CHAPTER 4

TORQUE AND STRESS BALANCE OF A MULTILAYERED STRAND

4.1 INTRODUCTION

Oceanography is one such field, where there are data gathering situations wherein it is desirable to tow or suspend instrumentation at greater depths, often measured in hundreds or thousands of meters with minimum rotational or angular displacement of instrument stations. In such applications where the cables are axially loaded in tension, there will be a torsional coupling that causes twisting of the cable. To prevent any cable twist, when found undesirable, external torque needs to be applied. Otherwise, suitable composition of layers should be designed, which inherently results in zero axial twist. This work addresses one such phenomenon making use of previously mentioned basic theoretical model and establishing mathematical relationships to find a suitable geometry between the adjoining layers of a multilayered stranded cable that yields a zero axial twist or a non rotating effect.

Long cables which are restrained from rotating may develop a sufficiently large induced torque that on slight relaxations of cable tension (momentary slack cable) can result in hockling (looping) due to instability. This when analysed with a stress balance situation in the adjoining layers, will yield a practical design condition of the cable. Knapp (1981) had considered the basic work in this direction by considering the effect of axial force in the
wire only. This work was further improved by Nabi et al. (1998) taking into account all the wire forces and couples, the effect of wire stretch on wire rotation as used earlier by Sathikh et al. (1996) and biaxial stress for strength consideration together with Possion’s effect on core and wire. The present work extends the findings of Nabi et al (1998) to consider the torque balance and the effects of the two other theories of failure namely maximum shear stress theory and distortion energy theory for two and three layered cable. Comparisons have been made with and without the effects of Poisson’s ratio.

4.2 TORQUE BALANCE

A multilayered strand with contra helically wound adjacent layer is shown in Figure 4.1. The helical armoring wires, which render the cable flexible, induce a torque as the helical wires try to “unwind” during axial loading.

![Figure 4.1 Multilayered strand (Two layered)](image)

The induced torque can be undesirable from several points of view. The cable rotation may loosen some wires and tighten others depending on
the direction of lay. This, of course, means that some wire layers will be stressed at higher levels than others. Thus, the efficiency of the cable is reduced and the breaking strength may be appreciably lowered. Long cables which are restrained from rotating may develop a sufficiently large induced torque that on slight relaxations of cable tension (momentary slack cable) can result in hockling (looping) due to instability. Upon reapplication of the cable load, the hockle radius may be decreased sufficiently to fail the armoring wires due to the large bending stresses. Furthermore, there is numerous cable applications which require torque-free performance, such as one in long oceanographic cables used for towed bodies.

Knapp (1981) in his work considered only the wire axial force and the effects of bending and wire twist were not considered together. Nabi et al (1998) considered the helical wires with all the wire forces and couples together with effect of wire stretch on wire rotation and Poisson’s effect on core and wire. The normal stress theory was used to predict the stress balance for a two layer cable. The present work extends the findings of Nabi et al (1998) in terms of torque balance considering up to three layers. The present thesis addresses the torque balance to consider its effects when Poisson’s ratio is included or omitted.

4.2.1 Conditions for Torque Balance

The condition for torque balance requires that no external torque be developed for a cable pulled in tension and restrained from rotating at both ends. For torque balance \( (\delta \phi / h) = 0 \) and \( M_x = 0 \). This requires also \( M_z = 0 \), which means that equation (3.52) is equal to zero.
4.2.2 Computational Procedure for Torque Balance

4.2.2.1 Two layered cable

Applying the condition for torque balance mentioned in section in 4.2.1 to the equilibrium equation of the strand in equation (3.1), the net torque in the strand must become zero. This demands the torque developed in the individual layers should be equal and opposite in direction. In other words of $M_1$ of layer 1 should be equal to $M_2$ of layer 2, in a two layered cable.

Numerical data pertaining to a two layer, contra helically armored KEVLAR EM cable used as a segment link between a surface support ship and a deep sea unmanned work system has been used. For the two layered cable shown in [Table 4.1], the suitable helix angle for the outer layer ($\alpha_2$) is computed for different helix angles of inner layer ($\alpha_1$). The relationship between $\alpha_1$ and $\alpha_2$ so obtained is shown in Figure 4.2.

<table>
<thead>
<tr>
<th>Table 4.1 Properties of a Two Layered cable</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameters</strong></td>
</tr>
<tr>
<td>Area of the wire, $A_i$ [$mm^2$]</td>
</tr>
<tr>
<td>Radius of the wire, $r_i$ [$mm$]</td>
</tr>
<tr>
<td>Helix radius, $r_i$ [$mm$]</td>
</tr>
<tr>
<td>Helix angle, $\alpha_i$ [deg]</td>
</tr>
<tr>
<td>Young’s modulus, $E_i$ [GPa]</td>
</tr>
<tr>
<td>Yield strength, $Sy_i$ [MPa]</td>
</tr>
<tr>
<td>Number of wires $m$</td>
</tr>
<tr>
<td>Radius of the core $R_c$ ($mm$)</td>
</tr>
</tbody>
</table>
The cable has been analysed for two different cases i.e. without and with Poisson’s effect. The required analytical formulations in these cases are shown in Appendix I.

4.2.2.2 Three layered cable

To demonstrate the effect of torque balance on a three layer cable, and All Alloy Aluminium Conductor (AAAC) used in overhead electrical power transmission has been chosen. Table 4.2 shows the properties of AAAC cable.

For torque balance for a three layered cable, the algebraic sum of torque induced in individual layers should be equal to zero. Mathematically it is given by

\[
(M_e)_{\text{layer 1}} + (M_e)_{\text{layer 2}} + (M_e)_{\text{layer 3}} = 0
\] (4.1)

The net torque induced in a three layered strand is dependent on helix angles, number of wires, diameter and material of the wires. For the three layered cable shown in [Table 4.2], the suitable helix angle for the outer layer \(\alpha_3\) is computed for a given helix angles of inner layer \(\alpha_1\), and middle layer \(\alpha_2\). The cable has been analysed for two different cases i.e. without and with Poisson’s effect. The required analytical formulations in these cases are shown in Appendix I.
Table 4.2 Properties of a Three Layered cable (AAAC)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Inner Layer</th>
<th>Middle Layer</th>
<th>Outer layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of the wire, $r_w$ [mm]</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Helix radius, $r_i$ [mm]</td>
<td>4</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Helix angle, $\alpha_i$ [deg]</td>
<td>82° RH Lay</td>
<td>79° LH Lay</td>
<td>77° RH Lay</td>
</tr>
<tr>
<td>Young’s modulus, $E_i$ [GPa]</td>
<td>58</td>
<td>58</td>
<td>58</td>
</tr>
<tr>
<td>Yield strength, $S_y$ [MPa]</td>
<td>236.25</td>
<td>236.25</td>
<td>236.25</td>
</tr>
<tr>
<td>Number of wires $m$</td>
<td>6</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Radius of the core $R_c$ (mm)</td>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

4.3 STRESS BALANCE

Strength balance implies that all wires between layers have equal strength. The maximum stress in a wire is influenced by the stresses induced in the wire due to its axial elongation, bending deformations and twisting effects, during loading. The wire stress is considered in these possible modes of deformations. The wire axial stress, wire bending stress and wire twisting stress are considered together to determine the maximum stress. In a wire in the layer $i$, the stresses are:

Axial stress $f_i = E_i \epsilon_{wi}$ (4.2)

Bending Stress $f_{bi} = E_i R_i \omega_{wi}$ (4.3)

Torsional stress $f_s = G_i R_i \omega_{si}$ (4.4)
The maximum principal stress is given by

\[ f_{\text{max}} = \frac{(f_i + f_{\text{bi}})}{2} + \left[ \left( \frac{f_i + f_{\text{bi}}}{2} \right)^2 + f_i^2 \right]^{0.5} \] (4.5)

The maximum shear stress is given by

\[ \tau_{\text{max}} = \frac{f_{\text{max}} - f_{\text{min}}}{2} \] (4.6)

where

\[ f_{\text{min}} = \frac{(f_i + f_{\text{bi}})}{2} - \left[ \left( \frac{f_i + f_{\text{bi}}}{2} \right)^2 + f_i^2 \right]^{0.5} \] (4.7)

and the von-Mises stress (bi-axial stress state) is given by

\[ f_{\text{Voni}} = \sqrt{f_{\text{max}}^2 + f_{\text{min}}^2 - (f_{\text{max}} \times f_{\text{min}})} \] (4.8)

For strength balance, Knapp (1981) considered only wire axial force for computing maximum stress in the wire and Nabi et al (1998) used the biaxial stress normal stress theory to predict the stress balance. In the present analysis the wire axial stress, wire bending stress and wire twisting stress (shear stress) are considered together to determine the maximum stress based on maximum principal stress, maximum shear stress and von – Mises stress theory. The present work is in the line with Nabi et al (1998) and extended upto three layers. The significance of Poisson’s effect with regard to achieving stress balance is also addressed.
4.3.1 Conditions for Stress Balance

For equal strength, it is not the wire stress in every layer that needs to be equal. What is required is that, for all layers the maximum induced stress in any wire in that layer should be equal to the yield stress of the material of the wire in that layer. This can be formulated based on the following theories of failure.

By maximum principal stress theory

\[ f_{\text{max}i} = S_{yi} \]  
(4.9)

where \( S_{yi} \) is the allowable design stress of the wire material of layer \( i \).

By maximum shear stress theory (for bi-axial stress state)

\[ f_{\text{max}i} - f_{\text{min}i} = S_{yi} \]  
(4.10)

By von Mises yield criterion

\[ f_{\text{von}i} = S_{yi} \]  
(4.11)

4.3.2 Computational Procedure for Stress Balance

The strength balance condition mentioned in section 4.3.1, the maximum stress in a wire in a layer for a two layered cable, is a function of strand strains and helix angle apart from the geometrical and material properties and is equal to the allowable yield stress of the material of the wire.
Using the equation (4.12) and equation (4.13), the helix angles required for the outer layer $\alpha_2$ is calculated for various helix angles of the inner layer ($\alpha_1$) for the data shown in [Table 4.1]. The required strand axial strain is computed from equation (4.12) and has been made use of in equation (4.13) to calculate $\alpha_2$.

The relationship between $\alpha_1$ and $\alpha_2$ that yields the strength balance for $\gamma$=0, is given in Figure 4.2.

In the case of a three layered cable equation (4.12 & 4.13) are valid for the first two layer while the maximum stress in the third layer is a function of the following.

$$f_{\text{max}1}(\varepsilon, \gamma, \alpha_1, r_1, R_1, E_1, G_1) = S_{y1} \quad (4.12)$$

$$f_{\text{max}2}(\varepsilon, \gamma, \alpha_2, r_2, R_2, E_2, G_2) = S_{y2} \quad (4.13)$$

$$f_{\text{max}3}(\varepsilon, \gamma, \alpha_3, r_3, R_3, E_3, G_3) = S_{y3} \quad (4.14)$$

Equation (4.12-4.14) can be used to find the suitable helix angles required for intermediate and outer layers ($\alpha_2$) and ($\alpha_3$) for a given helix angle ($\alpha_1$) of inner layer.

The cable has been analysed for two different cases i.e. without and with Poisson’s effect. The required analytical formulations in these cases are shown in Appendix I.
4.4 **TORQUE AND STRENGTH BALANCE**

In addition to the strength balance, if torque balance is also required, then the conditions stipulated in section 4.2 and 4.3 should be satisfied simultaneously. Such a condition will yield a perfect non rotating rope with equal strength between the layers. Figure 4.2 presents the results of helix angles required in the outer layer for achieving torque balance and stress balance independently. For various helix angles of the inner layer these studies have been done without and with Poisson’s effects. The point of intersection between the two graphs shown in Figure 4.2 yields the desired results to achieve torque and stress balance.

4.4.1 **Two layered cable**

![Figure 4.2 Torque Balance and Stress Balance of Two layered](image)
4.4.2 Three layered cable

![Graph showing torque balance for different layers](image)

![Graph showing stress balance for different layers](image)

**Figure 4.3** Three layered strand: (a) Torque Balance and (b) Stress Balance
4.5 ANALYSIS

A set of improved equations to determine the conditions of torque and stress balance of a multilayered cable have been derived. This should provide a useful design tool, particularly in a preliminary design situation, capable of revealing the physical characteristics of both torque balance and load-sharing among armoring wires.

The torque balance and strength analysis was carried out on a two and three layered cable respectively considering together with and without the Poisson’s ($\nu$) effect. In both the cases, the effect of bending and twisting of the wire in addition to the wire stretch on its rotations has also been considered. It is hoped that this improved torque and strength balance model should be helpful to designers in the field of cable and wire ropes. Oceanography is one such field, where there are data gathering situations wherein it is desirable to tow or suspend instrumentation at greater depths, often measured in hundreds or thousands of meters with minimum rotational or angular displacement of instrument stations. This invention may further said to reside in similar applications where such members are used.

Figure 4.2 compares the results of torque and stress balance for cases ($\nu=0$) and ($\nu=0.3$) for a two layered cable. In the case of torque balance curve, the effect of Poisson’s ratio is found to be significant upto a helix angle $\alpha_1 = 70^\circ$. However for $\alpha_1 > 70^\circ$ the curve is visibly insignificant. The stress balance curve however registers a continuous difference in the helix angle required for the outer layer, in the case of with and without Poisson effect. This variation is observed to be upto a maximum of 8% for $\alpha_1 = 55^\circ$ and 3.5% for $\alpha_1 = 80^\circ$. 
The required helix angles of adjoining layer of a two layered strand mentioned in (Table 4.1) are obtained as follows to get a non rotating effect with stress balance. For the KEVLAR EM cable, applying the maximum principal stress theory and for \( v=0 \), the helix angles of the inner and outer layer (\( \alpha_1 \) and \( \alpha_2 \)) are obtained as 74° RHL and 77° LHL respectively. Similarly, for the case \( v=0.3 \), the helix angles of the inner and outer layer (\( \alpha_1 \) and \( \alpha_2 \)) are obtained as 76° RHL and 79° LHL respectively. The other theories for strength were also considered but were found to produce results which have insignificant difference from those produced by the maximum principal stress theory. The following paragraphs address these considerations.

In general, for axial loading of the strand, the induced axial stress would dominate the other stresses, i.e. \( f_i >> f_s > f_{bi} \). The equation (4.7) indicates that for any shear stress contribution, the minimum principal stress will result in compression in nature, i.e. mathematically, when \( f_s \neq 0 \)

\[
f_{\text{mini}} < 0 \quad (4.15)
\]

as

\[
\frac{(f_i + f_{bi})}{2} < \left[ \left( \frac{f_i + f_{bi}}{2} \right)^2 + f_s^2 \right]^{0.5} \quad (4.16)
\]

For the geometric parameters considered, the magnitude of shear stress obtained is less than 1.0% of axial stress. This resulted in minimum principal stress nearing zero magnitude although compressive in nature. Therefore in the present analysis,

\[
\tau_{\text{max}} \equiv \frac{f_{\text{ maxi}} - f_{\text{ mini}}}{2} \equiv \frac{f_{\text{ maxi}}}{2} \equiv \frac{S_{yi}}{2} \quad (4.17)
\]
\[ f_{von} \equiv f_{max} \equiv S_{yi} \] (4.18)

For stress balance, the obtained helix angles of outer layer \((\alpha_2)\) for input helix angles of inner layer \((\alpha_1)\) remains same irrespective of the application of the above mentioned theories of strength. Annexure II is enclosed for the support of the same.

Figure 4.3(a) compares the results of torque balance for the cases \(\nu=0\) and \(\nu=0.3\) for a three layered cable. In the case of torque balance a nomogram was established between the helix angles for the three layers. The helix angle \((\alpha_1)\) for the inner layer and that of the middle layer \((\alpha_2)\) was limited to the range of 75° to 85° for computation of torque balance. The suitable helix angle for the outer layer \((\alpha_3)\) was computed for a given helix angles of inner layer \((\alpha_1)\), and middle layer \((\alpha_2)\). The torque balance curve had practically no difference (less than 0.3%) between the absence and inclusion of Poisson’s ratio in the above said range of helix angles. Any further increase in this range would contribute to \(\alpha_3\) becoming near straight wires in their winding for achieving torque balance.

Figure 4.3(b) compares the results of stress balance for cases \(\nu=0\) and \(\nu=0.3\) for a three layered cable. The stress balance between the layers were computed for various helix angle \(\alpha_1\) of the inner layer, ranging from 55° to 80°.

The stress balance curve however registers a continuous difference in helix angle required for the intermediate and outer layer in the case of with and without Poisson’s effect. This variation in the intermediate layer is observed to be 12% between the intermediate layer for \(\alpha_1 = 78°\). Likewise this
variation in the outer layer is observed to be 6% for $\alpha_1 = 78^\circ$ which is the operating range of helix angles used in applications of AAAC.

For stress balance, the obtained helix angles of outer layer ($\alpha_3$) and inner layer ($\alpha_2$) for input helix angles of inner layer ($\alpha_1$) remains the same irrespective of the application of the above mentioned theories of strength.