CHAPTER 6

ANALYTIC SOLUTION OF A FREE AND FORCED CONVECTION WITH SUCTION AND INJECTION OVER A NON-ISOTHERMAL WEDGE¹

6.1 INTRODUCTION

The effect of buoyancy forces on flow and heat transfer is usually ignored when a forced convection flow over a cooled or heated surface is studied. However, neglecting buoyancy effects cannot be justified, because under certain circumstances the buoyancy forces influences the flow and temperature functions despite the presence of forced convection flow.

The free and forced convection flow with suction and injection from a wedge is important in nature and in many practical experiments the steady and transient mixed convection has been investigated by many researchers. For the constant plate temperature the free and forced convection flows establish that the ratio of the Grashof number to the square root of Reynolds

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number is known as Richardson number. The forced convection is dominating for small values of Richardson number while free convection takes large values of Richardson number. The steady free and forced convection flows from a vertical plate has been investigated by Merkin (1969), Wilks (1973), Lloyd and Sparrow (1970), Gryzgoridis (1975) and Jaluria (1986). A combination of series expansion and numerical integration was applied by Merkin (1969) and he has solved the mixed convection problem along the isothermal wall for both aiding and opposing flows for $Pr=1$. Merkin (1969) has given that in the case of opposing flow and buoyancy effects increase the boundary layer separates from the plate at a Richardson number $= 0.192357$. A similar study has been carried out by Wilks (1973) and the results were obtained for $Pr=1$ for a constant heat flux boundary condition. Lloyd and Sparrow (1970) have obtained a complete solution for the aiding mixed convection flow from a vertical isothermal flat plate by employing local similarity method for different values of Prandtl number ranging from 0 to 4. The aiding mixed convection flow from a vertical plate has been experimentally investigated by Gryzgoridis (1975). A numerical study of aiding mixed convection flow over localized, finite-sized isoflux sources located on a vertical adiabatic vertical surface has been carried out by Jaluria (1986).

Merkin and Pop (2002) have shown that the similarity solution of the boundary layer flow equations describing mixed convection flow along a vertical plate exists if the difference between the temperature of the plate and the ambient temperature is inversely proportional to the distance from the leading edge of the plate.
Saeid (2004) has analyzed the periodic free convection flow from a vertical plate, and he has applied the laminar boundary layer theory to study the effects of periodic plate temperature oscillations with different amplitudes and frequencies on the free convection flow with different Prandtl numbers. Recently Saeid and Mohamad (2006) have considered the surface temperature oscillation in free convection flow from a vertical plate immersed in porous media. In these studies it is found that the free convection heat transfer from the vertical plate decreases with increase in either the amplitude or frequency of the surface temperature oscillation.

6.2 GOVERNING PRINCIPLE

The system of steady, two dimensional, incompressible and laminar fluid flow of constant transport coefficients over a non-isothermal wedge with apex angle $\pi \beta$ is considered. The free stream flow velocity $U_\infty$ and the surface temperature $T_0$ satisfy the power law

$$U_\infty(x) = ax^m \quad \text{and} \quad T_0 - T_\infty = bx^n,$$

where $U_\infty$ is obtained by the inviscid potential flow, “$a$” and “$b$” are constants, “$m$” is the wedge angle parameter and “$n$” is the wall temperature exponent. The coordinate axes along $x$ and $y$ directions are considered along and perpendicular to the surface, respectively. On applying the boundary layer approximations if viscous dissipation effect is ignored, the conservation equations of mass, momentum and energy are respectively given by
\[ u_x + v_y = 0 \] (mass),
\[ uu_x + vu_y = νυ_{yy} + U_{∞}U'_{∞} + gB(T - T_{∞}) \sin(\frac{π}{2}β/2) \]
\[ + \left[ \int_{y}^{∞} gB(T - T_{∞}) \cos(\frac{π}{2}β/2)dy \right] \] (momentum),
\[ uT_x + vT_y = α T_{yy} \] (energy).

The initial and boundary conditions of the system are
\[ y = 0; \quad u = 0, \quad v = v_0, \quad T = T_0, \]
\[ y = ∞; \quad u = U_{∞}, \quad T = T_{∞}(\text{uniform}). \] (6.3)

### 6.3 VARIATIONAL APPROACH

A system of two dimensional, laminar, inviscid potential flow past an unlimited wedge placed symmetrically in a stream with apex at the origin and the center line on the positive x-axis is considered. The wedge angle parameter “m” is connected with the apex angle πβ by the relation

\[ m = β/(2 - β) \quad \text{(or)} \quad β = 2m/(m + 1). \] (6.4)

To begin with the thermodynamic analysis, the trial functions for velocity and temperature fields inside their respective boundary layers are selected as

\[ u/U_{∞} = 3y/d_1 - 3y^2/d_1^2 + y^3/d_1^3, \quad (y < d_1) \]
\[ u = U_{∞}, \quad (y ≥ d_1) \]

and \[ (T - T_{∞})/(T_0 - T_{∞}) = 1 - 3y/2d_2 + y^3/2d_2^3, \quad (y < d_2) \]
\[ T = T_{∞}, \quad (y ≥ d_2) \] (6.5)
which satisfy the conditions,

\[ y = 0; \quad u = 0, \quad v = v_0, \quad T = T_0, \quad (\partial^2 T/\partial y^2) = 0, \]
\[ y = d_1; \quad u = U_\infty, (\partial u/\partial y) = 0, \quad (\partial^2 u/\partial y^2) = 0, \]
\[ y = d_2; \quad T = T_\infty, (\partial T/\partial y) = 0, \quad (\partial^2 T/\partial y^2) = 0. \quad (6.6) \]

The unknown quantities \( d_1 \) and \( d_2 \) are the extent of the hypothetical hydrodynamical and thermal boundary layer thicknesses respectively. These unknowns are to be determined from the present thermodynamic analysis.

The governing equations of motion (6.2) are written in the following balance forms:

\[ \nabla \cdot \tilde{V} = 0, \]
\[ \rho (\tilde{V} \cdot \nabla) \tilde{V} + \nabla \cdot \tilde{P} = g B_i (T - T_\infty) \sin(\pi \beta /2) \tilde{i} \]
\[ + \left[ \int_y^\infty g B_i (T - T_\infty) \cos(\pi \beta /2) dy \right]_x, \quad (6.7) \]
\[ \rho C_p (\tilde{V} \cdot \nabla) T + \nabla \cdot \tilde{J}_q = 0. \]

The constitutive relations connecting the independent forces and fluxes for the present two dimensional problem are

\[ P_{12} = -L_s (\partial u/\partial y) \quad \text{and} \quad J_q = -L_\lambda (\partial \ln T/\partial y). \quad (6.8) \]

The energy dissipation for this problem is

\[ T \sigma = -J_q (\partial \ln T/\partial y) - P_{12} (\partial u/\partial y). \quad (6.9) \]

The dissipation potentials in energy picture are

\[ \Psi^* = T \Psi = (1/2) \left[ L_\lambda (\partial \ln T/\partial y)^2 + L_s (\partial u/\partial y)^2 \right], \]
\[ \Phi^* = T \Phi = (1/2) \left[ R_\lambda J_q^2 + R_s P_{12}^2 \right]. \quad (6.10) \]
Using (6.9) and (6.10) the principle (2.12) is formulated in the form:

\[
\delta \int_0^1 \int_0^\infty \left[ -J_q(\partial nT/\partial y) - P_{12}(\partial u/\partial y) - \left( L_\lambda/2 \right)(\partial lnT/\partial y)^2 
- \left( (L_s/2)(\partial v/\partial y)^2 - (1/2)(J_q^2/L_\lambda) - (1/2)(P_{12}^2/L_s) \right) \right] dydx = 0, \quad (6.11)
\]

The transverse velocity component \( v \) is obtained from the mass balance Equation (6.2) as

\[
v = (mU_\infty/x)[-3y^2/2d_1 + y^3/d_1^2 - y^4/4d_1^3]
+ U_\infty[3y^2/2d_1^2 - 2y^3/d_1^3 + 3y^4/4d_1^4]d_1' + v_0. \quad (6.12)
\]

The velocity and temperature polynomial trial functions (6.5) and the boundary conditions (6.6) are used in the governing Equations (6.2) and on integration with respect to \( y \) with the help of their corresponding smooth-fit conditions \( (\partial u/\partial y) = 0 \) and \( (\partial T/\partial y) = 0 \), the momentum flux \( P_{12} \) and energy flux \( J_q \) for \( Pr \geq 1 \) \((ie) \quad d_1 \geq d_2 \) are obtained as follows

\[
-P_{12}/L_s = (\partial u^4/\partial y) = (U_\infty/d_1) + (mU_\infty^2/\nu x)[53d_1/160 - y
+ 3y^2/2d_1^2 - 3y^4/2d_1^3 + 3y^5/4d_1^4 - y^6/4d_1^5 + y^7/28d_1^6]
+ \left( U_\infty^2d_1'/\nu \right)[9/160 - 3y^3/2d_1^3 + 3y^4/d_1^4 - 9y^5/4d_1^5
+ 3y^6/4d_1^6 - 3y^7/28d_1^7] + [v_0U_\infty/\nu][-3/4 + 3y/d_1
- 3y^2/d_1^2 + y^3/d_1^3] + [[(gBd_1/\nu)(T_0 - T_\infty)][[\sin(\pi/\beta)/2]
(1/2 - d_1/4d_2 + d_1^2/40d_2^2 - y/d_1 + 3y^2/4d_1d_2 - y^4/8d_1d_2^2)]
+(nd_2/\pi)[\cos(\pi/\beta)/2][3/16 - d_1/6d_2 + d_1^2/16d_2^2 - d_1^4/240d_2^4
- 3y/8d_1 + y^2/2d_1d_2 - y^3/4d_1d_2^2 + y^5/40d_1d_2^4] + d_2'
[\cos(\pi/\beta)/2][3/16 - d_1^2/16d_2^2 + d_1^4/80d_2^4 - 3y/8d_1
+y^3/4d_1d_2^2 - 3y^5/40d_1d_2^4)](d_2 \leq y \leq d_1) \quad (6.13)
\]
and \(- J_q/L_N = (\partial T^*/\partial y) = (U_\infty/\alpha)(T_0 - T_\infty)\{d'_1[3d_2^4/10d_1^2 - 3d_2^3/12d_1^3] + 9d_2^2/140d_1^4 - 3y^3/4d_2^2d_2 + 3y^4/4d_1^3d_2 + 9y^5/20d_1^2d_2^3 - 9y^5/40d_1^2d_2 - 3y^6/6d_1^2d_2^3 + 9y^7/56d_1^2d_2^5 + d'_2[3d_2/5d_1 + 3d_2^2/8d_1^2 - 3d_2^3/35d_1^3 + 3y^3/2d_1d_2 - 9y^4/8d_1^2d_2^2 + 9y^4/10d_1^2d_2^2 + 3y^6/4d_1^2d_2^4 - 3y^7/14d_1^3d_2^6] + (md_2/x)[-3d_2/10d_1 + d_2^2/8d_1^2 - 3d_2^3/140d_1^3 + 3y^2/2d_1d_2 - 3y^3/2d_1d_2^2 - y^3/d_2^2d_2 + 9y^4/8d_1^2d_2^2 + y^4/4d_1^2d_2 + 3y^5/10d_1^2d_2 - 3y^5/10d_1^2d_2^2 - y^6/4d_1^2d_2^4 + y^7/14d_1^3d_2^6] + (md_2/x)[-3d_2/10d_1 + 3d_2^2/24d_1^2 - 3d_2^3/140d_1^3 + 3y^3/4d_1^2d_2^2 - 3y^4/8d_1^2d_2^2 + 3y^5/40d_1^3d_2^2 - 9y^5/20d_1d_2^5 + 3y^6/12d_1^2d_2^4 - 3y^7/56d_1^3d_2^6]\} + [v_0(T_0 - T_\infty)/\alpha] [1 - 3y/2d_2 + 3y^3/6d_2^3](0 \leq y \leq d_2) \quad (6.14)

Here the prime indicates partial differentiation with respect to \( x \). Using the expressions of momentum flux \( P_{12} \) and thermal flux \( J_q \) along with the velocity and temperature functions (6.5) the variational principle (6.11) is formulated. After carrying out the integration with respect to \( y \), the variational principle (6.11) is obtained as

\[
\delta \int_0^L L(d_1, d_2, d'_1, d'_2)dx = 0. \quad (6.15)
\]

The explicit expression for the above function is as follows,

\[
\delta \int_0^L \{[\frac{U_\infty^2}{d_1}(-0.4)] + (\frac{U_\infty^3}{\nu})(md_1/x)(0.124107145) + (d'_1) (0.03482143)] - (v_0 \frac{U_\infty^2}{\nu})[0.25] - (\frac{U_\infty^4}{\nu^2})(md_1^2/x^2)(0.009715542) + (d_1^2d_1)(0.000812448) + (md_1^3d_1/x)(0.00528494)] - (v_0 \frac{U_\infty^2}{\nu^2})(v_0)
\]
\[(0.040178571) + (mU^2 d_1/x)(-0.039387178) + (U_\infty d'_1)(-0.010409901)\]
\[(U_\infty d_1/\nu)gB(T_0 - T_\infty)\{\sin(\pi \beta/2)[0.25 - 0.05d^2_1/d^2_2 + 0.021428571 d^2_1/d^2_2] + (nd_2/x)\cos(\pi \beta/2)[0.09375 - 0.116666666d_1/d_2 + 0.05d^2_1/d^2_2]
-0.00372023d^2_1/d^2_2\} + (d'_2)\cos(\pi \beta/2)[0.09375 - 0.05d^2_1/d^2_2 + 0.01116071 d^2_1/d^2_2]\] - \[gB(T_0 - T_\infty)/\nu\{\sin(\pi \beta/2)(mU^3 d^3_1/d\nu \nu)[0.03793650]
-0.025744047d_1/d_2 + 0.003041802d^3_1/d^3_2\} - \sin(\pi \beta/2)(U^3 d'_1/d^3_2/\nu)\]
\[\{0.011458331 - 0.008482143d_1/d_2 + 0.0010829275d^3_1/d^3_2\} + (nd_2/x)\]
\[\cos(\pi \beta/2)(mU^2 d^3_1/d\nu \nu)[0.01419271 - 0.017162698d_1/d_2 + 0.007205434 d^2_1/d^2_2 - 0.00052216d^4_1/d^4_2\} + (nd_2/x)\cos(\pi \beta/2)(U^2 d'_1/d^2_2/\nu)[0.00429687
-0.005654762d_1/d_2 + 0.002490867d^3_1/d^3_2 - 0.00018947d^4_1/d^4_2\} + (d'_2)\]
\[\cos(\pi \beta/2)(mU^3 d^3_1/d\nu \nu)[0.014192709 - 0.00720542d^3_1/d^3_2 + 0.00153459 d^2_1/d^2_2\} + (d'_2)\cos(\pi \beta/2)(U^3 d'_1/d^3_2/\nu)[0.004296874 - 0.002490867 d^2_1/d^2_2 + 0.000568415d^4_1/d^4_2\} - (v_0 U_\infty d^2_1/d\nu)[gB(T_0 - T_\infty)/\nu\{\sin(\pi \beta/2)
-0.75 + 0.05d_1/d_2 - 0.0058035d^3_1/d^3_2\} + (nd_2/x)\cos(\pi \beta/2)[0.04583334
+0.03333333d_1/d_2 + 0.29136904d^3_1/d^3_2 + 0.017658728d^4_1/d^4_2\} + (d'_2)\]
\[\cos(\pi \beta/2)[-0.028125 + 0.013839285d^2_1/d^2_2 + 0.061309524d^4_1/d^4_2]\} - \[g^2 B^2(T_0 - T_\infty)^2/\nu^2\{d^4_1\{\sin^2(\pi \beta/2)[0.041666666 - 0.0625d_1/d_2
+0.0625d^2_1/d^2_2 + 0.008333333d^3_1/d^3_2 - 0.007142857d^4_1/d^4_2 + 0.0055555 d^6_1/d^6_2\} + (n^2 d^2_1/x^2)\cos^2(\pi \beta/2)[0.005859375 - 0.015625d_1/d_2
+0.018142361d^2_1/d^2_2 - 0.0143429165d^4_1/d^4_2 + 0.005859375d^6_1/d^6_2\}
+0.000868056d^2_1/d^2_2 - 0.0004340275d^4_1/d^4_2 + 0.00001941d^6_1/d^6_2\} + (d''_2)\]
\[\cos^2(\pi \beta/2)[0.005859375 - 0.00703125d^2_1/d^2_2 + 0.0041852675d^4_1/d^4_2
-0.001302083d^6_1/d^6_2 + 0.0000999315d^8_1/d^8_2]\} - \[gB(T_0 - T_\infty)/\nu\{nd_2/x\]
\[d_1^2 \sin(\pi/2) \cos(\pi/2) [-0.209821429 - 0.065104167 d_1/d_2]
+ 0.052083344 d_1^4/d_2^2 - 0.0125 d_1^3/d_2^3 - 0.001488045 d_1^4/d_1^5 + 0.003645834 d_1^6/d_2^6 + 0.004464285 d_1^2/d_2^2 - 0.00078125 d_1^4/d_1^2 - 0.00453125 d_1^6/d_2^6 \]

\[-[g B(T_0 - T_\infty)/\nu] (nd_2/x)(d_1^2/d_2^2) \cos^2(\pi/2) [0.0125 - 0.015625 d_1/d_2]
+ 0.0312499 d_1^4/d_2^2 - 0.00494791 d_1^4/d_2^2 - 0.00260416 d_1^6/d_2^6 + 0.0019965 d_1^6/d_2^6 - 0.000118371 d_1^6/d_2^6 \] + \lambda \{(T_0 - T_\infty)^2/d_2(0.6) + (U_\infty/\alpha) (T_0 - T_\infty)^2\}

\[
\{d_1^2 [0.0234375 d_2^5/d_1^2 - 0.210023808 d_2^3/d_1^3 + 0.05625 d_2^4/d_1^4]
+ d_2^2 [-0.46875 d_2/d_1 - 0.314285731 d_2^2/d_1^2 - 0.075 d_2^3/d_1^3] + (nd_2/x) [-0.16875 d_2/d_1 + 0.08452381 d_2^2/d_1^2 - 0.029464285 d_2^3/d_1^3] + (md_2/x) [-0.234375 d_2/d_1 + 0.04523788 d_2^2/d_1^2 - 0.018728571 d_2^3/d_1^3] \}
+ [v_0(T_0 - T_\infty)^2/\alpha(0.5) - (U_\infty/\alpha)^2 (T_0 - T_\infty)^2\}
\{d_1^2[d_1^2 + 0.030969959 d_2^6/d_1^2 - 0.010985331 d_1^6/d_1^6]
+ 0.0015763075 d_2^4/d_1^4 + (d_2^2/d_1^2) [0.092532468 d_2^6/d_1^2 - 0.123348214 d_2^6/d_1^3]
- 0.069983146 d_2^3/d_1^4 - 0.0197257645 d_2^3/d_1^4 + 0.0023601395 d_2^6/d_1^6]
+ (\nu^2 d_2^2/x^2) [0.0135551945 d_2^6/d_1^2 - 0.0134027385 d_2^6/d_1^3 + 0.005566938 d_2^4/d_1^4 - 0.0013174475 d_2^6/d_1^6 + 0.0001112075 d_2^6/d_1^6] + (m^2 d_2^2/x^2)
\{0.225633116 d_2^6/d_1^6 - 0.03274553 d_2^6/d_1^6 + 0.0078643215 d_2^6/d_1^6 \}
- 0.0018308891 d_2^6/d_1^6 + 0.0168269295 d_2^6/d_1^6 \} - [v_0^2(T_0 - T_\infty)^2/\alpha^2] d_2
+ 0.117857142 - (U_\infty/\alpha)^2 (T_0 - T_\infty)^2 (d_1^4 d_2^2) [-0.0025324665 d_2^2/d_1^2]
+ 0.0126562485 d_1^4/d_1^2 - 0.0207667335 d_2^6/d_2^6 + 0.021248403 d_2^6/d_1^6
- 0.0000963333 d_2^6/d_1^6] - (U_\infty/\alpha)^2 (T_0 - T_\infty)^2 (nd_2^2/x) d_1^6 [0.1236120135 d_2^6/d_1^6 + 0.036956847 d_2^6/d_1^6 - 0.1768204885 d_2^6/d_1^6 + 0.022813704 d_2^6/d_1^6
- 0.0021554985 d_2^6/d_1^6 - 0.13125 d_2^6/d_1^6] - (U_\infty/\alpha)^2 (T_0 - T_\infty)^2 (md_2^2/x) d_1
\[-0.0912662345d_2^3/d_1^3 + 0.089799102d_2^4/d_1^4 - 0.0330232185d_2^5/d_1^5 \\
+0.0288823365d_2^6/d_1^6 - 0.000885055d_2^7/d_1^7] - (U_\infty v_0/\alpha^2)(T_0 - T_\infty)^2 \\
(d'_1d_2)[-0.012499999d_2^2/d_1^2 - 0.087053568d_2^3/d_1^3 + 0.0229301925d_2^4/d_1^4 \\
+0.1125d_2^5/d_1^5] - (U_\infty/\alpha)^2(T_0 - T_\infty)^2(nd_2^2/x)d_2'[0.167775475 \\
d_2^2/d_1^2 + 0.529553572d_2^3/d_1^3 + 0.067779718d_2^4/d_1^4 - 0.009460421d_2^5/d_1^5 \\
+0.00010081d_2^6/d_1^6] - (U_\infty/\alpha)^2(T_0 - T_\infty)^2md_2^2/x)d_2'[0.092532466 \\
-0.102790176d_2^3/d_1^3 + 0.0495348375d_2^4/d_1^4 - 0.0115066965d_2^5/d_1^5 \\
+0.0011800725d_2^6/d_1^6] - (U_\infty v_0/\alpha^2)(T_0 - T_\infty)^2(d'_2d_2)[-0.20000003 \\
+0.33058036d_2^2/d_1^2 - 0.0305735935d_2^3/d_1^3] - (U_\infty/\alpha)^2(T_0 - T_\infty)^2 \\
(md_2^2/x^2)[0.033887985d_2^2/d_1^2 - 0.031614585d_2^3/d_1^3 + 0.013184004d_2^4/d_1^4 \\
-0.00027183195d_2^5/d_1^5 + 0.0002500685d_2^6/d_1^6] - (U_\infty v_0/\alpha^2)(T_0 - T_\infty)^2 \\
(nd_2^2/x)[-0.079761905d_2/d_1 + 0.038164644d_2^2/d_1^2 - 0.007021102d_2^3/d_1^3 \\
-(U_\infty v_0/\alpha^2)(T_0 - T_\infty)^2(md_2^2/x)[-0.099999999d_2/d_1 + 0.0435267825 \\
d_2^2/d_1^2 - 0.0076433985d_2^3/d_1^3]}dx = 0. \quad (6.16)

The boundary layer thicknesses \(d_1\) and \(d_2\) are the independent parameters which are to be varied and the Euler-Lagrange equations corresponding to these variational parameters are

\[
(d/dx)(\partial L/\partial d'_i) - (\partial L/\partial d_i) = 0, \quad i = 1, 2. \quad (6.17)
\]

Equations (6.17) are non-linear second order ordinary differential equations in terms of \(d_1\) and \(d_2\) whose coefficients are functions of \(Re, Pr, Gr, K, L,\) and \(n.\)
Where \( Re = U_\infty x/\nu \) (Reynolds number),
\( Pr = \frac{\mu C_p}{K} \) (Prandtl number),
\( Gr = gB(T_0 - T_\infty)x^3/\nu^2 \) (Grashof number),
\( K = \frac{Gr}{(Re)^{5/2}} \) (Buoyancy parameter),
\( L = \frac{\pi m}{(m + 1)} \),

and \( n = \) wall temperature exponent.

Although the Equations (6.17) can be solved directly by using a numerical method, a simple solution is obtained for the considered problem by employing the following transformations

\[
d_i = d_i^* \sqrt{\nu x/U_\infty}, \quad i = 1, 2. \tag{6.18}
\]

in the variational principle (6.15). Thus the Euler-Lagrange equations of the transformed principle assume the simple forms as

\[
\frac{\partial L}{\partial d_i^*} = 0. \quad i = 1, 2. \tag{6.19}
\]

Equations (6.19) are observed as coupled polynomial equations in non-dimensional boundary layer thicknesses \( d_1^* \) and \( d_2^* \) and the coefficients of these equations depend on the independent parameter \( Pr \), wedge angle parameter \( m \) and \( H \), where \( H \) is the non dimensional suction/injection speed and is given by

\[
H = \frac{[\nu_0/\nu] \sqrt{\nu x/U_\infty}}{}. \tag{6.20}
\]

Suction and injection are represented by \( H < 0 \) and \( H > 0 \) respectively. Equations (6.19) can be solved for any value of Prandtl number.
\( Pr \), suction/injection speed \( H \), wedge angle parameter \( m \) and the wall temperature exponent \( n \) and from the present variational procedure the non-linear partial differential equations governing the boundary layer flow are transformed into simple coupled polynomial equations which are of much useful for engineering applications.

### 6.4 DISCUSSION OF RESULTS

The main results of engineering interest are skin friction (shear stress) and heat transfer values (Nusselt number) and hence these two important characteristics for the present considered problem are analyzed here. After obtaining the simultaneous solutions of \( d_1^* \) and \( d_2^* \) for the the given combination of \( Pr, H, m \) and \( n \) the skin friction values and the local Nusselt number are calculated on using the following expressions respectively

\[
\tau_\omega = \sqrt{\nu_x/U_\infty^3} \left(-P_{12}/L_s\right)_{y=0}, \\
Nu_l = \sqrt{\nu_x/U_\infty(T_0 - T_\infty)^2} \left(-J_q/L_\lambda\right)_{y=0}.
\]

(6.21)

It is known that the Grashof number \( (Gr) \) and Reynolds number \( (Re) \) are the controlling parameters of free and forced convection flows, respectively. Thus, the buoyancy parameter \( K \) is for both free and forced convection flows. For very small values of \( K \), the forced convection predominates and the free convection becomes negligible. For large values of \( K \), the flow is controlled by the free convection, and the forced convection becomes decrease. When the surface is heated \( [(T_0 - T_\infty) > 0] \) then the buoyancy parameter becomes \( K > 0 \), and the flow has favourable pressure gradient.
The adjacent boundary layer is accelerated, and with the increasing value of $K$, the velocity and thermal boundary layer thicknesses $d_1^*$ and $d_2^*$ become decrease, while the skin friction and heat transfer values increase. When the surface is cooled, then the buoyancy parameter becomes $K < 0$ and the boundary layer is decelerated. Since the free and forced convections are in the opposite directions a flow separation occurs in the present study. This means that, the condition for the boundary layer separation from the wedge surface is occurred, when the flow becomes reverse at the interface. This phenomenon occurs when $(\partial u^*/\partial y)_{y=0} = 0$ at a certain negative buoyancy parameter $K$, for the case of $H = 0$.

When $\beta = 0.5$, $H = 0$, $n = 0$ and $Re = 100$ the boundary layer flow is separated for $Pr = 0.7, 1.0$ and $3.0$ at $K = -0.0952, -0.1007$ and $-0.1322$ respectively. It reveals that the separation is delayed with the increasing Prandtl number ($Pr$). It is also observed that, the flow separation is delayed with the increase of wall temperature exponent $n$. When $\beta = 0.5$, $H = 0$, $Re = 100$ and $Pr = 1$ the separation occurs at $K = -0.1007$ and $-0.1039$ when $n = 0$ and $n = 0.3333$ respectively.

Figures 6.1 and 6.2 represent the skin friction values for the cases of constant surface temperature and constant heat flux as a function of $K$, for three values of $\beta$ when $Pr = 0.7$, $H = 0$ and $Re = 100$ respectively. From these two figures it is observed that the vanishing of skin friction values is delayed by the increase of $\beta$. In Figures 6.3 and 6.4 the effect of $K$ for high Prandtl number $Pr = 100$, $H = 0$ and $Re = 100$ is analysed for constant surface temperature and constant surface heat flux respectively.
Figure 6.1  Skin Friction values for various $K$
when $Pr = 0.7$, $H = 0$ and $Re = 100$
(Constant surface temperature)
Figure 6.2 Skin Friction values for various $K$
when $Pr = 0.7$, $H = 0$ and $Re = 100$
(Constant surface heat flux)
Figure 6.3  Skin Friction values for various $K$
when $Pr = 100$, $H = 0$ and $Re = 100$
(Constant surface temperature)
Figure 6.4  Skin Friction values for various $K$
when $Pr = 100$, $H = 0$ and $Re = 100$
(Constant surface heat flux)
It is demonstrated that the skin friction values increase with the value of $K$ when the surface is heated. Figures 6.5 and 6.6 represent the local Nusselt number for $Pr = 0.7$ and $Pr = 100$ respectively, when the surface temperature becomes uniform.

The constant heat flux along the wedge is presented in Figures 6.7 and 6.8. When $K$ becomes negative the effect of buoyancy is to decrease the skin friction and heat transfer values. It is evident that the vanishing of local heat flux occurs beyond the point of zero skin friction. The skin friction is more strongly affected by buoyancy forces than in the heat transfer. It is noted that lower Prandtl number fluids are more sensitive to buoyancy effects. From the present analysis when $\beta = 0, 0.5, 1.0$ and 1.6 the surface heat transfer vanishes at $n = -0.5, -0.6666, -1.0$ and $-2.5$ respectively. This phenomenon occurs for any combination of the Prandtl number($Pr$) and the buoyancy parameter $K$ when $H = 0$.

When a new mathematical technique (present technique) is applied to a problem it is customary to compare the obtained results with the available solution in order to establish the accuracy of the results in the present analysis. Accordingly the present results are compared with series solutions of Sparrow and Minkowycz (1962) and Saeid (2005) when $\beta = 0$, $H = 0$ and $Pr = 0.7$ which are given in Table 6.1.
Figure 6.5  Heat transfer values for various $K$
($Pr = 0.7, n = 0, H = 0$ and $Re = 100$)
Figure 6.6 Heat transfer values for various $K$
($Pr = 100$, $n = 0$, $H = 0$ and $Re = 100$)
Figure 6.7  Buoyancy effect on heat transfer when \( Pr = 0.7, \ H = 0 \) and \( Re = 100 \) (Constant surface heat flux)
Figure 6.8  Buoyancy effect on heat transfer when $Pr = 100, H = 0$ and $Re = 100$
(Constant surface heat flux)
Table 6.1  Comparison of present results with series solution and Saeid (2005)

\((Pr = 0.7, H = 0 \text{ and } \beta = 0)\)

<table>
<thead>
<tr>
<th>(K)</th>
<th>Skin friction values</th>
<th>Heat transfer values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.3562</td>
<td>0.3493</td>
</tr>
<tr>
<td>0.05</td>
<td>0.4268</td>
<td>0.4182</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5049</td>
<td>0.5043</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5569</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7036</td>
<td>-</td>
</tr>
<tr>
<td>0.4</td>
<td>0.7398</td>
<td>-</td>
</tr>
<tr>
<td>0.5</td>
<td>0.8215</td>
<td>-</td>
</tr>
</tbody>
</table>

From Table 6.1, it is evident that the present values of skin friction and heat transfer are well comparable with the known series solution. One can also note that the order of accuracy remains same for very large and small Prandtl numbers for various \(H\) and \(\beta\).
6.5 CONCLUSION

This chapter presents an analytical result for free and forced convection with the effects of suction and injection over a non-isothermal wedge. The governing partial differential equations are reduced to coupled polynomial equations, the coefficients of which are functions of independent parameters $Pr$, $H$ and $m$. The great advantage involved in the present technique is that the results are obtained with remarkable accuracy and the cost of calculation is considerably less than that of numerical procedure.