CHAPTER 3

MATHEMATICAL MODELING OF GTA WELDING

3.1 INTRODUCTION

The various physical phenomena involved in gas tungsten arc (GTA) welding process model are shown in Figure 2.1. The weld pool formed below the welding torch is subjected to various forces such as buoyancy, electromagnetic and surface tension forces. These driving forces create weld pool convection. Experimental results show that the fluid flow is generally laminar and incompressible, except at very high heat input which results in a large weld pool. In general, to predict the weld pool size and temperature history for a given set of welding conditions, a heat and fluid flow model has to be developed to solve the coupled mass, momentum and energy transport equations within the weld pool.

Several numerical schemes have been developed for the simulation of both steady and unsteady viscous flows using various formulations of the problem. The main difficulties encountered in incompressible flow simulation arise from the absence of any obvious equation for pressure and the nature of spatial coupling between the pressure and velocity. For incompressible flow problems, pressure does not have the usual thermodynamical meaning; here it is a relative variable, which adjusts itself instantaneously in order for the condition of zero divergence to be satisfied at all times. This behaviour is related to the well known fact that in an incompressible fluid the speed of sound becomes infinite. As a consequence, the pressure field cannot be calculated by the explicit time-advancement procedure; instead, it requires an
implicit determination, which is able to take into account the coupling existing between the pressure and velocity fields, as well as the effects of the velocity boundary conditions. This aspect is the most distinctive feature of the primitive variable formulation of incompressible Navier Stokes equations.

3.1.1 Primitive Variable Solution Procedure

Primitive variable formulation is one of the solution methodologies, most widely used in pressure coupled equations (Ghoshdastidar 1998). In this method, the dependent variables are velocity components and pressure (i.e. u, v, w and p). For two dimensional and axisymmetric problems, two momentum equations are available and the continuity equation is used as a constraint for determining the pressure field. Pressure is indirectly estimated via the continuity equation i.e. when the correct pressure is substituted into the momentum equations, the resulting velocity field satisfies the continuity principle. This is the central idea of all primitive variable formulations.

3.1.2 Explicit Algorithms

The dependent variables at time (t + Δt) can be obtained explicitly from the known results at time (t) by explicit time marching algorithms. A number of explicit finite difference schemes have been developed such as MAC (Marker and Cell), in which the continuity equation is recast in the form of a Poisson equation for pressure correction. The MAC method was later modified as SMAC (Simplified Marker and Cell) in which the continuity equation is directly satisfied without converting it into a pressure equation. In a similar technique applied by Chorin (1967), the incompressible flow problem is solved by an asymptotic time solution of equations containing an artificial compressibility term. This term is designed to vanish as the steady state is reached. Hirt and Cook (1972) developed a simplified explicit flow solver for a wide variety of problems involving 3-D incompressible flows.
3.1.3 Implicit and Semi Implicit Schemes

The severe stability constraints on the time steps of the explicit algorithms have led to the use of implicit and semi-implicit algorithms. In the implicit approach, the equations are not only expressed in terms of the known quantities at time (t) but also in terms of unknown quantities at time (t + Δt). The Semi Implicit Pressure Linked Equations (SIMPLE) algorithm of Patankar and Spalding (1972) and its extensions (SIMPLER, SIMPLEC etc.) have provided a successful semi-implicit procedure with primitive variables. In order to extend the applicability of the above mentioned algorithms to complex geometries, non-orthogonal boundary fitted coordinate systems have been used. For complex geometries with flow, the finite volume method is attractive. Figure 3.1 shows the flow chart representing the SIMPLE algorithm.

In SIMPLE algorithm, a guess pressure is first used in the momentum equations to predict the approximate or guess velocities in x-direction (u*), and in y-direction (v*). The substitution of these velocities in continuity equation is tested for convergence. The error in mass balance is used to make corrections for pressure p as well as the velocity components u and v. The corrected pressure (pn) and velocity components (un,vn) are iteratively substituted until convergence is obtained. Once mass balance is achieved, the computation continues for the next time step.
3.2 MATHEMATICAL FORMULATION

In this section, the governing equations with the required boundary conditions describing the physical processes of the problem are presented. The formulation considers transient behaviour of weld pool and the
combination of buoyancy, electromagnetic and surface tension forces for melt circulation.

### 3.2.1 Theoretical Model

During linear welding, the weld pool is formed and extended in the direction of welding. It is well known that the depth and width of the pool are almost retained during the entire process and they represent the final depth and width of the bead. Modeling and simulation for a three dimensional transient problem requires large computing power, memory and computational time. An equivalent two dimensional model will require much less computational resources than the three dimensional approach and hence, it is preferable. In the present work, an equivalent two dimensional heat and fluid flow model has been developed with the following assumptions:

1. A Gaussian heat flux distribution is applied as shown in Figure 3.2.
2. Melting of metal occurs only in the region below the arc.
3. Due to constant values of traverse speed, welding current and voltage, the overall molten pool geometry is invariant in the electrode traverse direction.
4. After spatial averaging in the traverse direction, the resulting flow and thermal fields in the cross sectional (x-y) plane of the bead primarily determine the depth and width of the molten pool and the dimensions of the heat affected zone.

The Gaussian heat flux profile has a maximum value at the center of the arc and decreases along the radial direction. In the present work, the heat flux variation in the lateral (x) direction has been considered as a Gaussian profile, but it has been averaged into an equivalent constant value.
(maximum heat flux value) in the \(z\)-direction, leading to a reduced equivalent arc thickness in that direction (Figure 3.2c). The residence period of the arc at any location has been calculated using the equivalent thickness of the arc in the \(z\)-direction, as follows:

\[
\text{Arc residing period} = \frac{(C_{eq} \times \text{Diameter of Gaussian base circle})}{\text{Torch speed}}
\]  

(3.1)

where, \(C_{eq}\) is the fraction for reduced thickness in the \(z\)-direction and in the present study its value has been taken as 0.6 (Figure 3.3). Similar to the heat flux input, the current input and the associated self-induced magnetic field also need to be averaged spatially.

*Figure 3.2 Distribution of heat flux from the electric arc*
In a stationary arc (Figure 3.4 a), the current density vector \( \vec{J} \) will have two components in the radial (r) and axial (z) directions of the cylindrical polar co-ordinate system. The corresponding self-induced magnetic field vector \( \vec{B} \) will have non-zero component only in the azimuthal (\( \theta \)) direction, as shown in Figure 3.4a. However, for a moving arc, spatial averaging will result in cancellation of the lateral components of the magnetic field as shown in Figure 3.4b, and hence, the simplified magnetic field can be assumed to have non-zero component only in the z-direction (traverse direction) as shown in Figure 3.4c. Therefore, for the simplified 2-D model, the Lorentz force components in the x and y directions will be given by the equation \( \vec{F} = \vec{J} \times \vec{B} \). Under the condition of constant current and voltage inputs for a long plate specimen, the weld pool geometry can be taken as uniform in the z-direction, with simplified thermal and flow boundary conditions in the cross sectional (x-y) plane only.

Figure 3.3 Spatial averaging of heat flux from the electric arc
The following additional assumptions are made in the modeling of weld pool convection.

1. Flow is Newtonian, laminar and incompressible,

2. All physical properties are constant and independent of temperature, except for thermal conductivity (k) and surface tension (γ), and

3. The free surface of the weld pool is undeformed.
Assumptions of laminar flow and undeformed free surface are justifiable from the fact that the welding current considered here is in the low range of 60 A to 120 A (Kim and Na, 1992, Choo and Szekely, 1994), which is suitable for the autogenous welding of a 4.35 mm thick stainless steel plate. Given the fact that the coefficient of thermal expansion $\beta \sim 10^{-4}$ K$^{-1}$ and typical $\Delta T$ values across the molten metal pool are of the order of the 1000 K, the ratio of density variation $\Delta \rho$ with respect to the average density $\rho_{av}$ is given as $\Delta \rho / \rho_{av} \sim 10^{-4} \times 10^3 \sim 10^1$. For this reason, the variation of density can be accounted for in the buoyancy term only, treating density as constant for all other purposes. The resulting two-dimensional governing equations for mass, momentum and energy balance in the x and y coordinates are:

(A) Governing equations

continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (3.2)

x-momentum:

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x \beta(T - T_{amb}) + J_y B_z$$  \hspace{1cm} (3.3)

y-momentum:

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y \beta(T - T_{amb}) - J_x B_z$$  \hspace{1cm} (3.4)

energy:

$$\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$  \hspace{1cm} (3.5)
where \( \rho \) is the density, kgm\(^{-3}\),
\( C_p \) is the specific heat, Jkg\(^{-1}\) K\(^{-1}\),
\( k \) is the thermal conductivity in Wm\(^{-1}\) K\(^{-1}\).

For weld pool modeling,

\[
\begin{align*}
\rho &= \rho_{ss} \text{ for stainless steel} \\
C_p &= C_{pss} \text{ for stainless steel} \\
k &= k_{ss} \text{ for stainless steel (below solidus temperature)} \\
k &= k_{ls} \text{ for stainless steel (at and above liquidus temperature)}
\end{align*}
\]

where subscript ‘ss’ denotes stainless steel.

Here, the rates of mechanical work (kinetic energy, pressure work, electromagnetic work and viscous dissipation) are considered to be negligible, because of the low speed flow in the weld pool. The current density in the x-direction, at any location in the weld pool, can be written in terms of the potential (\( \phi \)) and electrical conductivity (\( \sigma \)) as,

\[
J_x = -\sigma \frac{\partial \phi}{\partial x} \quad (3.6)
\]

Similarly, the current density in the y-direction within the weld pool, can be written as,

\[
J_y = -\sigma \frac{\partial \phi}{\partial y} \quad (3.7)
\]

The distribution of electric potential (\( \phi \)) in the work piece can be evaluated from the equation,

\[
\nabla^2 \phi = 0 \quad (3.8)
\]

Symmetry about the bead mid-plane requires that \( B_z = 0 \) at \( x = 0 \). Therefore, the self induced magnetic field (\( B_z \)) is obtained from Ampere’s law as,
$B_z(x') = \left( \frac{\mu_m}{x'} \int_0^{x'} y x \, dx \right)$

(3.9)

where, $\mu_m$ is the magnetic permeability of weld metal in Hm$^{-1}$ and $x'$ is any location in the x-direction within the arc, measured from the bead mid-plane.

The electromagnetic force $J_x B$ in the weld pool is calculated from,

$\left( J_x B \right)_x = J_y B_z$

(3.10)

$\left( J_x B \right)_y = -J_z B_z$

(3.11)

The electromagnetic force thus calculated is substituted in the Equations (3.3 and 3.4).

For the in-house FVM code development, only laminar flow regime has been considered. For high current inputs where turbulence effects would become important, the modeling of turbulent flow in the melt pool needs to be performed.

(B) Boundary conditions

Due to symmetry about the centerline of the welding zone, one half plane region of the plate (Figure 3.2a) is considered for the computational study. The boundary conditions are prescribed as shown in Figure 3.5. The top, side and bottom surfaces of the plate are exposed to ambient conditions during welding. Hence, these surfaces are provided with convection boundary conditions. Since, the top surface attains higher temperature, radiation effect has also been considered for the top surface. The portion (AB) of the top surface that is exactly below the arc is subjected to prescribed heat flux and current density distributions. In addition, surface tension force is also applied at the top surface of the molten metal when the base metal melts during
welding. Symmetry about mid-plane is imposed on the left side boundary (AFE) and this implies no heat flow across the boundary.

\[ q_a(x) = \frac{3Q}{\pi r_b^2} e^{-\frac{3x^2}{r_b^2}} \]  

Figure 3.5 Boundary conditions

For the top surface AB under the welding beam, heat input occurs due to the heat generation in the arc zone. A Gaussian heat flux distribution, which has been frequently employed in many earlier studies (Pavelic et al 1969, Kim and Basu 1998) is considered for calculating the heat flux at each cell boundary under the arc. The heat flux \( q_a(x) \) at a distance \( x \) from the symmetry plane is written as,
where, $Q(= VI\eta)$ is the rate of heat input entering into the work piece, Watts

$\eta$ is the arc efficiency and

$r_b$ is the effective arc radius for heat flux distribution, m

The applied heat flux is transferred to the cells which are just below the welding arc through the boundary condition

$$q_a(x) = k_{eq} \left( \frac{\partial T}{\partial y} \right)$$  \hspace{1cm} (3.13)

where, $k_{eq}$ is the equivalent metal thermal conductivity which takes into account the effect due to liquid motion. In view of the low melt pool velocities considered in the present study, it is assumed that $k_{eq} = k_l$, thermal conductivity of the molten metal. In addition to the heat flux, the boundary condition for the current density is to be prescribed at the top surface. The Gaussian distribution of arc current density at a distance $x$, can be written as,

$$J_{surf}(x) = \frac{3I}{\pi r_j^2} e^{-\frac{3x^2}{r_j^2}} = -\sigma \frac{\partial \phi}{\partial y}$$  \hspace{1cm} (3.14)

where, $r_j$ is the effective arc radius for the current density distribution

Surface tension is the most dominant force among the primary forces which are causing the fluid flow. The fluid flow and heat transfer in the weld pool are influenced by the spatial variation of surface tension ($\gamma$), which in turn, is due to the variation of surface tension with temperature, governed by the coefficient $d\gamma/dT$. The surface shear stress is given by

$$\mu \left( \frac{\partial u}{\partial y} \right) = -\frac{\partial \gamma}{\partial T} \frac{\partial T}{\partial x}$$  \hspace{1cm} (3.15)
From the assumption of undeformed top surface, we get,

\[ v = 0 \]  \hspace{1cm} (3.16)

For the unheated top surface BC, the convective and radiative heat loss boundary condition can be expressed as,

\[ k \frac{\partial T}{\partial y} + h(T - T_{amb}) + \varepsilon \sigma_{rad} (T^4 - T_{amb}^4) = 0 \]  \hspace{1cm} (3.17)

where, \( h \) is the heat transfer coefficient, Wm\(^2\)K\(^{-1}\),
\( \sigma_{rad} \) is the Stefan Boltzman constant, Wm\(^2\)K\(^{-4}\),
\( \varepsilon \) is the emissivity of material and
\( T_{amb} \) is the ambient temperature, K.

For the flow region, the shear stress and undeformed surface condition (Equations 3.15 and 3.16) are prescribed on the top surface of the weld pool region and the no flow condition of \( u = v = 0 \) is prescribed in the solid region.

Assuming normal current flow through the boundaries at side surface CD and bottom surface DE,

\[ \phi = 0 \]  \hspace{1cm} (3.18)

\[ k \frac{\partial T}{\partial n} + h(T - T_{amb}) = 0 \]  \hspace{1cm} (3.19)

where, \( \frac{\partial T}{\partial n} \) is the temperature gradient in the normal direction at the boundary. Also at the solid boundaries,

\[ u = 0, v = 0 \]  \hspace{1cm} (3.20)
At the symmetry surfaces AF and FE

\[ \frac{\partial \phi}{\partial x} = 0, \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial v}{\partial x} = 0 \quad \text{and} \quad u = 0 \quad (3.21) \]

In general, both velocity components \( u \) and \( v \) are set to zero in the solid region (with temperature less than the solidus temperature). The release or absorption of latent heat due to phase change at the solid-liquid interface has been considered by accounting for latent heat in the form of an equivalent enhanced specific heat for the nodes lying in the phase change region.

(C) **Numerical solution procedure**

The computational domain, which is a rectangle, is divided into a number of rectangular cells of variable spacing. Finer grids are utilized nearer to the heat source, whilst further away a relatively coarser grid is employed. One half of the plate is considered as the computational domain due to the symmetry of the plate being welded. An overall 80 \( \times \) 80 non-uniform staggered grid system (Figure 3.6) has been used for the calculation of the temperature and velocity fields in the x-y plane. For the numerical solution of governing equations, the problem domain is covered by a set of rectangular control volumes. The discretization is performed using a staggered grid that consists of temperature/velocity nodes and pressure nodes, as discussed by Raghavan et al (2005).
Figure 3.6 Grid system

The staggered grid system eliminates the pressure-velocity decoupling problem to which incompressible flow solver algorithms are generally susceptible. A semi-implicit, control volume integration of the governing equations results in the discretized form given by,

$$a_p \phi_p = a_S \phi_S + a_N \phi_N + a_E \phi_E + a_W \phi_W + b$$  \hspace{1cm} (3.22)

where, $\phi$ is any flow variable or temperature and the subscripts $S$, $N$, $E$ and $W$ indicate neighbouring nodes for the node $P$ (Figure 3.7) under consideration. To solve the governing equations with the associated source terms numerically, a C++ code has been developed based on the SIMPLE
algorithm (Patankar 1980). The results obtained from the code are post-processed with TECPLLOT software. Figure 3.8 shows the flow chart of the newly developed algorithm.

The computational domain is created by prescribing the maximum values of x-distance (half width of the plate) and y-distance (thickness). The number of elements in the x-direction and y-direction are specified for both coarse regions with coarse grid and regions with fine grid. Material properties and values of constants used in the computations are furnished. The input variables such as welding current, voltage and welding speed are given. The parameters relevant to the boundary conditions are also included. Initial conditions are specified by initial temperature and pressure. Computational parameters such as time step, simulation time, residue level criterion and under relaxation parameter are provided. The computation for the first time step begins in the first cell and continues for each cell and ends in the last cell.
Start

Domain discretization, Material properties, Initial condition, boundary condition

In each cell, solve energy equation to find T*

Is T* > Tsolidus?

Yes

Initial guess pressure p*

Solve momentum equations to find u* and v*

p* = pn  u* = un  v* = vn

Solve pressure correction & velocity correction equation to find p', u' and v'

Find correct pressure and velocity
pn = p* + p'  un = u* + u'  vn = v* + v'

Solve the continuity equation to find residue

No

Is Residue < 0.1% ?

Yes

T = T*, p = p*, u = u*, v = v*

time = time + Δt

Is time < simulation time?

Stop

Figure 3.8 Flow chart of the algorithm for the newly developed code
Then, the next time step starts. Computation is carried out as per Gauss-Siedel point-by-point procedure. During the computation, the energy equation is solved in all the cells until the base metal melts. The solidus temperature ($T_{sol}$) is used as the reference for the melting of metal. Then the momentum and continuity equations are solved in all the cells which are in liquid state. Computation continues and total computational time increases, by the time step size until the total time becomes equal to the arcing time. At the end of computation the temperature and velocity fields are obtained as data files. The results have been converted into temperature and velocity plots using TECPLOT software.

### 3.2.2 Model Verification

The mathematical basis and the relevant finite volume formulation for the transient heat and fluid flow during the GTA welding problem have already been discussed in the previous sections. Based on this, a computer code with the capabilities for complete pre-processing, main computations and post-processing has been developed. Before applying the developed code for weld pool modeling, it is important to verify the validity of the results predicted by the code with benchmark solutions. In this section, a series of test cases, from simple lid-driven cavity flow to natural convective flow in a square cavity are modeled to check the correctness of the code and accuracy of the simulation. Basically, weld pool convection is driven by buoyancy, electromagnetic and surface tension forces. Initial studies have been made by considering all the three forces separately to understand the role of each individual driving force in controlling the free surface temperature and the weld pool geometry.
(A) **Lid driven laminar flow in a square cavity**

The steady state laminar flow in a square cavity filled with a fluid, driven by a moving top boundary with unit velocity is considered as the first benchmark problem. The configuration of the problem and associated boundary conditions are shown in Figure 3.9. The top moving boundary is moved with unit velocity. Fluid flow develops within the square cavity, starting from the top of the cavity. The predicted u-velocity profile along the center line \((x = 0.5)\) at steady state from this simulation is compared with those reported by Ghia et al (1982) for Reynolds number equal to 100. The predicted results are shown in Figures 3.10 and 3.11. It is observed that the predicted results of the present study agree well with those of Ghia et al (1982).

![Figure 3.9](image)

**Figure 3.9** Geometry and dimensionless boundary conditions for lid driven cavity problem
Figure 3.10  Dimensionless u-velocity variation in the lid driven cavity at x = 0.5

Figure 3.11  Dimensionless v-velocity variation in the lid driven cavity at y = 0.5
(B) Channel flow

The laminar flow development between two long plates having 2 m length, kept parallel to each other at a distance of 0.1 m has been simulated, to study the ability of the code with channel flow conditions. No slip boundary conditions are used on the plate surfaces. The inlet boundary condition for axial velocity is unity. The standard parabolic velocity profile of channel flow problem (Landau and Lifshitz 1987), demands that the maximum velocity at outlet is 1.5 times that of inlet velocity, whereas the predicted maximum velocity obtained by using the code is 1.502. Figure 3.12 shows the geometry and boundary conditions for channel flow and Figure 3.13 presents the predicted velocity profile for the channel flow.

\[ u = 1.0 \]
\[ u = 0; \ v = 0 \]

Figure 3.12 Geometry and dimensionless boundary conditions for channel flow
Figure 3.13 Results for dimensionless velocity profile in channel flow

(C) Natural convection in a square cavity

The code is validated for its ability to study buoyancy driven flow in a square cavity with vertical sides, which are differentially heated. Figure 3.14 shows the flow geometry with two isothermal and two adiabatic wall conditions. The predicted values of the present study for Rayleigh number equal to $10^6$ and Prandtl number equal to 0.71 (corresponding to air) have been compared with the results of Lewis et al (1995). Figure 3.15 shows the isothermal patterns for the present study and the results are compared with those of Lewis et al (1995). It is observed that the present predicted results for isothermal pattern are very close to the results of Lewis et al (1995). A non-uniform grid of 40 x 40 elements is used for this present simulation.
Figure 3.14 Geometry and boundary conditions for conduction and convection in a square cavity (length in m and velocity in m/s)

Figure 3.15 Results for Natural convection in a square cavity for $Ra = 10^6$
3.2.3 Mesh Refinement Study

In order to determine the optimal mesh size for accurate calculation of the temperature and velocity fields during GTAW with less computation time, grid refinement studies have been conducted in the computational domain. Essential terms for weld pool modeling have been included in the momentum equations and boundary conditions. Tests have been conducted with different sizes of (60 x 60, 70 x 70, 80 x 80 and 90 x 90) non-uniform grid system (Figure 3.6a) and the predicted results of grid independence test are given in Table 3.1. The peak temperature reduces by about 2.11 K (about 0.1 per cent) when the number of cells in the grid system is increased from 80 x 80 to 90 x 90. The peak temperature becomes almost constant if the grid size is further increased. Hence, an 80 x 80 grid system (Figure 3.6a) has been used for the simulation. The minimum size of grid along the x-direction is 0.1 mm and the minimum size of grid along the y-direction is 0.04 mm.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Element size</th>
<th>Peak temperature, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>60 x 60</td>
<td>2536.62</td>
</tr>
<tr>
<td>2.</td>
<td>70 x 70</td>
<td>2547.03</td>
</tr>
<tr>
<td>3.</td>
<td>80 x 80</td>
<td>2556.62</td>
</tr>
<tr>
<td>4.</td>
<td>90 x 90</td>
<td>2558.73</td>
</tr>
</tbody>
</table>

### Table 3.1 Results of Grid Independence Test
(Buoyancy force: Current =100 A, Voltage=13.5 V, Speed=3.6 mm/s)

3.3 PARAMETRIC STUDY FOR GTA WELDING

The effects of driving forces on the peak temperature, maximum velocity in the weld pool and bead dimensions have been studied by applying the buoyancy, electromagnetic and surface tension forces, individually and collectively. The properties of AISI 304 L stainless steel considered for the
The present work are given in Table 3.2. The computational domain is one half of the plate represented by a rectangle of size 40 mm x 4 mm. An 80 x 80 non-uniform grid system has been used for the simulation. The welding process parameters used in the computation are given in Table 3.3.

Table 3.2 Properties of AISI 304 L stainless steel

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbols</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity(solid)</td>
<td>$k_{ss}$</td>
<td>31.39 $W\cdot m^{-1}\cdot K^{-1}$</td>
</tr>
<tr>
<td>Thermal conductivity(liquid)</td>
<td>$k_{ls}$</td>
<td>15.48 $W\cdot m^{-1}\cdot K^{-1}$</td>
</tr>
<tr>
<td>Convection heat transfer coefficient</td>
<td>$h$</td>
<td>20 $W\cdot m^{-2}\cdot K^{-1}$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho_{ss}$</td>
<td>7200 $kg\cdot m^{-3}$</td>
</tr>
<tr>
<td>Specific heat</td>
<td>$C_{pss}$</td>
<td>753 $J\cdot kg^{-1}\cdot K^{-1}$</td>
</tr>
<tr>
<td>Latent heat</td>
<td>$\Delta H$</td>
<td>2.1E9 $J\cdot m^{-3}$</td>
</tr>
<tr>
<td>Solidus temperature</td>
<td>$T_{sol}$</td>
<td>1523 K</td>
</tr>
<tr>
<td>Liquidus temperature</td>
<td>$T_{liq}$</td>
<td>1723 K</td>
</tr>
<tr>
<td>Viscosity</td>
<td>$\mu$</td>
<td>0.005 $kg\cdot m^{-1}\cdot s^{-1}$</td>
</tr>
<tr>
<td>Coefficient of thermal expansion</td>
<td>$\beta$</td>
<td>$10^{-4}$ $K^{-1}$</td>
</tr>
<tr>
<td>Electrical conductivity</td>
<td>$\sigma$</td>
<td>7.7E5 $\Omega^{-1}\cdot m^{-1}$</td>
</tr>
<tr>
<td>Magnetic permeability of material</td>
<td>$\mu_{m}$</td>
<td>1.26E-6 $H\cdot m^{-1}$</td>
</tr>
</tbody>
</table>

Table 3.3 GTA welding process parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welding current, A</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>120</td>
</tr>
<tr>
<td>Welding voltage, V</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>13.5</td>
</tr>
<tr>
<td></td>
<td>14.0</td>
</tr>
<tr>
<td>Welding speed, mm/s</td>
<td>1.5 to 7.0</td>
</tr>
<tr>
<td>Arc gap, mm</td>
<td>2.0</td>
</tr>
</tbody>
</table>
3.3.1 Buoyancy Driven Flow

The temperature and velocity fields have been calculated by solving the governing equations after omitting the electromagnetic force term in the momentum equations, and applying all the boundary conditions except for surface tension and electromagnetic forces.

3.3.2 Electromagnetic Force Driven Flow

The temperature and velocity in the weld pool have been calculated by solving the governing equations, for all welding conditions used for buoyancy driven flow. The buoyancy term in the momentum equations and the boundary conditions for the surface tension force are omitted.

3.3.3 Surface Tension Force Driven Flow

The temperature and velocity in the weld pool have been calculated by solving the governing equations, for all welding conditions used for buoyancy driven flow. Both buoyancy and electromagnetic force terms are omitted from the momentum equations. The boundary conditions for electromagnetic force have also been neglected.

3.3.4 Combined Buoyancy, Electromagnetic and Surface Tension Forces Driven Flow

The complete governing equations and boundary conditions given by Equations (3.1 to 3.21) are considered for the computations when the driving forces of buoyancy, electromagnetic field and surface tension are included in the model to predict the temperature, velocity and weld bead geometry.

Figures 3.16 and 3.17 show the temperature and velocity fields in the GTA weld pool for a welding condition of 100 A, 13.5 V and 7.2 mm/s
when the buoyancy, electromagnetic and surface tension forces individually and collectively act on the weld pool. As the heat input is very low, at the rate of 0.1875 kJ/mm, a small weld pool is seen for all the cases. The weld pool dimensions are approximately 1.5 mm for half width and 0.6 mm for depth of penetration.

The fluid flow is found to be in the outward direction for buoyancy force (Figure 3.17a), and in the inward direction for electromagnetic or surface tension as well as for the combination of all forces (Figures 3.17b, c and d). The maximum fluid velocity is about 1 mm/s for buoyancy and 2 mm/s for electromagnetic force. But, it is in the order of 126 mm/s for the surface tension force and for the combined force situations. The maximum temperature in the weld pool is 2185.63 K (Figure 3.16a) for buoyancy driven flow. It is about 2190.75 K (Figure 3.16b) for electromagnetic force driven flow, which is slightly higher than that of buoyancy driven flow. The reason is the inward flow in the case of electromagnetic force which transfers heat from the outer periphery of the weld pool to the center of the weld pool. For surface tension force driven flow considered here (low current input), the flow is inward as the temperature is almost well below 2200 K in the major portion of the weld pool (Zacharia et al 1991a). The gradient of surface tension coefficient ($\partial \gamma / \partial T$) is positive and equal to $10^{-5}$ N/mK for temperatures below 2200 K, if the sulphur content in the base metal is about 80 ppm. Hence, the flow is inward. For the same reason, heat is carried by the fluid motion from the outer periphery to the center of the weld pool, resulting in a higher peak temperature of 2248.41 K (Figure 3.16c). The width of the weld pool is slightly smaller than that in the other cases. The depth of penetration is also the maximum for surface tension force driven flow. These can be attributed to the generation of stronger weld pool convection by the force due to surface tension gradient. In the case of combined all forces, the peak temperature and maximum velocity are 2243.42 K and 0.126 m/s (Figures 3.16d and 3.17d).
Figure 3.16 Effect of driving forces on temperature field during GTA welding at 100 A, 13.5 V, 7.2 mm/s with 0.1875 kJ/mm
Figure 3.17 Effect of driving forces on velocity field during GTA welding at 100 A, 13.5 V, 7.2 mm/s with 0.1875 kJ/mm
A relatively large weld pool is formed when the GTA welding is carried out at 100 A, 13.5 V and 3.6 mm/s. Figures 3.18 and 3.19 depict the temperature and velocity fields when the buoyancy, electromagnetic and surface tension forces individually and collectively drive the weld pool convection. It is clear from Figures 3.18a, 3.18b, 3.19a and 3.19b that the peak temperature and maximum velocity of molten metal in the weld pool are slightly higher for electromagnetic force driven flow than that of buoyancy driven flow. The peak temperatures are 2570.31 K for buoyancy force and 2601.23 K for electromagnetic force. The flow is outward with a velocity of 3 mm/s for buoyancy force and inward with a velocity of 5 mm/s for electromagnetic force. The depth of penetration and half width of the weld pool are 1.085 mm and 2.0 mm for buoyancy driven flow and 1.1 mm and 1.98 mm respectively for electromagnetic force driven flow.

Figures 3.18c and 3.19c show the temperature and velocity fields for the case of surface tension force driven flow. A positive surface tension coefficient value of $10^{-5}$ N/mK for temperatures below 2200 K and negative value of $10^{-5}$ N/mK for temperatures above 2200 K have been used in the computation. Hence, the flow is found to be both inward and outward at the top surface of the weld pool, forming a double loop fluid circulation in the weld pool. The outward flow is smaller than the inward flow loop for this case. Hence, the maximum velocity in the weld pool has the direction towards the center of the weld pool (inward) and a magnitude of 0.165 m/s. The radial distance of the outward flow is 0.6 mm. Since, the inward flow is dominating over the outward flow, the depth of penetration increases. The depth of penetration and half width of the weld pool are 1.15 mm and 1.97 mm respectively. But, the peak temperature (2587.5 K) is slightly less than that of electromagnetic force. The heat dissipation from the top surface is more for the case of surface tension force due to higher velocity (mass flow). In the case of combined forces, the peak temperature and maximum velocity are 2593.06 K and 0.173 m/s (Figures 3.18d and 3.19d).
Figure 3.18 Effect of driving forces on temperature field during GTA welding at 100 A, 13.5 V, 3.6 mm/s with 0.375 kJ/mm
Figure 3.19 Effect of driving forces on velocity field during GTA welding at 100 A, 13.5 V, 3.6 mm/s with 0.375 kJ/mm
The depth of penetration and half width of the weld pool are 1.25 mm and 1.96 mm respectively. The double loop circulation influences the contours of isotherms and the shape of the weld. The radial distance of the outward flow is 0.6 mm.

A very large weld pool is formed when the welding speed is reduced to 2.4 mm/s. Figures 3.20 and 3.21 present the temperature and velocity distributions in the GTA weld pool for the individual and combined effects of buoyancy, electromagnetic and surface tension forces at a welding condition of 100 A, 13.5 V and 2.4 mm/s. From Figures 3.20a, 3.20b and 3.20c, it is clear that the weld pool is shallow for the case of buoyancy force and deeper for electromagnetic force. The half width and depth of the weld pool are 2.35 mm and 1.55 mm for the case of buoyancy force only and 2.32 mm and 1.85 mm for the case of electromagnetic force only. The contours for isotherms are smooth and concentric for both buoyancy and electromagnetic forces. But, for the case of surface tension force (Figure 3.20c), the shapes of the contours seem to be irregular and the width of the weld pool is high. The half width and depth of the weld pool are 2.3 mm and 2.55 mm respectively. In the case of all forces combined (Figure 3.20d), a rounded weld pool is formed. The shapes of the isothermal contours are distorted. The peak temperature is very low (2618.9 K) as the outward flow dominates the inward flow (Figure 3.21d). Hence the maximum velocity in the weld pool for the combined force situation is 0.1776 m/s and it has an outward direction.

From Figures 3.21a and 3.21b, it is evident that the maximum velocity is lower for buoyancy (0.0058 m/s) than that of electromagnetic force (0.0075 m/s). At the same time, a high velocity (0.1828 m/s) is obtained for surface tension force, resulting in more dissipation of heat energy due to more circulation (Figure 3.21c).
Figure 3.20 Effect of driving forces on temperature field during GTA welding at 100 A, 13.5 V, 2.4 mm/s with 0.5625 kJ/mm
Figure 3.21 Effect of driving forces on velocity field during GTA welding at 100 A, 13.5 V, 2.4 mm/s with 0.5625 kJ/mm
Hence, the peak temperature for surface tension force is the lowest among all the cases. The inward-outward fluid flow in the weld pool develops a double loop circulation as in the previous cases. But, the outflow is dominating over the inward flow due to the fact that the major portion of the weld pool has a temperature above 2200 K. The radial distance of outward flow is 0.001 m and the maximum velocity is in the outward direction. In general, the loop formed by the outward flow may be called as main loop of the double circulation.

Figures 3.22 and 3.23 show the effects of welding parameters on peak temperature, maximum velocity, weld pool width and depth. From Figures 3.22a, 3.22b, 3.23a and 3.23b, it is clear that the peak temperature is steadily increasing for the cases of buoyancy and electromagnetic forces, as the flow is inward and velocity is very low. Particularly, for the case of electromagnetic force driven flow, the heat conducted to the periphery of the weld pool from the center of the weld pool is taken back to the weld center, resulting in low heat dissipation and the peak temperature is the maximum at low welding speeds. But, at higher welding speed, the peak temperature is low for all the cases. At high welding speed, the heat input is low and a major portion of the weld pool will have temperature less than 2200 K. Hence, the fluid flow is inward for the case with surface tension force also. Hence, the peak temperature is slightly high for the case of surface tension driven flow at high welding speed and low welding current. But, when the temperature in the major portion of the weld pool increases significantly above 2200 K, the fluid flow becomes inward and outward for the case of surface tension force. Hence, the outflow region transfers the heat from the center of the weld pool to the periphery resulting in the reduction of peak temperature for surface tension driven flow at low welding speed. This effect is also seen in the case of combined forces.
Figures 3.22b and 3.23b depict the effects of heat input on maximum velocity. The maximum velocity is steadily increasing when the welding speed is reduced. But, the magnitude is very low for both the cases of buoyancy and electromagnetic force driven flow. At the same time, the maximum velocity gradually increases for the case of surface tension driven flow when the welding speed is reduced until the temperature of the weld pool is considerably higher than 2200 K. The maximum velocity is found to be more or less remaining constant when the average temperature in the major portion of the weld pool is about 2200 K. The outward and inward flow exist at the top of the weld pool and their impingement reduces the magnitude of velocity. Hence, the maximum velocity is maintained constant for certain range of welding speed. When the average temperature at the top of the weld pool exceeds 2200 K, the maximum velocity again increases steadily. The same effect is seen in the case of combined driving forces.

Figures 3.22c and 3.23c show the effect of welding speed on the half width of the weld bead. The weld bead width is gradually increasing for all individual driving forces. At high welding speeds, buoyancy force shows relatively higher weld bead width because the flow is outward only for the buoyancy force. At low welding speeds, surface tension force creates (double loop circulation) both inward and outward flow in the same weld pool, leading to increase in the width. Hence, both buoyancy and surface tension forces increase the width at low welding speeds. Figures 3.22d, and 3.23d, show that maximum depth of weld pool is obtained for surface tension and combined driving forces at high and medium welding speeds because both electromagnetic and surface tension forces cause inward flow. The depth of penetration is more or less same for all cases at very low welding speeds.
In general, the weld pool formed by buoyancy force alone is shallow. The peak temperature is maximum for the electromagnetic force at higher currents and lower welding speeds. But, it is the maximum for the case of surface tension force driven flow at high welding speed due to the inward flow and it is the minimum at low welding speed due to the inward-outward

Figure 3.22 Effect of driving forces and welding speed on weld pool parameters – 100A and 13.5 V
Figure 3.23 Effect of driving forces and welding speed on weld pool parameters – 120A and 14 V
flow. The inward-outward flow with high velocity, in the range of few hundred millimeters per second, results in quicker dissipation of heat energy by increasing the mass flow rate of liquid metal exposed to the ambient conditions. Hence, the depth of penetration increases for high welding speed and the width of the weld pool increases for low welding speed. Results also show that the effect of electromagnetic force is stronger than that of the buoyancy force. The maximum velocity is few centimeters per second for electromagnetic force and few millimeters per second for buoyancy force. It is consistent with the observations of many researchers including Tsai and Kou (1990) and Woods and Milner (1971).

The weld pool development at various heat inputs for a given welding current has also been studied. Figures 3.24 to 3.29 present the temperature and velocity distributions during GTA welding of stainless steel with different heat inputs (different arcing times, 0.75 s, 1.0 s, 1.25 s, 1.5 s, 1.75 s and 2.0 s) at 80 A, 100 A and 120 A. Figure 3.24 shows the temperature fields for weld pool development at 80 A and 12 V. The weld bead geometry is determined by the contour for 1523 K (solidus temperature). The weld bead shape, size and aspect ratio are changing from time to time. The weld bead depth increases from 0.4 mm at 0.75 s, to 1.4 mm at 2.0 s. The width correspondingly increases from 1.0 mm to 1.8 mm. The aspect ratio (depth/width) is also found to be increasing. This is due to the effects of electromagnetic force and surface tension. The flow is inward and hence, the aspect ratio increases from 0.285 to 0.555. The peak temperature goes up to 2238.6 K. The corresponding velocity fields are shown in Figure 3.25. The fluid flow occurs in the inward direction for all the cases. The maximum velocity goes up to 0.121 m/s.
Figure 3.24 Weld pool development at 80 A and 12 V (Heating-temperature field)
Figure 3.25  Weld pool development at 80 A and 12 V (Heating-velocity field)
Similarly, Figures 3.26 and 3.28 show the temperature fields at 100 A and 120 A respectively. The aspect ratio increases from 0.61 to 0.77 for 100 A, 13.5 V and from 0.585 to 0.92 for 120 A, 140 V. The weld pool size increases with decrease in welding speed and the shape becomes a rounded one in the last stages. The corresponding velocity fields are shown in Figures 3.27 and 3.29. A double loop circulation is seen in the weld pool for both the cases of 100 A and 120 A.
Figure 3.26 Weld pool development at 100 A and 13.5 V (Heating-temperature field)
Figure 3.27 Weld pool development at 100 A and 13.5 V (Heating-velocity field)
Figure 3.28 Weld pool development at 120 A and 14 V (Heating-temperature field)
Figure 3.29  Weld pool development at 120 A and 14 V (Heating- velocity field)
3.4 SUMMARY

An equivalent 2-D heat and fluid flow model has been developed for linear GTA welding process, considering the spatial averaging of heat flux and current density in the z-direction (direction of welding). First order upwinding has been used to get convergence. The code validation is carried out at different levels. The benchmark problems such as lid driven cavity flow, 2-D channel flow and natural convection in square cavity have been solved. Grid independence test and time step independence test have been successfully carried out. The results obtained from the code match very well with the results given in the literature. A detailed parametric study has been made for the GTAW process. The results have been compared with the results given in literature. The maximum velocities in the weld pool for different forces have been reported to be in the order of few millimeters per second for buoyancy, few centimeters per second for electromagnetic induction and few hundred millimeters per second for surface tension. Moreover, the development of weld pool at constant welding current has been simulated for different heat inputs, considering buoyancy, electromagnetic and surface tension forces.