lin (1987) ($Nu_x$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m=0$</td>
<td>$m=1$</td>
</tr>
<tr>
<td>0.001</td>
<td>0.0056490</td>
<td>0.007990</td>
</tr>
<tr>
<td>0.001</td>
<td>0.017800</td>
<td>0.025240</td>
</tr>
<tr>
<td>0.01</td>
<td>0.054680</td>
<td>0.079086</td>
</tr>
<tr>
<td>0.1</td>
<td>0.146750</td>
<td>0.232550</td>
</tr>
<tr>
<td>1</td>
<td>0.344540</td>
<td>0.568590</td>
</tr>
<tr>
<td>10</td>
<td>0.778350</td>
<td>1.323490</td>
</tr>
<tr>
<td>100</td>
<td>1.71100</td>
<td>2.942610</td>
</tr>
<tr>
<td>1000</td>
<td>3.71945</td>
<td>6.427350</td>
</tr>
<tr>
<td>10000</td>
<td>8.04602</td>
<td>13.93341</td>
</tr>
</tbody>
</table>

Table 6.4  Comparison of Heat transfer values for various values of $Pr$, $Ec$ and $\xi$ when $m = 0$

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>$\xi$</th>
<th>$E_c = 0$</th>
<th>$E_c = 1$</th>
<th>$E_c = 0$</th>
<th>$E_c = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.733</td>
<td>0.0</td>
<td>0.302071269</td>
<td>0.19486313</td>
<td>0.297526</td>
<td>0.170272</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
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Skin friction values are presented graphically for various values of $\xi$ and $m$ in Figure 6.1. In this figure, the pressure gradient parameter $m$ ranges
from 0 to 9, and the magnetic parameter $\xi$ ranges from 0 to 10. Figures 6.2-6.4 represent the heat transfer values for various values of $m$ when $Ec = 0$, 0.5 and 1 for given value of $Pr = 0.1$.

Similarly for the cases of $Ec = 0$, 0.5 and 1 when $Pr = 1$ are given graphically in Figures 6.5-6.7 respectively. Figures 6.8-6.11, display the heat transfer values for various Prandtl numbers for given $Ec = 0.1$ when $m = 0$, 0.333333, 0.5 and 1. From these figures the local skin friction and heat transfer values increase as the pressure gradient parameter $m$ increases.

![Figure 6.1](image) **Figure 6.1** Skin friction as a function of $\xi$ for various values of $m$
Figure 6.2  Local Nusselt number as a function of $\xi$ for various $m$ when $Ec = 0$ and $Pr = 0.1$

Figure 6.3  Local Nusselt number as a function of $\xi$ for various $m$ when $Ec = 0.5$ and $Pr = 0.1$
Figure 6.4  Local Nusselt number as a function of $\xi$ for various $m$ when $Ec = 1$ and $Pr = 0.1$  

Figure 6.5  Local Nusselt number as a function of $\xi$ for various $m$ when $Ec = 0$ and $Pr = 1$
Figure 6.6  Local Nusselt number as a function of $\xi$ for various $m$ when $Ec = 0.5$ and $Pr = 1$

Figure 6.7  Local Nusselt number as a function of $\xi$ for various $m$ when $Ec = 1$ and $Pr = 1$
Figure 6.8  Local Nusselt number as a function of $\xi$ for various $Pr$ when $Ec = 0.1$ and $m = 0$

Figure 6.9  Local Nusselt number as a function of $\xi$ for various $Pr$ when $Ec = 0.1$ and $m = 0.333333$
Figure 6.10  Local Nusselt number as a function of $\xi$ for various $Pr$ when $Ec = 0.1$ and $m = 0.5$

Figure 6.11  Local Nusselt number as a function of $\xi$ for various $Pr$ when $Ec = 0.1$ and $m = 1$
6.5 CONCLUSION

This chapter deals with the effects of transverse magnetic field, viscous dissipation, stress work, shear stress and surface heat transfer over a non-isothermal wedge. Using simple transformation techniques the governing partial differential equations are simplified into much easier polynomial equations, the coefficients of which are functions of independent parameters $Pr$, $m$, $\xi$ and $Ec$. These equations offer a new way of obtaining skin friction and heat transfer values for any values of $Pr$, $m$, $\xi$, and $Ec$. In contrast to the conventional exact methods, the present variational technique yields the required results with more accuracy but with less calculations.