CHAPTER 7

ECONOMIC SUPPLY OF REACTIVE POWER SUPPORT REQUIRED FOR MEETING REACTIVE POWER DEMAND OF DISCOS AND EQUITABLE ALLOCATION OF REACTIVE POWER CHARGES AMONG DISCOS

7.1 INTRODUCTION

Reactive power support by generators are essential to meet reactive power demand and associated reactive power losses of the DISCOS. ISO should procure reactive power support required to meet the reactive power demand and associated losses of DISCOS at least cost. Each DISCO participant should be charged for reactive power according to their usage. ISO should allocate the reactive power charges among DISCOS in a transparent and equitable manner.

In this chapter, optimal procurement of reactive power support and allocation of charges among DISCOS is proposed. This is solved as a two step process. In the first step, optimal supply of reactive power support (and the associated charges) necessary for both the primary transactions and reactive power demands of the DISCOS is obtained through an optimization process. The reactive power support (associated charges) required for meeting reactive power demand alone and associated reactive power losses of DISCOs is obtained from the difference between the above and the reactive power support (charges) for real power transactions. In the second step the reactive power charges for meeting reactive power demand of DISCOs are allocated to
the DISCOs in a transparent and equitable manner using a sensitivity based mechanism.

The reactive power optimization problem is formulated with an objective of minimizing cost of reactive power support to be procured from willing reactive power support providers satisfying operating constraints. This is a Non Linear Programming Problem. This thesis proposes an algorithm which transforms the Non-Linear Programming problem into a sequence of Linear Programming (LP) problems using linearized models and solves it using Sequential Linear Programming (SLP) technique. A new sensitivity based mechanism for allocating the reactive power charges among DISCOS is proposed.

7.2 LEAST COST SUPPLY OF REACTIVE POWER SUPPORT REQUIRED FOR REAL POWER TRANSACTIONS AND REACTIVE POWER DEMAND OF DISCOS AND ASSOCIATED LOSSES

The objective is to minimize the total cost of reactive power support provided by the providers and to satisfy the operational constraints pertaining to the base case state. In deregulated power system the decision variables comprise generator’s reactive power and dependent variables comprise voltage magnitude of buses and line flows.

The above problem is similar to the one given in Chapter 6, section 6.2.1 except for the following difference. Load flow equations contain only the real power transactions whereas in the present problem the load flow equations contain real power transactions and reactive power demand of DISCOS.
The power flow equation (6.1) can be written by splitting into two groups as follows:

\[
\begin{bmatrix}
F_p(\theta, V) \\
F_q(\theta, V, QG)
\end{bmatrix} = \begin{bmatrix}
P(\theta, V) \\
Q(\theta, V)
\end{bmatrix} - \begin{bmatrix}
PI \\
QI(QG - QD)
\end{bmatrix} = \mathbf{0}
\] (7.1)

The injection vector \( Q_I \) in equation (7.1) consists both reactive power generation vector \( Q_G \) of reactive power support providers and reactive power demand vector \( Q_D \) of DISCOs of dimension ND, where ND is the number of DISCOS.

7.2.1 Problem Statement

The problem of least cost supply of reactive power support for real power transactions and reactive power demand of DISCOS and associated losses is stated as

**Determine** \( Q_G \):

**To minimize:**

The total cost of reactive power support

\[
C_{Q_T} = \sum_{i=1}^{NSP} C_{Q_i}(QG_i)
\] (7.2)

**Subject to**

Power flow constraints

\[
F(\theta, V, QG) = \mathbf{0}
\] (7.3)
Bus voltage constraints
\[ V^L \leq V \leq V^U \]  
(7.4)

MVA line flow constraints
\[ f(\theta, V) \leq P^R \]  
(7.5)

Limits on generator reactive power (decision vector)
\[ Q^L \leq Q_G \leq Q^U \]  
(7.6)

The OPF problem consists of minimizing objective function (7.2) subject to constraints given by equations (7.3) to (7.6).

### 7.2.2 Development of SLP Model

The Non Linear Programming problem stated in section 7.2.1 is solved using Sequential Linear Programming (SLP) approach (Sadasivam et al 1990). The procedure involved in the development of SLP model is similar to that explained in Chapter 6. The only difference is the operating state around which the model is linearized.

### 7.2.3 Linearized OPF Model

The OPF problem given in equations (7.2) to (7.6) is linearized around an operating state to obtain the following LP model.

**Objective function:**

\[ \text{To Minimize: } \Delta C{Q_T} \]  
(7.7)
Subject to

Bus voltage constraints

\[ \Delta V^L \leq [D_Q] \Delta QG \leq \Delta V^U \]  \hspace{1cm} (7.8)

MVA line flow constraints

\[ \Delta f = [K_G] \Delta QG \leq \Delta f^R \]  \hspace{1cm} (7.9)

Operating limits on decision vector

\[ \Delta QG^L \leq \Delta QG \leq \Delta QG^U \]  \hspace{1cm} (7.10)

7.2.4 Algorithm and Computational Details for Optimisation

The algorithm and computational details for least cost supply of reactive power support required for real power equations and reactive power demand of DISCOs are similar to that explained in the previous chapter.

7.2.5 Reactive Power Support Required for meeting Reactive Power Demand of DISCOs and Associated Reactive Power Losses

The difference between the reactive power support (charges) required for real power transactions and reactive power demand of DISCOs and the reactive power support (charges) required for real power transactions gives the reactive power support (charges) required to meet reactive power demand of DISCOs and associated losses.

Let \( QG_{i}^{*(T+QD)} \) and \( CQ_{i}^{*(T+QD)} \) be the optimal reactive power support and charges required for real power transactions and reactive power demand
of DISCOs and its associated reactive power losses obtained by the optimization process explained in the previous section.

\[ QG_i^{*(T)} \text{ and } CQ_i^{*(T)} \] be the optimal reactive power support and charges required for real power transactions alone obtained by the optimization process explained in section 6.2.1 of Chapter 6.

The optimal reactive power support and charges required for meeting reactive power demand of DISCOS and its associated losses, \( QD_i^{*(QD)} \) and \( CQ_i^{*(QD)} \) are given by

\[
\begin{align*}
QG_i^{*(QD)} &= QG_i^{*(T+QD)} - QG_i^{*(T)} \\
CQ_i^{*(QD)} &= CQ_i^{*(T+QD)} - CQ_i^{*(T)}
\end{align*}
\] (7.11)

where

\[ i=1\ldots\ldots\text{NSP} \]

### 7.3 ALLOCATION OF REACTIVE POWER SUPPORT CHARGES REQUIRED FOR REACTIVE POWER DEMAND OF DISCOS

The reactive power service providers are paid by ISO. The cost incurred by the ISO for supply of reactive power support should be allocated equitably to the DISCOS. This requires to find the contribution of each reactive power support providers to each DISCO. A sensitivity of reactive power injection at a generator bus to reactive power injection at a load bus is used to allocate the reactive power support charges among DISCOS. For allocation purpose, in the load flow equations all the generator buses except slack bus are kept as PV buses whereas in the optimization problem, these buses are kept as PQ buses. The voltage settings for generators corresponding
to the optimal reactive power generations obtained from reactive power optimization problem are taken for load flow analysis.

### 7.3.1 Sensitivity of Reactive Power Injection at a Generator Bus to Reactive Power Injection at a Load Bus

The linearized model of the power flow equation around the given operating point is represented by

\[
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix} =
\begin{bmatrix}
\Delta P_I \\
\Delta Q_I
\end{bmatrix}
\]

where \( [J] \) is the jacobian matrix,

The incremental change in the state vector is expressed in terms of the incremental real and reactive power injections as

\[
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix} = [J]^{-1} \begin{bmatrix}
\Delta P_I \\
\Delta Q_I
\end{bmatrix}
\]

(7.13)

The complex power injected at \( k^{\text{th}} \) bus is given by

\[
\bar{S}_k = V_k \bar{I}_k^* = V_k \sum_{i=1}^{N} Y_{ki}^* \bar{V}_i^*
\]

(7.14)

The complex power injected at bus \( k \) can be written as

\[
\bar{S}_k = P_k + j Q_k
\]

(7.15)
The reactive power $Q_k(\theta, V)$ is given by

$$Q_k(x) = V_k \sum_{i=1}^{N} V_i \left[ G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i) \right] \quad (7.16)$$

‘k’ may be any bus. If ‘k’ is a PV bus or Slack bus $V_k = V_k^\text{constant}$ and $\theta_k = 0$ (slack bus)

Linearising $Q_k$ i.e. allowing for small perturbation

$$\Delta Q_k = \begin{bmatrix} \frac{\partial Q_k}{\partial \theta}^T & \frac{\partial Q_k}{\partial V}^T \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \quad (7.17)$$

Substituting equation (7.13) in (7.17)

$$\Delta Q_k = \begin{bmatrix} \frac{\partial Q_k}{\partial \theta}^T & \frac{\partial Q_k}{\partial V}^T \end{bmatrix} \begin{bmatrix} [A_{11}]_{NP \times NP} & [A_{12}]_{NP \times NQ} \\ [A_{21}]_{NP \times NP} & [A_{22}]_{NQ \times NQ} \end{bmatrix} \begin{bmatrix} \Delta PI \\ \Delta QI \end{bmatrix} \quad (7.18)$$

where $A_{11}, A_{12}, A_{21}$ and $A_{22}$ are sub matrices of inverse Jacobian

As variation of reactive power alone are considered, real power injection are assumed as constant, i.e. $\Delta PI = 0$.

If ‘k’ is a generator bus and ‘m’ is a load bus, the injection at the $m^{\text{th}}$ load bus can be written as $\Delta Q_m$. 
Using the above notations, the equation (7.18) can be written as

$$\Delta Q_k = \begin{bmatrix} \frac{\partial Q_k}{\partial \theta} \end{bmatrix}^T + \begin{bmatrix} \frac{\partial Q_k}{\partial V} \end{bmatrix}^T \begin{bmatrix} A12_{1,m} \Delta Q_m \\ A12_{NP,m} \Delta Q_m \\ \vdots \\ A22_{1,m} \Delta Q_m \\ \vdots \\ A22_{NQ,m} \Delta Q_m \end{bmatrix}$$

(7.20)

$$\Delta Q_k = \left[ \sum_{i=1}^{NP} \frac{\partial Q_k}{\partial \theta_i} A12_{i,m} \right] + \left[ \sum_{j=1}^{NQ} \frac{\partial Q_k}{\partial V_j} A22_{j,m} \right] \Delta Q_m$$

(7.21)

Hence sensitivity of reactive power injection at generator bus $k$ to a reactive power injection at a load bus $m$ is given by

$$S_{km} = \frac{\Delta Q_k}{\Delta Q_m} = \frac{\partial Q_k}{\partial Q_m} = \left[ \sum_{i=1}^{NP} \frac{\partial Q_k}{\partial \theta_i} A12_{i,m} + \sum_{j=1}^{NQ} \frac{\partial Q_k}{\partial V_j} A22_{j,m} \right]$$

(7.22)

### 7.3.2 Allocation of Reactive Power Support Charges Among DISCOS

In order to allocate the reactive power support among DISCOs participants, cost allocation factor of DISCOS are used.

Let $QD_j; j=1, \ldots, ND$ be the reactive power demand of DISCOS participants where $ND$ is the number of DISCOS participants.
Let $S_{ij}$ be the sensitivity of bus reactive power injection of $i^{th}$ reactive power support provider to bus reactive power injection of $j^{th}$ DISCO participant calculated using equation (7.22).

The cost allocation factor of DISCO is given by

$$f_i^j = \frac{QD_j S_{ij}}{\sum_{k=i}^{ND} QD_k S_{ik}};$$

$$i = 1, \ldots, \text{NSP}$$

$$j = 1, \ldots, \text{ND}$$

(7.23)

The charge allocated to each DISCO participant is given by

$$CQ_i^j = f_i^j \left[ CQ_i^{*(QD)} \right]$$

$$i = 1, \ldots, \text{NSP}$$

$$j = 1, \ldots, \text{ND}$$

(7.24)

7.3.3 Algorithm and Computational Details for Allocation

The algorithm for allocation of reactive power charges among DISCO participants is as follows.

1. Run power flow solution. The voltage settings for generators corresponding to the optimal reactive power generations obtained from reactive power optimization problem are taken for power flow analysis.
2. Calculate the cost of reactive power support required for meeting the reactive power demand of DISCOS using equation (7.11)

3. Compute the jacobian matrix \( J \).

4. Obtain jacobian inverse whose sub-matrices are \( A_{11}, A_{12}, A_{21} \) and \( A_{22} \).

5. Calculate sensitivity of reactive power injection at a generator bus to a reactive power injection at a load bus by using equation (7.22).

6. Calculate the cost allocation factor for allocating the reactive power support among DISCO participants using equation (7.23).

7. Calculate reactive power charge allocated to the DISCO participants by using equation (7.24).

A computer package for the allocation of reactive power charges among DISCOS is developed in MATLAB.

**7.4 SYSTEM STUDY AND RESULTS**

In order to demonstrate the effectiveness of the proposed method, the Modified IEEE 24 Bus Reliability Test System is used. Both real power transactions and reactive power demand of DISCOS are considered. The reactive power demand of DISCOS is given Appendix A3.8.
7.4.1 Least Cost Supply of Reactive Power Support for Real Power Transactions and Reactive Power Demand of DISCOS - Modified IEEE 24 Bus Reliability Test System

The algorithm for least cost supply of reactive power support for real power transactions and reactive power demand of DISCOS is applied to Modified IEEE 24 Bus RTS System. Table 7.1 shows the reactive power generation of reactive power support providers in each LPMOVE. The convergence of cost of reactive power support of each reactive power support provider is shown in Table 7.2. It is observed from Table 7.3 and Table 7.4 that the generator and load bus voltage are within their limits. Table 7.5 shows the MVA line flows of the lines which were found to be critical in congestion management problem explained in previous chapter. It is seen from Table 7.5 that the rescheduling of reactive power generation does not cause any overloads in critical lines. The MVA line flows are within their ratings.

![Figure 7.1 Convergence of Total Cost of Reactive Power Support - Modified IEEE 24 Bus RTS System](image-url)
It is seen from Figure 7.1 that the total cost of reactive power support decreases in each LPMOVE. The above problem takes four LPMOVEs to converge. It is seen from Table 7.2 that the total cost of reactive power incurred by ISO in providing reactive power support for real power transactions is 7,01,010 Rs/hr.

<table>
<thead>
<tr>
<th>Reactive Power Support Provider</th>
<th>LPMOVE 0</th>
<th>LPMOVE 1</th>
<th>LPMOVE 2</th>
<th>LPMOVE 3</th>
<th>LPMOVE 4</th>
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<td>1</td>
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<td>160.3</td>
<td>139.6</td>
<td>164.6</td>
<td>152.1</td>
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<tr>
<td>2</td>
<td>135.9</td>
<td>110.9</td>
<td>135.9</td>
<td>120.2</td>
<td>132.7</td>
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<tr>
<td>3</td>
<td>94.82</td>
<td>119.8</td>
<td>129</td>
<td>128.3</td>
<td>131.2</td>
</tr>
<tr>
<td>4</td>
<td>158.3</td>
<td>133.3</td>
<td>108.3</td>
<td>83.29</td>
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<td>80.85</td>
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<td>80.85</td>
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<td>6</td>
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<td>39.13</td>
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<td>76.63</td>
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<td>7</td>
<td>151.8</td>
<td>126.8</td>
<td>105.4</td>
<td>80.42</td>
<td>67.92</td>
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<tr>
<td>8</td>
<td>89.21</td>
<td>64.21</td>
<td>89.21</td>
<td>67</td>
<td>79.5</td>
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<tr>
<td>9</td>
<td>9.455</td>
<td>34.46</td>
<td>59.46</td>
<td>84.46</td>
<td>71.96</td>
</tr>
<tr>
<td>10</td>
<td>179.4</td>
<td>154.4</td>
<td>129.4</td>
<td>104.4</td>
<td>91.94</td>
</tr>
</tbody>
</table>
Table 7.2  Cost of Reactive Power Support in Rs/hr - Modified IEEE 24 Bus RTS System

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>LPMOVE</th>
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<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>139315.2</td>
</tr>
<tr>
<td>2</td>
<td>173082</td>
</tr>
<tr>
<td>7</td>
<td>67113.2</td>
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<tr>
<td>14</td>
<td>176235.2</td>
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<tr>
<td>15</td>
<td>24512</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>18</td>
<td>147072</td>
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<td>21</td>
<td>38052</td>
</tr>
<tr>
<td>22</td>
<td>456.8</td>
</tr>
<tr>
<td>23</td>
<td>129194.8</td>
</tr>
<tr>
<td>Total Reactive Cost (Rs/hr)</td>
<td>8,95,037.2</td>
</tr>
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</table>
Table 7.3  Generator Bus Voltages in p.u -Modified IEEE 24 Bus RTS System

<table>
<thead>
<tr>
<th>Generator Bus No</th>
<th>Voltage Magnitude in p.u</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max : 1.1 p.u</td>
</tr>
<tr>
<td></td>
<td>Min : 1.0 p.u</td>
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<tr>
<td></td>
<td>LPMOVE</td>
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<td>13</td>
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<td>14</td>
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<td>1.0806</td>
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<tr>
<td>23</td>
<td>1.0591</td>
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Table 7.4  Load Bus Voltages in p.u - Modified IEEE 24 Bus RTS System

<table>
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<th>Load Bus No.</th>
<th>Voltage Magnitude in p.u</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Max : 1.05 p.u</td>
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<tr>
<td></td>
<td>Min : 0.95 p.u</td>
</tr>
<tr>
<td></td>
<td>LPMOVE</td>
</tr>
<tr>
<td></td>
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</tr>
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<td>3</td>
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<td>1.0383</td>
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<tr>
<td>20</td>
<td>1.0487</td>
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<tr>
<td>24</td>
<td>0.9917</td>
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Table 7.5  MVA Line Flow of Critical Lines - Modified IEEE 24 Bus RTS System

<table>
<thead>
<tr>
<th>Critical Lines</th>
<th>MVA Rating</th>
<th>MVA Line Flow</th>
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<td></td>
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<tr>
<td>8-9</td>
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<td>17-18</td>
<td>160</td>
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7.4.2  Reactive Power Support (Charges) Required for meeting Reactive Power Demand of DISCOs and Associated Reactive Power Losses - Modified IEEE 24 Bus RTS System

The optimal reactive power support required for both real power transactions and reactive power demand of DISCOS for the modified IEEE 24 Bus RTS system obtained by the optimization process explained in section 7.2.1 is shown in column 2 of Table 7.6.

The column 3 of Table 7.6 shows the optimal reactive power support for real power transactions for the modified IEEE Bus RTS system obtained by the optimization process explained in section 6.2.1 of Chapter 6.
Table 7.6  Reactive Power Support - Modified IEEE 24 Bus RTS System

<table>
<thead>
<tr>
<th>Reactive Power Support Providers</th>
<th>Optimal Reactive Power Support for Real PowerTransactions and Reactive Power Demand of DISCOS ( Q_{G_i}^{(T-QD)} ) (MVAR)</th>
<th>Optimal Reactive Power Support for Real Power Transactions ( Q_{G_i}^{(T)} ) (MVAR)</th>
<th>Reactive Power Support required for meeting reactive power demand of DISCOS ( Q_{G_i}^{(QD)} ) (MVAR)</th>
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<tr>
<td>1</td>
<td>152.1</td>
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### Table 7.7  Least Cost of Reactive Power Support - Modified IEEE 24 Bus RTS System

<table>
<thead>
<tr>
<th>Reactive Power Support Providers</th>
<th>Least Cost of Reactive Power Support for Real Power Transactions and Reactive Power Demand of DISCOS $CQ_i^{(T,QD)}$ (Rs/hr)</th>
<th>Least Cost of Reactive Power Support for Real Power Transactions $CQ_i^{(T)}$ (Rs/hr)</th>
<th>Least Cost of Reactive Power Support for Reactive Power Demand of DISCOS $CQ_i^{(QD)}$ (Rs/hr)</th>
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<td>23,260</td>
<td>1,36,771</td>
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<td>11,148</td>
<td>1,52,854</td>
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<td>33,774</td>
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</table>
The optimal reactive power support required for meeting the reactive power demand of DISCOS for the modified IEEE Bus RTS system obtained by using equation (7.11) is shown in column 4 of Table 7.6.

The least cost of reactive power support required for both real power transactions and reactive power demand of DISCOS for the modified IEEE 24 Bus RTS system obtained by the optimization process explained in section 7.2.1 is shown in column 2 of Table 7.7.

The column 3 of Table 7.7 shows the least cost of reactive power support for real power transactions for the modified IEEE Bus RTS system obtained by the optimization process explained in section 6.2.1 of Chapter 6.

The least cost of reactive power support required for meeting the reactive power demand of DISCOS for the modified IEEE Bus RTS system obtained by using equation (7.11) is shown in column 4 of Table 7.7.

7.4.3 Allocation of Reactive Power charges among DISCOS - Modified IEEE 24 Bus Reliability Test System

The reactive power service providers are paid by ISO. The cost incurred by the ISO for supply of reactive power support for meeting reactive power demand of DISCOS should be allocated equitably to the DISCOS. In order to allocate the reactive power support among DISCOs participants, cost allocation factor of DISCOS are used.

The cost allocation factors of DISCOS of Modified IEEE 24 Bus RTS System are shown in Table 7.8. The equitable allocation of reactive power support charges among DISCOS Modified IEEE 24 Bus RTS System is shown in Table 7.9.
Table 7.8  Cost allocation factor of DISCOS- Modified IEEE 24 Bus RTS System

<table>
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<tr>
<th>Reactive Power Support Providers</th>
<th>DISCO 1</th>
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<th>DISCO 3</th>
<th>DISCO 4</th>
<th>DISCO 5</th>
<th>DISCO 6</th>
<th>DISCO 7</th>
<th>DISCO 8</th>
<th>DISCO 9</th>
<th>DISCO 10</th>
<th>DISCO 12</th>
<th>DISCO 13</th>
<th>DISCO 14</th>
<th>DISCO 15</th>
<th>DISCO 16</th>
<th>DISCO 17</th>
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### Table 7.9 Allocation of Reactive Power Charges among DISCOS - Modified IEEE 24 Bus RTS System

<table>
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<tr>
<th>Reactive Power Support Providers</th>
<th>DISCO 1</th>
<th>DISCO 2</th>
<th>DISCO 3</th>
<th>DISCO 4</th>
<th>DISCO 5</th>
<th>DISCO 6</th>
<th>DISCO 7</th>
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7.5 CONCLUSION

An SLP based optimization method for least cost supply of reactive power support for meeting the reactive power demand of DISCOS has been proposed. A computer package is developed in MATLAB for the proposed method and its effectiveness is tested using Modified IEEE 24 Bus Reliability Test System. Results obtained confirm that the proposed algorithm has reliable and monotonic convergence satisfying effectively all the operating constraints. The results are obtained in 4 LPMOVES. The resultant charges are allocated to the DISCOS in a transparent and equitable manner.