CHAPTER 4

CONJUGATE HEAT TRANSFER STUDY: IMPINGING SLOT JET FLOWS

4.1 INTRODUCTION

Most of the applications involve the base plate thickness. The two-dimensionality in the slab side heat transfer influences the heat transfer in the fluid region. In the present work, the conjugate heat transfer in connection with nozzle to plate distance, Reynolds number, Prandtl number, solid slab thickness, and solid to fluid thermal conductivity ratio are investigated. The conduction in the slab and convection heat transfer in the fluid are computed for various parameters.

4.2 MATHEMATICAL FORMULATION

An impinging slot jet configuration sketch together with the definition of the relevant coordinates is shown in Figure 4.1. The jet comes from the top with uniform velocity and impinges on hot wall located at the bottom. A Cartesian coordinate system centered at the stagnation point on impingement slab is used. The solid slab bottom is kept at constant temperature and sidewalls of the slab are insulated. Only the right half of the domain is considered as the problem configuration is symmetrical with the axis of the rectangular slot.
The governing equations for two-dimensional laminar incompressible impinging slot jet flow using conservation of mass, momentum and energy can be written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4.1}
\]

\[
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{4.2}
\]

\[
u \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial \eta} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \tag{4.3}
\]

\[
u \frac{\partial T_f}{\partial x} + \frac{\partial T_f}{\partial y} = \alpha \left( \frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right) \tag{4.4}
\]

where \(x\) and \(y\) are the distances measured along the horizontal and vertical directions, respectively; \(u\) and \(v\) are the velocity components in the \(x\)- and \(y\)- directions, respectively; \(T_f\) denotes the temperature; \(\nu\) and \(\alpha\) are kinematic viscosity and thermal diffusivity, respectively; \(p\) is the pressure.
and $\rho$ is the density; $T_h$ and $T_c$ are the temperatures at hot bottom wall of the slab and inlet temperature of the jet, respectively.

The heat transfer in the solid plate is described by the steady-state heat conduction equation,

$$\frac{\partial^2 T_s}{\partial X^2} + \frac{\partial^2 T_s}{\partial Y^2} = 0$$

(4.5)

where $T_s$ denotes the temperature of the solid slab.

The scaling parameters are chosen as the uniform velocity at the jet exit, $V_j$, the hydraulic diameter of the jet, $D_h$, and $\rho V_j^2$ as the characteristic pressure. Using the following non-dimensional variables:

$$X = \frac{x}{D_h}, \quad U = \frac{u}{V_j}, \quad \theta_j = \frac{T_j - T_c}{T_h - T_c}, \quad Re = \frac{V_j D_h}{\nu},$$

$$Y = \frac{y}{D_h}, \quad V = \frac{v}{V_j}, \quad \theta_s = \frac{T_s - T_c}{T_h - T_c}, \quad Pr = \frac{v}{\alpha}, \quad P = \frac{p}{\rho V_j^2}$$

(4.6)

the governing equations (4.1) - (4.5) reduce to non-dimensional form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad \text{(4.7)}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad \text{(4.8)}$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) \quad \text{(4.9)}$$

$$U \frac{\partial \theta_j}{\partial X} + V \frac{\partial \theta_j}{\partial Y} = \frac{1}{Re Pr} \left( \frac{\partial^2 \theta_j}{\partial X^2} + \frac{\partial^2 \theta_j}{\partial Y^2} \right) \quad \text{(4.10)}$$
\[
\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = 0
\]  

(4.11)

Here \( X \) and \( Y \) are dimensionless coordinates along horizontal and vertical directions, respectively; \( U \) and \( V \) are, dimensionless velocity components in the \( X \)- and \( Y \)- directions, respectively; \( \theta \) is the dimensionless temperature; \( P \) is the dimensionless pressure; \( Re \) and \( Pr \) are Reynolds and Prandtl numbers, respectively.

The boundary conditions at the jet exit are specified as \( U = 0 \) and \( V_j = -1 \), and \( \theta_j = 0 \). The rest of the top wall is adiabatic, \( U = V = 0 \), and \( \partial \theta_j / \partial Y = 0 \). The bottom wall conditions are specified as \( U = V = 0 \). The left wall is symmetry plane, \( U = 0 \), \( \partial V / \partial X = 0 \), and \( \partial \theta_j / \partial X = 0 \). Constant pressure conditions are applied at the right exit boundary, where other variables are extrapolated from inside.

The conjugate boundary conditions along the interface \(( Y = 0, 0 \leq X \leq L \) ) are

\[
k_f \left( \frac{\partial \theta_j}{\partial Y} \right)_{y=0} = k_s \left( \frac{\partial \theta_s}{\partial Y} \right)_{y=0}, \quad \theta_j = \theta_s
\]  

(4.12)

where \( k_f \) and \( k_s \) denote the thermal conductivities of the fluid and of the solid slab, respectively. The solid slab bottom is kept at constant temperature \(( \theta_s = 1 \) \) and sidewalls of the slab are insulated \(( \partial \theta_j / \partial X = 0 \).

The local Nusselt number is obtained at the bottom plate using the following equation.

\[
Nu(X) = -\left. \frac{\partial \theta_j}{\partial Y} \right|_{y=0}
\]  

(4.13)
The average Nusselt number is obtained by numerical integration using trapezoidal rule.

\[ Nu_{avg} = \frac{1}{L} \int_{0}^{L} Nu(X) dX \] (4.14)

4.3 NUMERICAL PROCEDURE

The governing equations are resolved by the control-volume based finite-difference method with SIMPLE algorithm as explained in section 3.3.1. The energy equation in the fluid and the heat conduction equation in the solid are solved simultaneously. The validation of the present computer code is verified for non-conjugate case and given in section 3.3.2.

In \( X \) direction, up to jet width, the grids are arranged uniformly after that they are clustered. In \( Y \) direction, grids are clustered near top and bottom walls. Typical grids are shown in Figure 4.2 for fluid region and solid slab.

![Figure 4.2 Typical grid arrangement: (a) Fluid region (b) Solid slab](image)

Systematic grid refinement study has been carried out with \( 37 \times 15, 51 \times 21, 65 \times 25, 81 \times 31, \) and \( 95 \times 35 \), (Table 3.3) for non-conjugate heat transfer study. It is observed that grid refinement level 4 \((81 \times 31)\) produced almost grid independent results and thus grid refinement level 4 is used for the entire computations in the fluid region. Since the interface is common for the solid as well as fluid region, in \( X \) direction the same clustered grids are
used for the solid slab. However, in the Y direction grids are clustered near top and bottom walls of the slab (Kanna and Das 2005b). Grid points 15, 21, and 25 are tested for $Re = 100, 500$ and $S = 2$. It is seen from Table 4.1 that the variation in average Nusselt number is very less and the grid point 21 is selected for all the computations.

Table 4.1 Grid independence study: Average Nusselt number for $S = 2$

<table>
<thead>
<tr>
<th>Grids in Y</th>
<th>$Re = 100$</th>
<th>$Re = 500$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.6775</td>
<td>1.2550</td>
</tr>
<tr>
<td>21</td>
<td>0.6780</td>
<td>1.2548</td>
</tr>
<tr>
<td>25</td>
<td>0.6780</td>
<td>1.2548</td>
</tr>
</tbody>
</table>

4.4 RESULTS AND DISCUSSION

The jet width, $W$, is taken as 0.5 for the computations, so that the hydraulic diameter, $D_h$, becomes 1. The dimensionless domain length and height, $L$ and $H$, are taken as 15 and 1.0 - 2.5 respectively. The computations are done for different jet exit Reynolds numbers, which are defined in terms of the hydraulic diameter and uniform exit velocity of the jet. The conjugate heat transfer in connection with five parameters, nozzle to plate distance ($H = 1.0 - 2.5$), Reynolds number ($Re = 100 - 500$), Prandtl number ($Pr = 0.01 - 100$), solid slab thickness ($S = 0.5 - 2.0$), and solid to fluid thermal conductivity ratio ($k_r = 1 - 100$) are investigated.

4.4.1 Effect of Nozzle to Plate Distance

Figures 4.3 - 4.6 show the effect of nozzle to plate distance on heat transfer in the fluid region and solid slab, Conjugate interface temperature and local Nusselt number for $Re = 400$, $Pr = 1.0$, $k_r = 10.0$, and $S = 0.5$. 
Figure 4.3(a)-(d) show the effect of $H$ on heat transfer in fluid region. The change in the isotherms is noticed as the length of impingement and wall jet regions increase with increase in $H$ for same $Re$. The length of thermal boundary layer increases when $H$ increases. The secondary vortex is formed at higher $Re$ when $H$ is increased. The isotherms are uncompressed near bottom impingement wall in the secondary vortex region (Figure 4.3(c,d)). Figure 4.4(a)-(d) show the effect of $H$ in the solid slab heat transfer. The change in the isotherms are noticed as the length of thermal boundary layer increases when $H$ increases. The isotherm with $\theta = 0.9$ moves downstream when $H$ is increased.
The effect of nozzle to plate distance on interface temperature is shown in Figure 4.5. It is noticed that the interface temperature is not affected in the impingement region. The interface temperature is affected in the wall jet region as the length of wall jet region increases with increase in $H$. The effect of nozzle to plate distance on $Nu$ is shown in Figure 4.6. It is noticed that the local Nusselt number is not affected in the impingement region. The local Nusselt number is affected in the wall jet region as the length of wall jet region increases with increase in $H$. The minimum local Nusselt number occurs at the end of the wall jet region and its value decreases with increase in $H$. There is a rise in $Nu$ after wall jet region due to the formation of the secondary vortex. Further downstream $Nu$ decreases gradually.

![Figure 4.5](image)

**Figure 4.5** Conjugate interface temperature ($\theta_i$): Effect of $H$ ($Re = 400$, $Pr = 1.0$, $k_r = 10.0$, and $S = 0.5$)
Figure 4.6  Local Nusselt number ($Nu$): Effect of $H$ ($Re = 400$, $Pr = 1.0$, $k_r = 10.0$, and $S = 0.5$)

It is noticed from Table 4.2, the average Nusselt number is increased when $H$ is decreased. Though the stagnation Nusselt number is not affected by $H$ (Figure 4.6), local Nusselt number distribution in the wall jet and further downstream regions affects the average Nusselt number. The nozzle to plate distance must be small for higher overall heat removal rate. The average Nusselt number is decreased by 12 % with non-conjugate case for the considered range of $H$ (Table 4.2). The reduction in fluid region heat transfer with non-conjugate case is same for any particular nozzle to plate distance within the considered range.

Table 4.2 Average Nusselt number for various $H$

<table>
<thead>
<tr>
<th>$H$</th>
<th>Conjugate case</th>
<th>Non-conjugate case</th>
<th>% reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>2.1252</td>
<td>2.4264</td>
<td>12.4</td>
</tr>
<tr>
<td>2.0</td>
<td>2.1423</td>
<td>2.4495</td>
<td>12.5</td>
</tr>
<tr>
<td>1.5</td>
<td>2.1994</td>
<td>2.5110</td>
<td>12.4</td>
</tr>
<tr>
<td>1.0</td>
<td>2.4230</td>
<td>2.7513</td>
<td>11.9</td>
</tr>
</tbody>
</table>
4.4.2 Effect of Reynolds Number

Figures 4.7 - 4.10 show the effect of Reynolds number on heat transfer in the fluid region and solid slab, Conjugate interface temperature and local Nusselt number for \( H = 2.5 \), \( Pr = 1 \), \( k_r = 5 \) and \( S = 1 \). Figure 4.7(a)-(e) show the effect of \( Re \) on heat transfer in fluid region.

![Isotherm contour in the fluid region: Effect of Re](image)

**Figure 4.7 Isotherm contour in the fluid region: Effect of Re \( (H = 2.5, Pr = 1, k_r = 5 \text{ and } S = 1) \) (a) \( Re = 100 \); (b) \( Re = 200 \); (c) \( Re = 300 \); (d) \( Re = 400 \); (e) \( Re = 500 \)**

The isotherm changes due to the changes in size and position of the primary and secondary vortices with \( Re \). The length of wall jet region increases as Reynolds number increases which in turn increase the length of thermal boundary layer. It is noticed that the isotherms are compressed against the bottom impingement wall in the impingement and wall jet regions as Reynolds number increases. The secondary vortex is formed at high Reynolds number and the isotherms are uncompressed in the secondary
vortex region (Figure 4.7(c)-(e)). The clockwise circulation causes more heat to be removed in the secondary vortex region. The thermal boundary layer thickness decreases in the impingement and wall jet regions as Reynolds number increases and hence the heat transfer is increased in that region. Figure 4.8(a)-(e) show the effect of $Re$ in the solid slab heat transfer. The flow property $Re$ affects the heat transfer in the solid slab. When $Re$ increases, the heat removal rate increases due to increase in convection mode of heat transfer. The magnitudes of isotherm are reduced at any point in the solid slab as Reynolds number increases. This indicates the increase in heat transfer in the solid slab.

![Isotherm contour in the solid slab](image)

**Figure 4.8** Isotherm contour in the solid slab: Effect of $Re$ ($H = 2.5$, $Pr = 1, k_r = 5$ and $S = 1$) (a) $Re = 100$; (b) $Re = 200$; (c) $Re = 300$; (d) $Re = 400$; (e) $Re = 500$

It is noticed that the temperature gradient is high in the impingement and wall jet regions. Further downstream the temperature gradient is less due to low heat removal rate. It is also observed that the minimum temperature value occurs at the stagnation point (Left-top of the slab). Figure 4.9 shows the variation of interface temperature ($\theta_i$) with $Re$. The interface temperature is minimum at the stagnation point and increases monotonically in the impingement and the wall jet regions. Further downstream it increases gradually at low $Re$. It is noticed that a drop in
interface temperature in case of high $Re$. This is due to the formation of secondary vortex, which increases the heat transfer in that region.

When Reynolds number is increased, the interface temperature is decreased. This means that, when Reynolds number is increased, the convection heat transfer is increased according to Newton’s law of cooling. Figure 4.10 shows the variation of local Nusselt number with $Re$. The local Nusselt number increases with increase in $Re$. The peak value of $Nu$ occurs at stagnation point. $Nu$ decreases monotonically in the impingement region and further downstream decreases gradually at low $Re$. $Nu$ gets a second peak value in the secondary vortex region due to contribution of secondary vortex in heat transfer at high $Re$.

**Figure 4.9  Conjugate interface temperature ($\theta_i$): Effect of $Re$ ($H = 2.5$, $Pr = 1$, $k_r = 5$ and $S = 1$)**
It is noticed from Table 4.3, the average Nusselt number is increased when $Re$ is increased. The average Nusselt number is decreased by 16.6% with non-conjugate case when $Re = 100$. When $Re$ is increased to 500, average Nusselt number is decreased by 39.2% with non-conjugate case. When $Re$ is increased the effect of conjugate parameters on average Nusselt number gets increased with decreasing rate.

Table 4.3 Average Nusselt number for various $Re$

<table>
<thead>
<tr>
<th>$Re$</th>
<th>Conjugate case</th>
<th>Non-conjugate case</th>
<th>% reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.8004</td>
<td>0.9590</td>
<td>16.6</td>
</tr>
<tr>
<td>200</td>
<td>1.0975</td>
<td>1.4928</td>
<td>26.5</td>
</tr>
<tr>
<td>300</td>
<td>1.3330</td>
<td>1.9690</td>
<td>32.3</td>
</tr>
<tr>
<td>400</td>
<td>1.5482</td>
<td>2.4264</td>
<td>36.2</td>
</tr>
<tr>
<td>500</td>
<td>1.7404</td>
<td>2.8637</td>
<td>39.2</td>
</tr>
</tbody>
</table>
4.4.3 Effect of Prandtl Number

Figures 4.11 - 4.14 show the effect of Prandtl number on heat transfer in the fluid region and solid slab, Conjugate interface temperature and local Nusselt number for $H = 2.5$, $Re = 100$, $k_r = 7.5$, and $S = 1.5$. Figure 4.11(a)-(e) show the effect of $Pr$ on heat transfer in fluid region.

![Isotherm contour in the fluid region: Effect of Pr (H = 2.5, Re = 100, kr = 7.5, and S = 1.5) (a) Pr = 0.01; (b) Pr = 0.1; (c) Pr = 1.0; (d) Pr = 10.0; (e) Pr = 100.0](image)

Figure 4.11 Isotherm contour in the fluid region: Effect of $Pr$ ($H = 2.5$, $Re = 100$, $k_r = 7.5$, and $S = 1.5$) (a) $Pr = 0.01$; (b) $Pr = 0.1$; (c) $Pr = 1.0$; (d) $Pr = 10.0$; (e) $Pr = 100.0$

Figure 4.11(a) shows the isotherm contour when $Pr = 0.01$. The heat transfer is primarily due to conduction. The isotherms are dense near the jet inlet which may indicate lower heat transfer from bottom impingement wall to fluid. The isotherms are smooth curves and self similar. All isotherms reach the top confinement wall. As $Pr$ increases from 0.01 to 1, the isotherms are concentrated towards bottom impingement wall (Figure 4.11(c)). The isotherms with $\theta \geq 0.4$ are not reaching the top confinement wall. The heat
transfer is primarily due to convection. The thermal boundary layer thickness is small at the stagnation point and hence the heat transfer is maximum at that point. When the \( Pr = 100 \), the isotherms are highly dense near the bottom impingement wall which indicate a lower heat transfer at the top confinement wall and inlet of the jet (Figure 4.11(e)). The thermal boundary layer thickness is reduced and heat transfer is increased.

Figure 4.12(a)-(e) show the effect of \( Pr \) in the solid slab heat transfer for \( H = 2.5, \ Re = 100, k_r = 7.5, \) and \( S = 1.5 \). It is observed that the magnitudes of isotherm are reduced at any point in the solid slab as \( Pr \) increases. This indicates the increase in heat transfer in the solid slab.

![Figure 4.12 Isotherm contour in the solid slab: Effect of \( Pr \) (\( H = 2.5, Re = 100, k_r = 7.5, \) and \( S = 1.5 \))](image)

\( (a) \ Pr = 0.01; \ (b) \ Pr = 0.1; \ (c) \ Pr = 1.0; \ (d) \ Pr = 10.0; \ (e) \ Pr = 100.0 \)

The effect of \( Pr \) on interface temperature is shown in Figure 4.13 for \( H = 2.5, Re = 100, k_r = 7.5, \) and \( S = 1.5 \). The heat transfer rate increases when \( Pr \) is increased. And hence the interface temperature decreases with increase in \( Pr \) at any particular location. The interface temperature reaches the higher value at low \( Pr \). The effect of secondary vortex is increased with
increase in $Pr$. The drop in interface temperature is noted at $Pr = 100$ after wall jet region. This drop in interface temperature is due to the effect of secondary vortex.

![Figure 4.13 Conjugate interface temperature ($\theta_i$): Effect of $Pr$ ($H = 2.5$, $Re = 100$, $k_r = 7.5$, and $S = 1.5$)](image)

The effect of $Pr$ on $Nu$ is shown in Figure 4.14 for $H = 2.5$, $Re = 100$, $k_r = 7.5$, and $S = 1.5$. The heat transfer rate increases when $Pr$ is increased. And hence the local Nusselt number increases with increase in $Pr$ at any particular location. An increase in $Nu$ is noted at $Pr = 100$ after wall jet region. This is due to the effect of secondary vortex. And its effect is decreased with decreases in $Pr$. 
Figure 4.14  Local Nusselt number (\(Nu\)): Effect of \(Pr\) (\(H = 2.5, Re = 100, k_r = 7.5\), and \(S = 1.5\))

It is noticed from Table 4.4, the average Nusselt number is increased when \(Pr\) is increased. The conjugate parameters have no effect on average Nusselt number up to \(Pr = 0.1\). When \(Pr\) is 1.0, average Nusselt number is decreased by 16.5 % with non-conjugate case. When \(Pr\) is increased to 100, average Nusselt number is decreased by 55.8 % with non-conjugate case. When \(Pr\) is increased the effect of conjugate parameters on heat transfer gets increased.

<table>
<thead>
<tr>
<th>(Pr)</th>
<th>Conjugate case</th>
<th>Non-conjugate case</th>
<th>% reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0392</td>
<td>0.0393</td>
<td>0.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1560</td>
<td>0.1573</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8012</td>
<td>0.9590</td>
<td>16.5</td>
</tr>
<tr>
<td>10.0</td>
<td>1.5831</td>
<td>2.4911</td>
<td>36.5</td>
</tr>
<tr>
<td>100.0</td>
<td>2.5001</td>
<td>5.6504</td>
<td>55.8</td>
</tr>
</tbody>
</table>
4.4.4 Effect of Thermal Conductivity Ratio

Figures 4.15 - 4.18 show the effect of thermal conductivity ratio on heat transfer in the fluid region and solid slab, Conjugate interface temperature and local Nusselt number for $H = 1.5$, $Re = 300$, $Pr = 1.0$, and $S = 2.0$. Figure 4.15(a)-(f) show the effect of $k_r$ on heat transfer in fluid region.

Figure 4.15 Isotherm contour in the fluid region: Effect of $k_r$ ($H = 1.5$, $Re = 300$, $Pr = 1.0$, and $S = 2.0$) (a) $k_r = 1.0$; (b) $k_r = 2.5$; (c) $k_r = 5.0$; (d) $k_r = 7.5$; (e) $k_r = 10.0$; (f) $k_r = 100.0$

The isotherms look qualitatively similar as $k_r$ increases. The magnitudes of isotherm are increased in the fluid region with increase in $k_r$. This indicates the increase in heat transfer. Figure 4.16(a)-(f) show the effect of $k_r$ in the solid slab heat transfer. Although the magnitudes of isotherm and thermal gradient across the slab are reduced, the heat transfer increases in the
solid slab with increase in $k_r$. When $k_r$ is increased, the thermal conductivity of the slab is increased for same fluid, which causes the increase in heat transfer in the solid slab.

Figure 4.16  Isotherm contour in the solid slab: Effect of $k_r$ ($H = 1.5$, $Re = 300$, $Pr = 1.0$, and $S = 2.0$) (a) $k_r = 1.0$; (b) $k_r = 2.5$; (c) $k_r = 5.0$; (d) $k_r = 7.5$; (e) $k_r = 10.0$; (f) $k_r = 100.0$

Figure 4.17 depicts the variation of interface temperature with $k_r$. The interface temperature is increased when $k_r$ is increased. Figure 4.18 depicts the variation of $Nu$ with $k_r$. It is observed that $Nu$ increases when $k_r$ is increased as heat transfer increases. However, it is noticed that the increase in heat transfer is sensitive in the impingement and wall jet regions. Further downstream the variation of $Nu$ is less at high $k_r$. 
Figure 4.17  Conjugate interface temperature ($\theta_i$): Effect of $k_r$ ($H = 1.5$, $Re = 300$, $Pr = 1.0$, and $S = 2.0$)

Figure 4.18  Local Nusselt number ($Nu$): Effect of $k_r$ ($H = 1.5$, $Re = 300$, $Pr = 1.0$, and $S = 2.0$)
It is noticed from Table 4.5, the average Nusselt number is increased when \( k_r \) is increased. The average Nusselt number is decreased by 81% with non-conjugate case when \( k_r = 1.0 \). When \( k_r \) is increased to 100, average Nusselt number is decreased by 3.6% with non-conjugate case. It can be noticed that the effect of \( k_r \) on heat transfer is decreased when \( k_r \) is increased. When \( k_r \) is increased the average Nusselt number moves close to non-conjugate case.

**Table 4.5 Average Nusselt number for various \( k_r \)**

<table>
<thead>
<tr>
<th>( k_r )</th>
<th>Conjugate case</th>
<th>Non-conjugate case</th>
<th>% reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.3997</td>
<td>2.1169</td>
<td>81.1</td>
</tr>
<tr>
<td>2.5</td>
<td>0.7694</td>
<td>2.1169</td>
<td>63.7</td>
</tr>
<tr>
<td>5.0</td>
<td>1.1168</td>
<td>2.1169</td>
<td>47.2</td>
</tr>
<tr>
<td>7.5</td>
<td>1.3186</td>
<td>2.1169</td>
<td>37.7</td>
</tr>
<tr>
<td>10.0</td>
<td>1.4520</td>
<td>2.1169</td>
<td>31.4</td>
</tr>
<tr>
<td>100.0</td>
<td>2.0404</td>
<td>2.1169</td>
<td>3.6</td>
</tr>
</tbody>
</table>

### 4.4.5 Effect of Solid Slab Thickness

Figures 4.19 - 4.22 show the effect of solid slab thickness on heat transfer in the fluid region and solid slab, Conjugate interface temperature and local Nusselt number for \( H = 2.0 \), \( Re = 200 \), \( Pr = 1 \), and \( k_r = 10.0 \). Figure 4.19(a)-(d) show the effect of \( S \) on heat transfer in fluid region. The change in the isotherms is noticed in the top of the fluid region where the temperature gradient is small. Although the isotherms look qualitatively similar as \( S \) increases from 0.5 to 2, the magnitudes of isotherm are reduced in the fluid region due to reduction in heat transfer in the solid slab. Figure 4.20(a)-(d) show the effect of \( S \) in the solid slab heat transfer.
When solid slab thickness increases, the heat transfer rate in the solid slab is decreased. The temperature gradient decreases with increases in $S$ in the impingement and wall jet regions. The solid slab thickness acts as a resistance to the heat transfer. Hence increase in $S$ decreases the heat transfer.

Figure 4.19  Isotherm contour in the fluid region: Effect of $S$ ($H = 2.0$, $Re = 200$, $Pr = 1$, and $k_r = 10.0$) (a) $S = 0.5$; (b) $S = 1.0$; (c) $S = 1.5$; (d) $S = 2.0$

Figure 4.20  Isotherm contour in the solid slab: Effect of $S$ ($H = 2.0$, $Re = 200$, $Pr = 1$, and $k_r = 10.0$) (a) $S = 0.5$; (b) $S = 1.0$; (c) $S = 1.5$; (d) $S = 2.0$

The effect of solid slab thickness on interface temperature is shown in Figure 4.21. The interface temperature is reduced when solid slab thickness is increased due to increase in thermal resistance.
Figure 4.21 Conjugate interface temperature ($\theta_i$): Effect of $S$ ($H = 2.0$, $Re = 200$, $Pr = 1$, and $k_r = 10.0$)

The effect of solid slab thickness on $Nu$ is shown in Figure 4.22. It is noticed that the local Nusselt number is affected only in the impingement and wall jet regions though the interface temperature is affected in the entire length (Figure 4.21). The local Nusselt number is reduced when solid slab thickness is increased due to increase in thermal resistance in the impingement and wall jet regions. Further downstream it becomes invariant with solid slab thickness though the interface temperature is reduced. The effect of solid slab thickness on $Nu$ is maximum in the stagnation point.
Figure 4.22  Local Nusselt number (Nu): Effect of $S$ ($H = 2.0$, $Re = 200$, $Pr = 1$, and $k_r = 10.0$)

It is noticed from Table 4.6, the average Nusselt number is decreased when $S$ is increased. The average Nusselt number is decreased by 8 % with non-conjugate case when $S = 0.5$. When $S$ is increased to 2, average Nusselt number is decreased by 26 % with non-conjugate case. When $S$ is decreased the average Nusselt number goes near non-conjugate case. The heat transfer in the solid slab increases with increase in $k_r$ and decrease in $S$. This affects the heat transfer in the fluid region. The influence of $Re$ and $Pr$ is increased with increase in solid slab heat transfer.

<table>
<thead>
<tr>
<th>$S$</th>
<th>Conjugate case</th>
<th>Non-conjugate case</th>
<th>% reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.4383</td>
<td>1.5606</td>
<td>7.8</td>
</tr>
<tr>
<td>1.0</td>
<td>1.3251</td>
<td>1.5606</td>
<td>15.1</td>
</tr>
<tr>
<td>1.5</td>
<td>1.2335</td>
<td>1.5606</td>
<td>20.1</td>
</tr>
<tr>
<td>2.0</td>
<td>1.1568</td>
<td>1.5606</td>
<td>25.9</td>
</tr>
</tbody>
</table>
4.5 CONCLUSIONS

Conjugate heat transfer of a two-dimensional, incompressible impinging slot jet is solved by the control-volume based finite-difference method with SIMPLE algorithm. Effects of nozzle to plate distance \((H)\), Reynolds number \((Re)\), Prandtl number \((Pr)\), solid slab thickness \((S)\), and solid to fluid thermal conductivity ratio \((k_r)\) on thermal characteristics are studied in details.

- The average Nusselt number is increased when nozzle to plate distance is decreased. It is noticed that the heat transfer is not affected in the impingement region when nozzle to plate distance is changed.

- The local Nusselt number shows a strong dependence on the Reynolds number. As Reynolds increases, the hydrodynamic boundary layer thickness decreases and local Nusselt number increases.

- The local Nusselt number also shows a strong dependence on the Prandtl number. As Prandtl number increases, the thermal boundary layer thickness decreases and Nusselt number increases.

- The interface temperature is minimum at stagnation point. The conjugate interface temperature at stagnation point is decreased with increase in \(Re\), \(Pr\) and \(S\). It is increased when \(k_r\) is increased and remains unchanged when \(H\) is varied.

- The heat transfer increases with increase in \(Re\), \(Pr\) and \(k_r\). It decreases with increase in \(H\) and \(S\).
• The effect of $k_r$ on heat transfer is decreased when $k_r$ is increased.

• The prediction of heat transfer for non-conjugate case is high when compared to conjugate cases in the impingement and wall jet regions.