CHAPTER 3

VEDIC MATHEMATICS

3.1 INTRODUCTION

The work, Vedic Mathematics or ‘Sixteen Simple Mathematical Formulae from the Vedas’ was written by His Holiness Jagadguru Sankaracharya Sri Bharati Tirthaji Maharaja of Govardhana Matha, Puri (1884 -1960). The very word ‘Veda’ has a derived meaning, ie the fountainhead and unlimited store-house of all knowledge. The Vedas are well known and are four in number Rig, Yajur, Sama and Atharva, but, they have also, the four Upavedas or the six Vedangas, all of which form an indivisible corpus of divine knowledge as it once was and as it may be revealed.

<table>
<thead>
<tr>
<th>Veda</th>
<th>Upaveda</th>
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<tr>
<td>Rigveda</td>
<td>Ayurveda</td>
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<tr>
<td>Samaveda</td>
<td>Gandharvaveda</td>
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<tr>
<td>Yajurveda</td>
<td>Dhanurveda</td>
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<tr>
<td>Atharvaveda</td>
<td>Sthapatyaveda</td>
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</table>

In this list, the Upaveda ‘Sthapatya’ or engineering comprises all kinds of architectural and structural human endeavour and all visual arts. Mathematics or the science of calculations falls under this category. The sixteen Sutras presented here form part of a ‘Parisista’ of the Atharva veda.
3.2 LIST OF SUTRAS

Twelve of the sixteen sutras alone (relevant sutras) are presented in table 3.1 along with their application in brief.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Name of the Sutra</th>
<th>Explanation</th>
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<tbody>
<tr>
<td>1.</td>
<td>Ekadhikena Purvena’</td>
<td>By one more than the previous one-to convert vulgar fractions to their equivalent decimal form.</td>
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<td>2.</td>
<td>Nikhilam Navathascaramam Dasatah</td>
<td>All from nine and the last from ten-multiplication of numbers by taking as base for calculations, that power of 10 which is nearest to the numbers to be multiplied.</td>
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<td>3.</td>
<td>Urdhva-tiryagbhyan</td>
<td>Vertically and crosswise-applicable to all cases of multiplication and found useful in division of large number by another number.</td>
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<td>4.</td>
<td>Paravartya Yojayet</td>
<td>Transpose and apply – the transposition rule enjoins invariable change of sign with every change of side. Thus + becomes - and * becomes / and conversely.</td>
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<td>5.</td>
<td>Sunyam Samyasamuccaye</td>
<td>Samuccaya can take different meanings in different contexts. For instance it can be a term, which occurs as a common factor in all the terms concerned, or the product of the independent terms, or the sum of the denominators of two fractions having the same numerical numerator etc.</td>
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<tr>
<td>6.</td>
<td>(Anurupye) Sunyamanyat</td>
<td>Proportionately. When neither the multiplicand nor the multiplier is sufficiently near a convenient multiple or sub-multiple of a</td>
</tr>
<tr>
<td>S.No.</td>
<td>Name of the Sutra</td>
<td>Explanation</td>
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<tr>
<td>40</td>
<td></td>
<td>suitable base, perform the necessary operation with its aid and then multiply or divide the result proportionately.</td>
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<td>7.</td>
<td>Sankalana-vyavakalanabhyam</td>
<td>used for finding the highest common factor of two or more given expressions. This sutra adopts one of alternate destruction of the highest and the lowest powers by a suitable multiplication of the coefficients and the addition or subtraction of the multiples.</td>
</tr>
<tr>
<td>8.</td>
<td>Puranapuranabhyam</td>
<td>completion or non-completion (of the square, the cube, the fourth power etc) – For solving the general form of quadratic equation by the method of argumentation and factorization.</td>
</tr>
<tr>
<td>9.</td>
<td>Calana-kalanabhyam</td>
<td>The Sutra means 'Sequential motion' or Differential Calculus'. Every Quadratic can thus be broken down into two binomial factors.</td>
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<td>10.</td>
<td>Yavadunam</td>
<td>It so happens that squaring, cubing etc., of numbers have a particular entity and individuality of their own; Thus yavadunam sutra in the further outlook can be similar to binomial theorem.</td>
</tr>
<tr>
<td>11.</td>
<td>Ekanyunena Purvena</td>
<td>One less than the Previous. It provides for multiplications wherein the multiplier digits consist entirely of nines. It is useful in solving Special Multiplications like 777 X 999, 123456789X 999 999 999.</td>
</tr>
<tr>
<td>12.</td>
<td>Gunitasamuccayah</td>
<td>The whole product is the same. This means that the product of the sum of the coefficients in the factors is equal to the sum of the coefficients in the product.</td>
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</tbody>
</table>
The Urdhva Tiryakbhyam Sutra, which is the vertical and crosswise multiplication algorithm, is mainly used in this work. In the proposed architecture, grouping of 4-bits is done for both the multiplicand and multiplier. Thus the whole operation is decomposed into 4 x 4 bit multiplication modules.

3.3 URDHVA TIRYAKBHYAM SUTRA

This sutra has been identified for use in the present work since it gives a general formula that is applicable to all cases of multiplication (large bit multiplication, small bit multiplication and modular multiplication) and is also very compact in the division of a large number by another large number, for example division of a 15 digit number by a 5 digit number. The algebraic principle involved is explained as follows

3.3.1. Multiplication Using Urdhva Tiryakbhyam Sutra

Suppose we have to multiply \((ax+b)\) by \((cx+d)\). The product is \(acx^2 + x(ad+bc) + bd\). This can be obtained as follows:

Step 1: The coefficient of \(x^2\) is obtained by the vertical multiplication of \(a\) and \(c\)

Step 2: The coefficient of \(x\) is obtained by the crosswise multiplication of \(a\) and \(d\) and of \(b\) and \(c\) and the addition of the two products

Step 3: The independent term is arrived at by vertical multiplication of the absolute terms \(b\) and \(d\).
3.3.2 Example of 8 X 8 Bit Multiplication

Let A be the 8 bit multiplicand and B be the 8 bit multiplier. These can be further divided into 4 bit terms as shown below:

\[
A = A_7A_6A_5A_4 \quad A_3A_2A_1A_0
\]

\[
B = B_7B_6B_5B_4 \quad B_3B_2B_1B_0
\]

So \( A = X_1 X_0 \) (8 bit Multiplicand)

\( B = Y_1 Y_0 \) (8 bit Multiplier)

where \( X_1, X_0, Y_1, Y_0 \) are each of 4-bits. Multiplying, we get a 16 bit product, which is further divided into 4 four bit terms, F, E, D, C.

\[
X_1X_0 \times Y_1Y_0 = F \quad E \quad D \quad C
\]

1. \( CP = X_0 \times Y_0 = C \)

2. \( CP = X_1 \times Y_0 + X_0 \times Y_1 = D \)

3. \( CP = X_1 \times Y_1 = F \quad E \)

where \( F \) is the carry of the product of \( X_1 \times Y_1 \) and \( CP \) is the cross product.

Note:

1. Each Multiplication operation is an embedded parallel 4 x 4 multiply module.

2. The carry generated in each of the multiplication modules is propagated to the next module.
This multiplier architecture has the advantage compared of minimal gate delays and improved regularity of structure.

The process is further explained with the help of examples. Two digit and three digit multiplication examples are explained using decimal numbers and the multiplication process is shown with the help of lines. The digits on either side of the line are multiplied and the result is added to the previous carry and the process is continued.

Some examples illustrating the process of multiplication using this sutra is given in Appendix I.

3.4 SUMMARY

The Sutra ‘Urdhva Tiryakbhyam’ is identified and is used in this work. The steps involved are explained in detail. The proposed algorithms can be extended for any number of digits. Since the identified Sutra of Vedic Mathematics works with an ability similar to that of human cognitive thinking, it is highly suitable for implementation in embedded architectures and ASICs.