CHAPTER 3

DESIGN OF COMPOSITE DRIVE SHAFT

3.1 INTRODUCTION

Laminated composite, circular cylindrical hollow shafts are used extensively as primary load-carrying structures in many applications under various loading configurations, due to their higher specific stiffness. Polymer matrix composites such as carbon/epoxy or glass/epoxy offer better fatigue characteristics, as micro cracks in the resin do not grow further like metals, but terminate at the holes of the fibers. Generally, composites have less susceptibility to the effects of stress concentration, such as those caused by notches and holes, than metals (Jones 1990). These composite materials are ideally suited for long, power drive-shaft applications (Shokrieh et al 2004). Composite drive shafts offer the potential of lighter and longer drive train with higher critical speed. Drive shafts as power transmission tubing, are used in many applications, including cooling towers, pumpsets, aerospace, trucks and automobiles. The first application of a composite drive shaft to an automotive was the one developed by Spicer U-joint divisions of Dana Corporation for the Ford econoline van models in 1985 (Rastogi 2004).

3.2 METALLIC DRIVE SHAFT - LIMITATION

Almost all automobiles (at least those which correspond to a design with Front engine installation and rear wheel drive) have transmission shafts as shown in Figure 3.1.
Figure 3.1 Conventional two piece driveshaft for rear wheel drive

The conventional steel drive shaft consists of two pieces connected by a center supported bearing and universal joint, and hence the overall weight of the assembly will be more. Also it has a less specific modulus, specific strength, and its corrosion resistance is less as compared with that of composite materials. A metallic drive shaft has the limitations of weight, low critical speed and vibrational characteristics (Hibbeler 2003). Several solutions such as flywheels, harmonic dampers, vibration shock absorbers and multiple shafts with bearings, couplings, and heavy associated hardware have shown limited success in overcoming the problems (Badie et al 2006). Composite drive shafts can solve many such automotive and industrial problems that accompany the usage of the conventional metal ones, because their performance is limited due to lower critical speed, weight, fatigue and vibration. A drive shaft of composites offers excellent vibration damping, cabin comfort, reduction of wear on drive train components and increasing tyres traction. In addition, the use of a one piece torque tube reduces assembly time, inventory cost, maintenance, and part complexity.
3.3 COMPOSITE DRIVE SHAFT

In the study, a hollow cylindrical shaft made up of high modulus carbon–epoxy composite material is chosen. In general, a hollow shaft is stronger than a solid shaft for the same specific weight. The stress distribution in the case of the solid shaft is zero at the center and maximum at the outer surface, while in a hollow shaft the stress variation is smaller. In solid shafts the material close to the center is not fully utilized.

The torque transmission capability of the drive shaft used in automotive applications should be larger than 3500 Nm ($T_{\text{min}}$) and the fundamental natural bending frequency of the drive shaft should be higher than 110 Hz ($F_{\text{cr}}$) to avoid whirling vibration. The drive shaft’s outer diameter $d_0$ should not exceed 100 mm due to space limitations. Here, the outer diameter $d_0$ and length $L$ of the shaft considered are 90 mm and 1250 mm respectively.

The properties of the high modulus carbon-epoxy materials considered for RBDO of composite drive shafts are given in Table 3.1. (Rangaswamy and Vijayarangan 2004).
Table 3.1 Properties of HM Carbon – Epoxy Composite Material

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal modulus ($E_{11}$)</td>
<td>190 GPa</td>
</tr>
<tr>
<td>Transverse modulus ($E_{22}$)</td>
<td>7.7 GPa</td>
</tr>
<tr>
<td>Shear modulus ($G_{12}$)</td>
<td>4 GPa</td>
</tr>
<tr>
<td>Minor Poisson’s ratio ($\nu_{12}$)</td>
<td>0.3</td>
</tr>
<tr>
<td>Longitudinal tensile stress ($X_T$) &amp; compressive stress ($X_C$)</td>
<td>870 MPa</td>
</tr>
<tr>
<td>Transverse tensile stress ($Y_T$) &amp; compressive stress ($Y_C$)</td>
<td>54 MPa</td>
</tr>
<tr>
<td>Shear stress ($\tau_{12}$)</td>
<td>30 MPa</td>
</tr>
<tr>
<td>Density ($\rho$)</td>
<td>1600</td>
</tr>
<tr>
<td>Volume fraction ($V_f$)</td>
<td>0.6</td>
</tr>
</tbody>
</table>

3.3.1 Assumptions made

The following assumptions are made in this study:

- The shaft rotates at a constant speed about its longitudinal axis.
- The shaft has a uniform circular cross section.
- The shaft is perfectly balanced, i.e., at every cross section, the mass centre coincides with the geometric centre.
- All damping and non-linear effects are excluded.
- The stress-strain relationship for composite material is linear elastic; hence Hooke’s law is applicable for composite materials.
- Since the lamina is thin and no out-of-plane loads are applied, it is to be considered under the plane stress condition.
In this study, the mathematical model of the composite drive shaft developed by Rangaswamy and Vijayarangan (2004) is considered for the RBDO analysis. However, the formulae used for predicting the buckling torque have been modified to have better conformance with the published experimental data.

### 3.3.2 Nomenclature

The following are the notations used in the mathematical model of the composite drive shaft.

- $m$: Mass of the drive shaft (kg)
- $\rho$: Density of the material (kg/m$^3$)
- $d_o$: Outer diameter of the shaft (m)
- $d_i$: Inner diameter of the shaft (m)
- $T$: Torque transmission capacity of the shaft (Nm)
- $T_b$: Buckling torque (Nm)
- $F_{cr}$: Fundamental natural frequency (Hz)
- $\theta_k$: Orientation angle of $k^{th}$ ply (Deg)
- $t_k$: Thickness of $k^{th}$ ply (m), $k = 1, 2, \ldots, n$
- $n$: Number of plies
- $\sigma_1$: Longitudinal stress along fiber directions (N/m$^2$)
- $\sigma_2$: Transverse stress along fiber directions (N/m$^2$)
- $\tau_{12}$: Shear stress along fiber directions (N/m$^2$)
- $\varepsilon_1$: Longitudinal strain along fiber directions
- $\varepsilon_2$: Transverse strain along fiber directions
- $\gamma_{12}$: Shear strain along fiber directions
- $\sigma_x$: Longitudinal stress along X directions (N/m$^2$)
- $\sigma_y$: Transverse stress along Y directions (N/m$^2$)
\( \tau_{xy} \quad \text{Shear stress along XY directions (N/m}^2) \)

\( \varepsilon_x \quad \text{Longitudinal strain along X directions} \)

\( \varepsilon_y \quad \text{Transverse strain along X directions} \)

\( \gamma_{xy} \quad \text{Shear strain along XY directions} \)

\( \omega \quad \text{Angular velocity of the shaft (rad/sec)} \)

\( X_T \quad \text{Strength of unidirectional fiber composite under tension along fiber direction (N/m}^2) \)

\( X_C \quad \text{Strength of unidirectional fiber composite under compression along fiber direction (N/m}^2) \)

\( Y_T \quad \text{Strength of unidirectional fiber composite under tension perpendicular to fiber direction (N/m}^2) \)

\( Y_C \quad \text{Strength of unidirectional fiber composite under compression perpendicular fiber direction (N/m}^2) \)

### 3.4 MECHANICS OF COMPOSITES

The material properties of the drive shaft are analyzed through the classical lamination theory. This theory, which deals with the linear elastic response of the laminated composite, incorporates the assumption of Kirchhoff-Lsove for bending and stretching of thin plates, besides the assumption that each layer is in a state of plane stress (Herakovich 1998).

The following Sections explain the fundamental relationship of the composite laminates based on the lamination theory.
3.4.1 Stress - Strain relations of unidirectional lamina

It is possible to reduce the 3-D problem into a 2-D problem since the lamina is assumed to be thin. For a unidirectional 2-D lamina, the stress-strain relationship in terms of principal material directions is given by:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{21} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
\]

(3.1)

From the properties of the composite materials at fibers direction, the first step is the construction of the reduced stiffness matrix. The expressions of the reduced stiffness coefficients \(Q_{ij}\) in terms of engineering constants are as follows:

\[
Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}
\]

(3.2)

\[
Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}
\]

(3.3)

\[
Q_{12} = Q_{21} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}
\]

(3.4)

\[
Q_{66} = G_{12}
\]

(3.5)

3.4.2 Stress-Strain Relations in Arbitrary Direction

The stress strain relationship for an angle-ply lamina, where fibers are oriented at an angle with the positive X-axis (longitudinal axis of shaft) is given by
\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

(3.6)

\(\bar{Q}\) matrix denotes the transformed reduced stiffness of the lamina. Its terms are individually calculated by:

\[
\bar{Q}_{11} = Q_{11} C^4 + Q_{22} S^4 + 2(Q_{12} + 2Q_{66})S^2C^2
\]

(3.7)

\[
\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})S^2C^2 + Q_{12}(C^4 + S^4)
\]

(3.8)

\[
\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})C^3S - (Q_{22} - Q_{12} - 2Q_{66})CS^3
\]

(3.9)

\[
\bar{Q}_{22} = Q_{11}S^4 + Q_{22}C^4 + 2(Q_{12} + 2Q_{66})S^2C^2
\]

(3.10)

\[
\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})CS^3 - (Q_{22} - Q_{12} - 2Q_{66})C^3S
\]

(3.11)

\[
\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{66})S^2C^2 + Q_{66}(C^4 + S^4)
\]

(3.12)

where

\[
C = \cos \theta \quad \text{and} \quad S = \sin \theta
\]

(3.13)

Since the lamina considered is thin, laminate strains are linearly related to the distance from the mid plane.

\[
\varepsilon_x = \varepsilon_0 + h \frac{k}{x}
\]

(3.15)
\[ \varepsilon_y = \varepsilon_0^y + h k_y \]  
\[ \gamma_{xy} = \gamma_{0xy}^y + h k_{xy} \]  

where

\[ \varepsilon_0^x, \varepsilon_0^y \] = midplane normal strains in the laminate

\[ \gamma_{xy}^0 \] = midplane shear strain in the laminate.

\[ k_x, k_y \] = bending curvatures in the laminate

\[ k_{xy} \] = twisting curvature in the laminate

\[ h \] = distance from the midplane in the thickness direction.

### 3.4.3 Force and Moment Resultants

The second step is to construct the extensional stiffness matrix \([A]\). This matrix is the summation of the products of the transformed reduced stiffness matrix \([\overline{Q}]\) of each layer and the thickness of this layer as:

The extensional stiffness matrix is

\[
[A] = \sum_{i=1}^{n} [\overline{Q}_i] \times [h_i - h_{i-1}] 
\]  

The matrix is in (N/m) and the thickness of each ply is calculated with reference to its coordinate location in the laminate.

The coupling matrix is

\[
[B] = \frac{1}{2} \left[ \sum_{i=1}^{n} [\overline{Q}_i] \times [h_i^2 - h_{i-1}^2] \right] 
\]  

(3.18)  

(3.19)
The Bending matrix is

\[
[D] = \frac{1}{3} \sum_{i=1}^{n} [\overline{Q}]_i \times [h_i^3 - h_{i-1}^3]
\]  
(3.20)

Applied force and moment resultants on a laminate are related to the midplane strains and curvatures by the following equations:

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} = [A] \begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} + [B] \begin{bmatrix}
k_x \\
k_y \\
k_{xy}
\end{bmatrix}  
(3.21)
\]

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = [B] \begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} + [D] \begin{bmatrix}
k_x \\
k_y \\
k_{xy}
\end{bmatrix}  
(3.22)
\]

where

- \( N_x \) = normal resultant force in the x direction (per unit width)
- \( N_y \) = normal resultant force in the y direction (per unit width)
- \( N_{xy} \) = shear resultant force (per unit width)
- \( M_x \) = resultant bending moment in the yz plane (per unit width)
- \( M_y \) = resultant bending moment in the xz plane (per unit width)
- \( M_{xy} \) = resultant twisting moment (per unit width)

For a symmetric laminate (considered in this study), the \( B \) matrix vanishes and the in-plane and bending stiffness are uncoupled. For a symmetric laminate
\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix}
\] (3.23)

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} =
\begin{bmatrix}
D_{11} & D_{12} & D_{16} \\
D_{12} & D_{22} & D_{26} \\
D_{16} & D_{26} & D_{66}
\end{bmatrix}
\begin{bmatrix}
\kappa_x^0 \\
\kappa_y^0 \\
\kappa_{xy}^0
\end{bmatrix}
\] (3.24)

### 3.5 DESIGN VARIABLES

In this study a hollow cylindrical shaft made up of High Modulus Carbon – epoxy composite material is considered. An efficient design of a composite drive shaft could be achieved by selecting the proper variables, which can be identified for safe structure against failure and to meet the performance requirements. As the length and outer radius of the drive shafts in automotive applications are limited due to spacing, the design variables include the inside radius, layer thickness, number of layers, fiber orientation angle and the stacking sequence. In an optimal design of the drive shaft, these variables are constrained by the lateral natural frequency, torsional strength and torsional buckling. The tailorability of elastic constants in composites provides numerous alternatives for the variables to meet the required stability and strength of a structure. The number of plies required depends on the design constraints, allowable material properties, thickness of plies and the stacking sequence. The design variables considered in this study are: the Number of plies \((n)\), the Thickness of the each ply \((t_k)\) and Orientation angle of each ply \((\theta_k)\). The limiting values of the design variables are:

\[0 < n \leq 32, \quad -90^\circ \leq \theta_k \leq 90^\circ\] and \(0.1 \text{ mm} \leq t_k \leq 0.5 \text{ mm}\).
3.6 DESIGN CONSTRAINTS

Due to the physical geometry (larger radius) of the drive shafts used in the mentioned applications, including automotive applications, the shear strength which specifies the load carrying capacity, is of minor design importance since the failure mode is dominated by buckling; therefore, the main design factors are the bending natural frequency and the torsional buckling strength, which are functions of the longitudinal and hoop bending stiffness, respectively. The variable of the laminate thickness has a big effect on the buckling strength, and a slight effect on the bending natural frequency. A discrete variable optimization algorithm could be employed for optimization of ply thickness and orientation. The torque transmission capability of the drive shaft used in automotive applications should be larger than 3500 Nm \((T_{\text{min}})\) and the fundamental natural bending frequency of the drive shaft should be higher than 110 Hz \((F_{\text{cr}})\) to avoid whirling vibration. The drive shaft’s outer diameter \(d_o\) should not exceed 100 mm due to space limitations. Here, the outer diameter \(d_o\) and length \(L\) of the shaft considered are 90 mm and 1250 mm respectively.

3.6.1 Torque Transmission Capacity of Composite Drive Shaft

The composite drive shaft is designed to carry the torque without failure. The torsional strength or the torque at which the shaft fails is directly related to the laminate shear strength through

\[
T = 2\pi r_m^2 t \tau_{xy}
\]  

(3.25)
where

\[ T = \text{Torque transmission capacity of the shaft (Nm)} \]
\[ \tau_{xy} = \text{Allowable shear stress (Nm}^2\text{)} \]
\[ r_m = \text{Mean radius (m)} \]
\[ t = \text{Laminate Thickness (m)} \]

Since the laminate is assumed to be failed at the failure of the first ply, the maximum stress failure criterion could be used after finding the in-plane stresses at every ply to specify a factor of safety for the torque transmission capacity. The first step is to construct the transverse of the extensional stiffness matrix \([A]\) and after solving the overall strains, the stresses in each layer can be examined by transforming these stresses to the direction of the fibers in each layer. The layers of fiber direction ±45° are of special concern, since they make a substantial contribution to the load carrying. The axial force \(N_x = 0\), the centrifugal force \(N_y = 0\) are neglected and \(N_{xy}\) is the resultant shear force. The torque \(T\) is the peak torque, if the design involves fatigue considerations.

### 3.6.2 Torsional buckling capacity of composite drive shaft

For very thin-walled tubes, the possibility of torsional buckling exists. For symmetrically laminated hollow shafts of moderate lengths, the critical buckling torque equation used by Rastogi (2004) has been modified in this study as

\[
T_b = 2\pi r_m^2 \left[ K_{cr} \frac{\pi^2 D^*_{xy} L}{L^2} \right]
\]  

(3.26)
where

\[ K_{xx} = 0.925Z_t^{0.75} \]  \hspace{1cm} (3.27)

\[ Z_t = \frac{E_{xx}}{r_m} \sqrt{ \frac{A_{11}A_{22} - A_{12}^2}{12A_{22}D_{22}^*} } \]  \hspace{1cm} (3.28)

\[ [D'] = [D] - [B][A]^{-1}[B] \]  \hspace{1cm} (3.29)

where

\[ A_{11} = \text{Stiffness matrix element for the laminate (N/m)} \]

\[ D_{22} = \text{Bending matrix element for the laminate (N-m)} \]

### 3.6.3 Fundamental Natural Frequency of Composite Drive Shaft

The drive shaft is designed to have a critical speed (60 times the natural frequency), that is high enough to exceed the rotational speed. If the rotating speed of the shaft equals the critical speed, a large amplitude vibration (whirling) occurs. The drive shaft is idealized as simply supported at its ends or pinned-pinned beam. The fundamental natural frequency corresponding to the critical speed of the rotating shaft is given by

\[ F = \frac{\pi}{2} \left[ \frac{r_m}{L^2} \right] \sqrt{ \frac{E_{xx}}{2\rho} } \]  \hspace{1cm} (3.30)

where

Axial modulus \( E_{xx} = \frac{A_{11}A_{22} - A_{12}^2}{tA_{22}} \)  \hspace{1cm} (3.31)

where \( t \) is the laminate thickness = \( \sum_{k=1}^{n} t_k \)  \hspace{1cm} (3.32)

\( r_m \) is mean radius of the shaft
3.7 TSAI-WU FAILURE CRITERION

Several failure theories are available in the literature to predict the failure behavior of laminates, such as the Maximum stress theory, the Maximum strain theory, the Azzi-Tsai-Hill theory, the Azzi-Wu theory, the Tsai-Hill theory and the Tsai-Wu Theory (Mallick and Newman 1990). Among these failure theories the Tsai-Wu criterion is more conservative and found to be more accurate (Kaw 2005). This theory is based on Goldenblat and Koponov’s work (Jones 1990). Tsai and Wu modified it assuming the existence of the failure surface in stress space, and in plane shear strength similarity, and formulated the failure condition as,

\[ f_1 \sigma_1 + f_2 \sigma_2 + f_{11} \sigma_1^2 + f_{22} \sigma_2^2 + f_{66} \tau_6^2 + 2f_{12} \sigma_1 \sigma_2 = 1 \]  \hspace{1cm} (3.33)

\[ f_1 = \frac{1}{X_t} - \frac{1}{X_c}; f_{11} = \frac{1}{X_t X_c}; f_2 = \frac{1}{Y_t} - \frac{1}{Y_c}; f_{22} = \frac{1}{Y_t Y_c}; f_{66} = \frac{1}{S_{12}}; f_{12} = -\frac{1}{2} \left[ f_{11} f_{22} \right]^{1/2} \]  \hspace{1cm} (3.34)

Using the above, the equation can be rewritten as,

\[ \left( \frac{\sigma_1}{X_t X_c} \right)^2 + \left( \frac{\sigma_2}{Y_t Y_c} \right)^2 + \left( \frac{1}{X_t} - \frac{1}{X_c} \right) \sigma_1 + \left( \frac{1}{Y_t} - \frac{1}{Y_c} \right) \sigma_2 + \frac{2f_{12} \sigma_1 \sigma_2}{\sqrt{X_t X_c Y_t Y_c}} + \frac{\tau_{12}^2}{S_{12}} = 1 \]  \hspace{1cm} (3.35)

The Tsai-Wu Criterion accounts for tensile and compressive stress through linear terms; it is readily amenable to computational procedure, and it uses stress invariants. With these advantages, this is the most widely used theory. In this composite drive shaft design optimization study, the Tsai-Wu failure criterion is used to find the failure of the laminate.
When the shaft is subjected to torque $T$, the resultant forces $N_x, N_y, N_{xy}$ induced in the laminate due to the effect of centrifugal forces are

\begin{align*}
N_x &= 0 \quad (3.36) \\
N_y &= 2 \rho tr^2 \omega^2 \quad (3.37) \\
N_{xy} &= \frac{T}{2 \pi r^2} \quad (3.38)
\end{align*}

Midplane strains in the reference surfaces are given by

\[
\begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} =
\begin{bmatrix}
 a_{11} & a_{12} & a_{16} \\
 a_{21} & a_{22} & a_{26} \\
 a_{61} & a_{62} & a_{66}
\end{bmatrix}
\begin{bmatrix}
 N_x \\
 N_y \\
 N_{xy}
\end{bmatrix}
\quad (3.39)
\]

where

\[
\begin{bmatrix}
 a_{11} & a_{12} & a_{16} \\
 a_{21} & a_{22} & a_{26} \\
 a_{61} & a_{62} & a_{66}
\end{bmatrix} = \begin{bmatrix}
 A_{11} & A_{12} & A_{16} \\
 A_{21} & A_{22} & A_{26} \\
 A_{61} & A_{62} & A_{66}
\end{bmatrix}^{-1}
\quad (3.40)
\]

Now the stresses in each ply are calculated using the equation

\[
\begin{bmatrix}
 \sigma_x \\
 \sigma_y \\
 \tau_{xy}
\end{bmatrix}_k =
\begin{bmatrix}
 \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\
 \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\
 \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66}
\end{bmatrix}_k
\begin{bmatrix}
 \varepsilon_x^0 \\
 \varepsilon_y^0 \\
 \gamma_{xy}^0
\end{bmatrix}
\quad (3.41)
\]

The stresses along the X, Y direction are transformed into fiber direction 1, 2 using the following relation.
\[
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{pmatrix}_k =
\begin{bmatrix}
C^2 & S^2 & 2CS \\
S^2 & C^2 & -2CS \\
-CS & CS & (C^2 - S^2)
\end{bmatrix}_k
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix}
\]

where \( C = \cos \theta \), \( S = \sin \theta \) \( \theta \) = Fiber orientation

Figure 3.2 shows the relation between the material coordinate system 1,2 and geometric coordination system X, Y.

![Diagram showing the relation between material and geometric coordinate systems](image)

**Figure 3.2 Relation between the material and Geometric coordinate system**

Knowing the stresses in each ply, the failure of the laminate is determined using the first ply failure criterion. According to the Tsai Wu failure criterion the laminate is said to be failed if the following condition is satisfied.

\[ M \geq 1 \] (3.43)

where

\[
M = \left( \frac{\sigma_1}{X_1 Y_1} \right)^2 + \left( \frac{\sigma_2}{Y_1 Y_2} \right)^2 + \left( \frac{1}{X_1} - \frac{1}{X_2} \right) \sigma_1 + \left( \frac{1}{Y_1} - \frac{1}{Y_2} \right) \sigma_2 + \frac{2F_{12} \sigma_1 \sigma_2}{\sqrt{X_1 X_2 Y_1 Y_2}} + \left( \frac{\tau_{12}}{S_{12}} \right)^2
\] (3.44)
Since the fiber orientation angle ($\theta$) that offers the maximum bending stiffness which leads to the maximum bending natural frequency is to place the fibers longitudinally at zero angle from the shaft axis, on the other hand, the angle of $\pm 45^\circ$ orientation realizes the maximum shear strength and $90^\circ$ is the best for buckling strength. The main design goal is to achieve the minimum weight while adjusting the variables to meet a sufficient margin of safety, which is translated in a critical speed (natural frequency) higher than the operating speed, a critical torque higher than the ultimate transmitted torque and a nominal stress (the maximum at fiber direction) less than the allowable stress after applying any of the failure criteria like the maximum stress criteria.

Due to the physical geometry (larger radius) of the drive shafts considered in the study for automotive applications, the shear strength which specifies the load carrying capacity, is of minor design importance since the failure mode is dominated by buckling; therefore, the main design factors, viz., the bending natural frequency and the torsional buckling strength, are functions of the longitudinal and hoop bending stiffness, respectively. The variable of the laminate thickness has a big effect on the buckling strength and a slight effect on the bending natural frequency. A discrete variable optimization algorithm could be employed for the optimization of ply thickness and orientation.

Several alternative stacking sequences are possible with different orientation angles and different positions of each ply of the composite laminate, and so the number of possible solutions in this design optimization problem is quite large. Hence, in this study, an attempt is made to explore the use of search heuristics such as the Genetic Algorithm, Particle Swarm Optimization algorithm and Evolutionary Programming Algorithm to
determine the optimal parameters required for the design of a Composite Drive shaft.

3.8 SUMMARY

The composite drive shaft has better Noise, Vibration and Harshness (NVH) characteristics when compared to the metallic one. The High Modulus Carbon-Epoxy composite drive shaft is considered in the design optimization study. The fundamental relationships of composite laminates are explained based on the lamination theory. The detailed mathematical model of the composite drive shaft considered for study, is presented. The equation used to predict the Torsional buckling, to include the influence of External stiffness [A] matrix and coupling [B] matrix.